**Prime Fuzzy Submodules and Strongly Prime**

**Fuzzy Submodules Over Near-ring**

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**الخلاصة**

الغرض من هذا البحث هو توسيع المفاهيم (الاعتيادية ) للموديولات الأولية إلى الموديولات الضبابية الأولية في الحلقة القريبة . قادتنا دراستنا لهذا المفهوم لتقديم ودراسة المفاهيم الضبابية القريبة منه والتشاكل الضبابي عليه

كذلك قمنا بدراسة مفهوم (الاعتيادية ) الموديولات الأولية القوية وتحويله إلى الموديولات الضبابية الأولية القوية في الحلقة القريبة . قادتنا دراستنا لهذا المفهوم لتقديم ودراسة المفاهيم الضبابية القربية منه والتشاكل الضبابي عليه .

بالإضافة لذلك , درسنا علاقة الموديولات الضبابية الاولية في الحلقة القريبة مع الموديولات الضبابية الأولية القوية (الشديدة ) في الحلقة القريبة .

**ABSTRACT**

The main aim of this paper is to extend and study the notions of (ordinary ) prime submodules into prime fuzzy submodules over near-ring .This lead us to introduce and study other properties of fuzzy submodules of near-ring and fuzzy homomorphism about it .

Also , we study the notions of (ordinary ) strongly prime submodules into strongly prime fuzzy submodules over near-ring .This lead us to introduce and study other properties of fuzzy submodules of near-ring and fuzzy homomorphism about it .

Moreover , we study the relationships of prime fuzzy submodules of near-ring and strongly prime fuzzy submodules of near-ring .

**INTRODUCTION**

The present paper introduces and studies prime fuzzy submodules of a near-ring and strongly prime fuzzy submodules of a near-ring .

Near-rings are one of the generalized structures of rings . The study and research near-ring is very systematic and continuous . In 1905 L.E. Dickson began the study of near-ring and later 1930 Wieland has investigated it . Further , material about a near-ring can be found . In 1965 Zadeh introduced the concept of fuzzy subset . Fuzzy ideal of ring were introduced by Liu 1982 . In 1990 , Mlik D.S. and Mordeson J. N. introduced the concept fuzzy ideals of ring and prime fuzzy ideals of a ring . In 1997 D.T.K. and Biswas introduced the concept fuzzy ideal of near-ring . In 2001 , Jari R.H. introduced the concept fuzzy submodule and prime fuzzy submodule . In 2009 , Naghiipour A.R. introduced the concept prime submodule and strongly prime submodule .

In section one , some basic definitions and results are recalled which will be needed later. In section two , We introduce the concept of the prime fuzzy submodules of a near-ring and we study some properties and theorems about it . In section three, we introduce the concept of the strongly prime fuzzy submodules of a near-ring. Throughout this paper N is commutative near-ring with unity , and every fuzzy subset A of a near-ring N is nonempty fuzzy subset .

**S . 1 PRELIMINARY:**

In this section , some basic definitions and results which we will be used in the next section .Let (N,+,٠) be a nonempty set where N is a set and (+) and (⋅) are any binary operations on N such that (1) (N,+) is a group (2) multiplication is associative (3) left distributive law n⋅ (b +c) = n⋅b + n⋅c , for all n , b, c ∈ N. This **near-ring** will be termed as right near-ring if (b +c) ⋅n = b⋅n + c⋅n , for all n , b, c ∈ N , **([AL-Abege A.M.-H , 2010],[ Burton D.M. , 1967]).**If 1⋅n= n (n⋅1 = n) then N has a left identity (right identity) , if (N, + ) is abelian , we call N **an abelian near-ring** **,([ Choudhary S.C. and Gajendra Purohit , 2009],[ Vasantha W.B. , 2002,]) .** If (N, ⋅) is commutative we call N itself a commutative near-ring **,([ Choudhary S.C. and Gajendra Purohit , 2009],[ Vasantha W.B. , 2002,]) .**

Clearly if N is commutative near-ring then left or right distributive law is satisfied and 1⋅n = n⋅1 = n , N is called **unital commutative near-ring** , **([D.T.K .and Biswas , 1997],[ Zadeh L.A., 1965]) .**

