**Sequential Procedure to Derive Minimal Cut Sets for aComplex System Directly from its Fault Tree**

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**Abstracts:**

 In this paper a sequentially method applied a minimal cut sets from fault trees is presented. Since for more complicated fault trees it will not to be possible to determine the cut sets by observation so amore structured method will be necessary. The presented method is illustrated through three example for different size of fault trees. Conclusions are listed at the end.

**الخلاصة**

 تم في هذا البحث عرض طريقة تتابعية لغرض ايجاد مجاميع القطع الاقصر مباشرة من شجرة فشلها .اذانه فيحالة الانظمة المعقدة تكون هذه الطريقة اكثر قبولاً من طريقة الملاحظة والتي تكون مطولة وتأخذ وقتاً طويلاً .وتم توضيح ذلك من خلال ثلاثة امثلة والنتائج ارفقت في نهاية البحث

**1. Introduction:**

 There are several methods presented by many researchers by which a minimal cut sets can be determined. For complex systemsome of them based on prior knowledge of graph theory and Boolean algebra, while others depend on set theory and probability…etc. see Lars. H (2011),Massimo Lazzarone, L. Cristaldi, L. Peretto, P. Rinaldi and M. Catelani(2011) and Nikolaos Limnios (2007).

The procedure inquestion uses the logical gets AND and OR to develop the minimal cut sets from a fault tree directly and involves numbering each gate and event in the fault tree with the fact that each input to an OR gate identifies a separable cut set and that every AND gate identifies just one cut set.

**2.Some Definitions and Concepts:**

**Definition (1), Fault tree [Nikolaos Limnios (2007)] :**

The fault tree is a graphic model of the various parallel and sequential combinations of the fault that can lead to the occurrence of the predefined event or top event.

**Definition (2), basic event [Lars. H (2011)]:**

 The basic of a fault tree are those events, which for one reason or another, have not been further developed.

**Definition (3), Intermediate Event [Nikolaos Limnios (2007)]:**

 An intermediate event is a fault event which occurs because of one or more antecedent causes acting through logic gates. All intermediate event are symbolized by rectangles.

**Definition (4), Minimal cut set [M. Stamatelatos, W. Vesely, J. Caraballo, J. Dugan, J. Fragola, J. Minarick and J. Railsback(2002)]:**

 A minimal cut set is a cut which there is no specific of components whose failure alone will cause the system to fail.

There are two basic types of fault tree gates:

1. The OR which is used to show that the output event occurs only if one or more of the input events occur.
2. The AND gate which is used to show that the input fault occur.

The symbols of AND and OR gates are shown in the following figure respectively. For more detail see [Marko Cepin(2011)], [W. E. Vesely, F.F. Goldberg, N. H. Roberts and D. F. Haasl (2002)]

(b) OR gate

1. AND gate

Figure (1) show the AND and OR [W. E. Vesely, F.F. Goldberg, N. H. Roberts and D. F. Haasl (2002)]

**3.The suggested method:**

 After number each gate and basic events state with the top gate on the fault tree. The method consists of a stagewise vertical listing of all basic failure events and gates of the next lower state leading to the failure event at the stage under consideration then we sequentially work our way down until were ach all the base or primary event of the considered the system.

**4.Illustrative Examples:**

 To illustrate the method above, consider the three following examples:

**Example (1):**

 Consider the fault tree shown in the figure (2). To derive the cut sets, the steps are as follows:

 The first step was to list vertically all the basic events and gates leading to the first level OR gate . In the second step we expand second level gates and by vertically listing all basic events or third level gates leading to them. Whenever an AND gate is met, basic event and leading to it are listed horizontally replace each gate by its input basic events or gates until all fault tree gates are replaced with the basic event entries. The final minimal cut listed under step3 consists of event 1,2,7 and 8 joint event 3,4,5 and 6.

Figure(2). FT of example(1)

T

3

1

2

G111

4

5

6

7

8

G444

G

G6666

G22

G3







|  |  |  |
| --- | --- | --- |
| Step 1 | Step 2 | Step 3 |
| 1 | 1 | 1 |
|  | 2 | 2 |
|  | 3, 4 |
|  |  | 5,6 |
|  | 78 |

Table(1)

If in a cut set in which all gates have been eliminated a basic event is both member of the cut set to drive the minimal cut set . In other words, a single events are a more basic event that a joint event made up of such single events .

**Example(2):**

 Consider a system which consists of eight components as shown in the following network. A bored circuit of motors control consist of eight components, which is taken from [Farnkle. E. G (1987)]

8

7

4

2

5

1

6

3

Figure (3): A system network

The corresponding fault tree of system above is shown in the fig. (4) below

5

3

6

7

4

2

1

8

Figure(4): A fault tree of figure(3)

T

Now

Now the top gate is an OR gate with three inputs. In step (1) , we list vertically all basic events and gates 1 , 8 and . In the second step we replace by its input , horizontally , leading first level . in the next step we expand gates by vertically listing all primary events or gates leading to them . In step (4) we replace gate horizontally by its basic events 5 ,6 . Now all of the gates have been replaced by their inputs , so in the last step we list all the minimal cut sets this mean it consists of events 1,8 and joint events ,(2,3),(4,3), (7,3) ,(2,5,60, (4,5,6) and (7,5,6) as shown in the table (2) bellow



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Step 1 | Step 2 | Step 3 | Step 4 | Step5 |
| 1 | 1 | 1 | 1 | 1 |
|  |  | 2 , 34 , 7  | 2 , 34 , 5,67  | 2,34,37,32,5,64,5,67,5,6 |
| 8 | 8 | 8 | 8 | 8 |

**Table (2)**

**Example (3):**

 Consider the system in the figure(5) below, which is taken from [W. E. Vesely, F.F. Goldberg

 N. H. Roberts and D. F. Haasl (2002)] as a question.

S1

K2

KT1

KT2

KT3

S1

K1

K3

K5

K1

KT2

Figure (5): A fault tree of a system

T

1. The solution for determine minimal as shown in [W. E. Vesely, F.F. Goldberg, N. H. Roberts and D. F. Haasl (2002)].

From above fault tree we have :-

Now

(

[

The minimal cut are :-

1. The solution by the suggested procedure.

Now in step (1) we put

in step (2) we listed horizontally

in step (3) we replace and by their inputs

in step (4) we replace by its input horizontal and by its input vertically

in step (5) we replace vertically by its inputs .

in step (6) we replace horizontally by its inputs then in the last step we list all minimal cut sets there are : and . as shown in the table(3) .



|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Step (1) | Step (2) | Step (3) | Step (4) | Step (5) | Step (6) | Step (7) |
|  |  |  |  |  |  |  |

Table(3)

**Note:-**

For more detail see [W. E. Vesely, F.F. Goldberg, N. H. Roberts and D. F. Haasl (2002)]

is not minimal cut because is minimal cut

**5.Conclusions:**

1. An OR gate always increase the number of cuts sets . (three will be a separate cut set for each OR gate input ).
2. An AND gate always increase the size of a cut set. (there will be one cut set for an AND gate, and each input increase the size of the cut set ).
3. The presented method can be used :

A – to obtain the cut set for any size fault tree , since it will not to be possible to derive the cut set by observation for more complicated fault trees .

B – For both manual and computer analysis of fault trees .

1. It is simpler than the algebra reduction method which presented in [W. E. Vesely, F.F. Goldberg, N. H. Roberts and D. F. Haasl (2002)] because the later requites to translate.

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