A **fuzzy subset of N** is a function from N into [0,1], ( [10],[ 18] ). Let A and B be fuzzy subset of N. We write **A ⊆ B** if A(x) ≤ B(x) , for all x ∈ N. If A ⊆ B and there exists x ∈ N such that A(x) < B(x) , then we write **A ⊂ B** and we say that A is a proper fuzzy subset of B , [**Dixit V.N., Kumar R. and Ajmal N., 1991**]. Note that **A = B** if and only if A(x) = B(x), for all x ∈ N , **[Kumbhojkar H.V. and Bapat M.S., 1993].**

Let A be a fuzzy subset of N, A(0) ≥ A(x) , for all x ∈ N, **([ Bae J.Y. and Kim H. , 2002],[ Liu W.J., 1982],[ Vasantha W.B. , 2002]) .**Let f : N→ N' ,A and B be two fuzzy subsets of N and N' respectively ,the fuzzy subset f(A) of N' defined by : f(A)(y) = sup A(y) if f(y) ≠ 0 , y∈ N' and f(A)(y) = 0 , otherwise .It is called **the image of A** **under f** and denoted by f(A) . The fuzzy subset f -1 (B) of N defined by : f -1 (B)(y) = B(f(x)) ,for all x ∈N .Is called **the inverse image of B** and denoted by f -1 (B) **, ([Dixit V.N., Kumar R. and Ajmal N., 1991],[ Kumbhojkar H.V. and Bapat M.S., 1991],[ Martines L., 1995]) .**

The characteristic function of N denoted by λN which define by : ,it is called **singular fuzzy set in N** **,[ Zadehi M .M .and Mashinchi M. , 1992,] .**

Let N , N' be any sets and f : N→ N' be any function .A fuzzy subset A of N is called **f-invariant** if f(x) = f(y) implies A(x) =A(y), where x ,y ∈N **,( [Kumar R.,1991] ,[ Kumbhojkar H.V. ,1997],[ Liu W.J., 1982]) .** For each t ∈ [0,1], the set **At** = { x ∈ N | A(x) ≥ t } is called **a level subset of N** and A=B if and only ifAt = Bt for all t ∈ [0,1] , **( [Dixit V.N., Kumar R. and Ajmal N., 1991],[ Malik D.S. and Mordeson J.N. , 1990],[ Martines L., 1995] ).**

Let x ∈ N and t ∈ [0,1], let xt denote the fuzzy subset of N defined by :

 , ∀y ∈ N . xt is called **a fuzzy singleton, [Malik D.S. and Mordeson J.N. , 1990].** If xt and ys are fuzzy singletons, then xt + ys = (x + y)λ and xt  ys = (x **.** y)λ , where λ = min { t ,s } , **( [Kumbhojkar H.V. and Bapat M.S., 1993],[ Malik D.S. and Mordeson J.N. , 1990]).** Let {Ai **|** i ∈ Λ} be a collection of fuzzy subset of N . Define the fuzzy subset of N (**intersection**) by () (x) = inf { Ai(x) **|** i ∈ Λ }, for all x ∈ N**,([ Dixit V.N., Kumar R. and Ajmal N., 1991] ,[ Malik D.S. and Mordeson J.N. , 1990]).** Let φ be denote φ(x) = 0 for all x ∈ N, **the empty fuzzy subset** of N**,([ Kumbhojkar H.V. and Bapat M.S., 1993],[ Swamy U.M. and Swamy K.N., 1988] ).**

Let A and B be a fuzzy subsets of N , **the product AB** define by : AB (x) = sup { min {A(y), B(z) }**|** x = y ⋅ z}} y, z ∈ N , for all x ∈ N , **[Mukherjee T.K. and Sen M.K.,1989] .The addition A + B** define by(A + B) (x) = sup {min {A(y), B(z) **|** x = y + z}} y, z ∈ N , for all x ∈ N , **[Mukherjee T.K. and Sen M.K.,1989]** .

1. Let A be a fuzzy subset of N, A is called **a fuzzy subnear-ring** of N if for all x, y ∈ N , A(x + y) ≥ min {A(x), A(y)} and A(x) = A(-x**),([ AL-Abege A.M.-H , 2010],[ Bae J.Y. and Kim H. , 2002],[ Bae J.Y. and Kim H. , 2002],[ Jun Y.B. and Kim H.S., 2002]) .**

Let A be a fuzzy subset of N , A is called **a fuzzy ring of N** if for all x, y ∈ N , A(x – y) ≥ min {A(x), A(y)} and A(x ⋅ y) ≥ min {A(x), A(y)}, ([12],[23],[25],[26]).

1. A non empty fuzzy subset A of N is called **a fuzzy ideal of N** if and only if A(x - y) ≥ min {A(x), A(y)} ; A(x ⋅ y) ≥ min {A(x), A(y)}; A(y+x - y) ≥ A(x) ; and A(x⋅y) ≥ A(y) , for all y, x ∈ N , **([Jun Y.B. and Kim H.S., 2002],[ Vasantha W.B. , 2003])** .

Let X be a fuzzy ring of N and A be a fuzzy ideal of N such that A ⊆ X , then A is a fuzzy ideal of X, **[Vasantha W.B. , 2002].** Let A be a fuzzy ideal of N . If for all t ∈ [0, A(0)], then At is an ideal of N and **( [Jun Y.B. and Kim H.S., 2002] , [Swamy U.M. and Swamy K.N., 1988] ).**Let X be a fuzzy ring of N . A be a fuzzy subset of X is a fuzzy ideal of X if and only if At is an ideal of Xt ,for all t ∈ [0,A(0)] **, ([Jun Y.B. and Kim H.S., 2002],[ Mordeson J.N. and Malik D.S.,1998],[ Swamy U.M. and Swamy K.N., 1988]).** Let A and B be fuzzy ideals of N, then AB is a fuzzy ideal of N**,([ Jun Y.B. and Kim H.S., 2002],[ Swamy U.M. and Swamy K.N., 1988] ) .**Let A and B be fuzzy ideals of N, then A  B, A + B are fuzzy ideals of N **,([ Jun Y.B. and Kim H.S., 2002],[ Swamy U.M. and Swamy K.N., 1988] ).**

**PROPOSITION 1. 1 [ Abou-Draeb A.T. 2011]**

Let A and B be two fuzzy subsets of N .Then :

1. A****B ⊆ A∩B .
2. (A****B) t = At∙ Bt , t∈ [0,1] **,[ Dixit V.N., Kumar R. and Ajmal N., 1991]** .
3. (A∩B) t = At **∩** Bt , t∈ [0,1].

**PROPOSITION 1 .2 ([Kumar R., 1992],[ Kumar R.,1991]) :**

Let f : N1→ N2 be a homomorphism of rings . Let A and B be two fuzzy subsets of N1 and C and D be two fuzzy subsets of N2 .Then :

1. A ⊆ f -1(f(A)) and A = f -1(f(A)) whenever A is f-invariant .
2. A = f -1(f(A)) , whenever A is f-invariant .
3. f(A∩B) ⊆ f(A) ∩ f(B) .
4. f( f -1(C)) = C , f -1(C∩D) = f -1(C) ∩ f -1(D) .
5. A ⊆ B ⇒ f(A) ⊆ f(B) .
6. C ⊆ D ⇒ f-1 (C) ⊆ f-1 (D) .

**PROPOSITION 1. 3 ( [Abou-Draeb A.T. 2011 ],[ Kumbhojkar H.V. and Bapat M.S., 1991]) :**

Let A : N → [0,1] a fuzzy ideal of N , B : N**׳**→ [0,1] be a fuzzy ideal of Nˊ, and f : N → N**׳** be homomorphism between them, then :

**1.** f (A) is a fuzzy ideal of Nˊ.

**2.** f -1(B) is a fuzzy ideal of N.

**PROPOSITION 1. 4 ([Abou-Draeb A.T. 2011],[ Kumbhojkar H.V. and Bapat M.S., 1991]) :**

If X : N → [0,1], Y : N**׳**→ [0,1] are fuzzy rings , let f : N → N**׳** be homomorphism between them and A : N → [0,1] a fuzzy ideal of X , B : N**׳**→ [0,1] a fuzzy ideal of Y, then :

**1.** f (A) is a fuzzy ideal of Y.

**2.** f -1(B) is a fuzzy ideal of X.

**PROPOSITION 1.5 ([Mukherjee T.K. and Sen M.K., 1987],[ Zadehi M .M .and Mashinchi M. , 1992, ]) :**

Let {Ai **|** i ∈ Λ} be a family of fuzzy ideals of N. Then is a fuzzy ideal of N . **PROPOSITION 1. 6 ( [Abou-Draeb A.T. 2011 ],[ Kumbhojkar H.V. and Bapat M.S., 1991,]) :**

Let A and B be two fuzzy subsets of N and f : N → N**׳** is inverse image function of B .Then :

1. f (A)∩ f (B)= f (A∩B) .
2. f (A) **** f (B)= f (A****B) .
3. f ( At) = (f (A))t .
4. f -1(At) = (f -1(A))t .

**PROPOSITION 1 .7 ([Abou-Draeb A.T. 2011],[ Liu W.J., 1982]) :**

Let A and B be two fuzzy ideals of a fuzzy ring X of N1 and C and D be two fuzzy ideals of a fuzzy ring Y of N2 . Let f : N1→ N2 be a homomorphism . Then :

1. f -1(C∩D) = f -1(C) ∩ f -1(D) .
2. f (A∩B) = f (A)∩ f (B) , where f is a monomorphism .

**PROPOSITION 1 .8 ([Kumbhojkar H.V. and Bapat M.S.,1991],[ Liu W.J., 1982,],[ Martines L., 1995,]) :**

A is a fuzzy ideal of N if and only if λN is a fuzzy ideal of N.

**DEFINITION 1.9 ([AL-Abege A.M.-H , 2010],[ Mordeson J.N. and Malik D.S.,1998]) :**

Let (M,+,⋅) be a group . Suppose we have a map N × M → M written as a scalar multiplication (n, m)→ n⋅m satisfying :

(1) n⋅m ∈M ; (2) (n1 + n2 ) ⋅m = n1 ⋅m+ n2 ⋅m ; (3) (n1 ⋅ n2 ) ⋅m = n1 (n2 ⋅m) ; for all n, n1 , n2 ∈ N and m ∈ M; then m is the structure of a near left N-module , and we can define the concept of a near right N-module . If M is a near left N-module and a near right N-module then M is a near-module ( **M is a near-ring module**) .

**REMARK 1.10 [Vasantha W.B. , 2003] :**

Let A and B be fuzzy near-modules of an N-module M **.** Define **AB** by : AB (x) = sup {min { min{A(yi), B(zi) }**|** i=1,.. , n}| yi, zi ∈ M, i=1,.. , n}| x = ∑i=1n yi ⋅zi ,n∈ Ɲ}, for all x ∈ M, ([26],[27]) . Let B be fuzzy submodule of an N-module M and xt is fuzzy singleton of N , **the product xt B** define by : xt B (a) = sup { xt (y), B(z) }**|** a = y ⋅z } y, z ∈ M , ∀ a ∈ M .

**PROPOSITION 1.11 [Vasantha W.B. , 2003] :**

Let W and K be near-modules over a fuzzy near-ring A in a near-ring N and f a homomorphism of W into K .

1. Let W be a fuzzy near-module in K , then the inverse image f-1 (B) of B is a fuzzy near-module in W .
2. Let K be a fuzzy near-module in W that has the sup property , then the image f (B) of B is a fuzzy near-module in K .

**DEFINITION 1.12 [Vasantha W.B. , 2003] :**

**An N-homomorphism** f of an N-module M into an N-module M′ is a mapping from M to M′ such that (s+n)f = sf + nf and (mf)r = (mr)f for all s ,n , m ∈ M and for all r ∈ N .

Now , we give some properties and theorems of fuzzy submodules .

**DEFINITION 1.13 [Vasantha W.B. , 2003] :**

A non empty fuzzy subset A of an additive group G is called **a fuzzy normal subgroup of G** if (1) A(x+y) ≥ min { A(x) , A(y)} for all x, y in G ; (2) A(-x) = A(x) for all x ∈ G ; (3) A(y+x-y) = A(x) for all x , y in G .

**DEFINITION 1.14 [Vasantha W.B. , 2003] :**

Let A be a non empty fuzzy subset of an N-module M .Then A is called a **fuzzy submodule of M** if (1) A is a fuzzy normal subgroup of M and (2) A[(x+y)r-xr] ≥ A(y) for all x , y ∈ M ,r ∈ N .

**PROPOSITION 1.15 [Vasantha W.B. , 2003] :**

For a non empty fuzzy subset A of an N-module M . A is a fuzzy submodule of an N-module M if and only if At is a submodule of M , for all t ∈ [0, 1] .

**DEFINITION 1.16 ([Kumar R.,1991 ], [Vasantha W.B. , 2003]) :**

Let X be a non empty fuzzy module of an N-module M , and A be a non empty fuzzy subset of an N-module M .Then A is called a **fuzzy submodule of X** if A is a fuzzy submodule of a fuzzy module X of an N-module M .

**PROPOSITION 1 .17 ([Kumbhojkar H.V. and Bapat M.S., 1991],[ Liu W.J., 1982],[ Martines L., 1995]) :**

A is a fuzzy submodule of N-module M if and only if λN is a fuzzy submodule of N-module M  **.**

**PROPOSITION 1. 18 ( [AL-Abege A.M.-H , 2010 ],[ Mukherjee T.K. and Sen M.K.,1989]) :**

Let A : M → [0,1] a fuzzy submodule of N-module M , B : M**׳**→ [0,1] be a fuzzy submodule of N-module Mˊ, and f : M →M**׳** be homomorphism between them, then:

**1.** f (A) is a fuzzy submodule of N-module Mˊ.

**2.** f -1(B) is a fuzzy submodule of N-module M .

**DEFINITION 1.19 [Jun Y.B. and Kim H.S., 2002]) :**

Let A be a fuzzy ideal of a near-ring N .Then A is called **a prime fuzzy ideal of N** if A is not a constant function and for any fuzzy ideals B and C of N ,if BC ⊆ A , then either B ⊆ A or C ⊆ A .

**THEOREM 1.20 [Jun Y.B. and Kim H.S., 2002 ] :**

A is a fuzzy ideal of N .Then A is a prime fuzzy ideal if and only if At is a prime ideal of N for all t ∈ [0,A(0) ] .

**DEFINITION 1.21 [Zadehi M .M .and Mashinchi M. , 1992] :**

Let A and B be two fuzzy submodule of N-module M . We define (A:B) by :

(A: B) = { rt |rt fuzzy singleton of N such that rt B ⊆ A } .Note that : (A: B) (r) = sup { t ∈ [0, 1] | rt B ⊆ A, for all r ∈ N} .And if B = (bk),then :(A: (bk)) = { rt |rt fuzzy singleton of N such that rt bk ⊆ A } .

**DEFINITION 1.22 [Zadehi M .M .and Mashinchi M. , 1992] :**

Let A be a fuzzy submodule of X where X be a fuzzy module of N-module M and I be a fuzzy ideal of N , define (A:I) by (A: I) = {ak | ak ⊆ X , ak I ⊆ A} .Note that :(A:X I) (x) = sup {t| t ∈ [0, 1] , Ixt ⊆ A } ,∀ x ∈ M . And if I = (rt),then (A: (rt))={ ak | ak ⊆ X ; rt ak ⊆ A } .

**PROPOSITION 1.23 [Zadehi M .M .and Mashinchi M. , 1992 ] :**

Let A and B be two fuzzy submodule of X where X be a fuzzy module of N-module M and I be a fuzzy ideal of N such that I(0) = 1, then :

1. (A:X I) is a fuzzysubmodule of N-module M **.**
2. (A: B) is a fuzzy ideal of N .

**PROPOSITION 1.24 ( [D.T.K .and Biswas , 1997 ],[ Martinez L., 1999]) :**

Let A be a fuzzy submodule of N-module M . |A is torsion-free fuzzy submodule if and only if At is a torsion-free submodule of M for all t ∈ (0, 1] .

**PROPOSITION 1.25 ( [D.T.K .and Biswas ],[ Martinez L., 1999]) :**

Let A be a fuzzy submodule of N-module M . |A is divisible fuzzy submodule if and only if At is a divisible submodule of M for all t ∈ (0, 1] .

**S.2 PRIME FUZZY SUBMODULES OF NEAR-RING :**

We shall fuzzify this concept in to prime submodule of near-ring by [**Kumar R.,1991**] . Some basic definitions and results of prime fuzzy submodule s of near-ring are recalled which will be needed later as **([Abou-Draeb A.T. 2012],[ Jun Y.B. and Kim H.S., 2002],[ Kash F. , 1982]) .**

**DEFINITION 2 .1 :**

Let X be a fuzzy module of an N-module M and A be a proper fuzzy submodule of X .Then A is called **a prime fuzzy submodule of an N-module M**  if rt at ⊆ A for fuzzy singleton rt of N and at ⊆ X , then either rt ⊆ (A:NX) or at ⊆ A .Note that : (A:NX) = { rt | rt X ⊆ A , rt ⊆ X }.

**THEOREM 2 .2 :**

Let X be a fuzzy module of an N-module M and A be a proper fuzzy submodule of X .Then A is called **a prime fuzzy submodule of an N-module M**  if for all fuzzy submodule B of X and for any fuzzy ideal I of N , if IB ⊆ A , then B ⊆ A or I ⊆ ( A:X) .Note that : (A: X) = { rt | rt X ⊆ A , rt fuzzy singleton of N }.

**REMARK 2 .3 :**

The fuzzy ideal P = (A: (ak)) , for all ak ⊆ X such that ak ⊄ A , where

(A: (ak)) = { rt | rt ak ⊆ A , rt is a fuzzy singleton of N } , then P = (A:NX) .

**PROPOSITION 2 .4 :**

The definition (2.1) and theorem (2.2) are equivalent.

**Proof:**

If definition (2.1) is true , to prove definition (2.2) .If B is a fuzzy submodule of X and I is a fuzzy ideal of N such that IB ⊆ A . Suppose B ⊄ A , then there exists bk ⊆ B but bk ⊄ A , hence for all rt ⊆ I , rt bk ⊆ IB ⊆ A . So that rt ⊆ (A :X) Since A is a prime fuzzy submodule and bk ⊄ A . Therefore , I ⊆ (A:X) .If theorem (2.2) is true , to prove definition (2.1) .

Let rt at ⊆ A and at ⊄ A , then rt ⊆ (A: (ak)) . But (A: (ak)) = P = (A: NX) by remark (2.3) .Therefore , rt ⊆ (A: N X) . A is a prime fuzzy submodule of X .

**PROPOSITION 2 .5 :**

If A is a prime fuzzy submodule of X where X is a fuzzy module of an N-module M , then (A: X) is a prime fuzzy ideal of N .

**Proof:**

By proposition (1.23) , (A: X) is a fuzzy ideal of N. To prove (A: X) is a prime fuzzy ideal of N . Let rt , yk be two fuzzy singletons of N such that rt yk ⊆ (A: X) . Then rt yk X ⊆ A and rt yk ms ⊆ A for all ms ⊆ X implies that rt (yk ms) ⊆ A , so either yk ms ⊆ A or rt ⊆ (A: X) Since A is a prime fuzzy submodule . If yk ms ⊆ A , then yk ⊆ (A: X), then either rt ⊆ (A: X) or yk ⊆ (A: X). Therefore , (A: X) is a prime fuzzy ideal of N .

**REMARK 2 .6 :**

The converse of proposition (2.5) is not true in general for example :

Let M = Z⊕Z as a Z-module , let X: M → [0, 1] , A:M→ [0,1] defined by :



It is clear that X is a fuzzy module of M and A is a fuzzy submodule of X .Since 21/2 (2, 0)1/2 = (4, 0)1/2 and (4, 0) 1/2 ⊆ A Since A(4, 0) = 1 > 1/2 , (2, 0) 1/2 ⊆ X Since X(2, 0) = 1 > 1/2 and (2, 0) 1/2 ⊄ A Since A(2, 0) = 0≠ 1/2 .

Since 21/2 ⊄ (A:X) Since there exists (2, 3) 1/2 ⊆ X and 21/2 (2, 3) 1/2 = (4, 6) 1/2 ⊄ A because A(4, 6) = 0 ≠ 1/2 .Then A is not a prime fuzzy submodule of X .Since (At:Xt) = (4Z⊕0 , 2Z⊕Z) = {0} , for all t > 0 .Hence (A:X)t = {0} for all t > 0 . and (A:X)(r) =  = 01 , which can be easily shown a prime fuzzy ideal of Z .

**REMARK 2 .7 :**

The converse of proposition (2.7) is not true in general for example :

Let M = N = Z , let X: M → [0, 1] , defined by :

X is a fuzzy module of M . A:M→ [0,1] defined by : . A is a fuzzy submodule of N-module M .

A is not a prime fuzzy submodule of X , since 51/3 42/3 = 201/3 ⊆ A and 42/3 ⊆X . But A(4) = 1/2 ≠ 2/3 and 42/3⊄ A , 51/3 ⊄ (A:X) ,Since there exists 31/4 ⊆ X , but 51/331/4 = 151/4 ⊄ A .We have A1/2 = 2Z and X1/2 = 2Z , At = {0} and Xt = 4Z , for all t > 1/2 .And At = 2Z and Xt = Z , for all t < 1/2 .At ≠ Xt and At is a prime submodule of Xt .

**PROPOSITION 2 .8 :**

A is a prime fuzzy submodule of N-module M if and only if λN is a prime fuzzy submodule of N-module M.

**Proof:**

By proposition (1.18) , A is a fuzzy submodule of N-module M if and only if λN is a fuzzy submodule of N-module M . Suppose that A is a prime fuzzy submodule of N-module M, we must prove that λN is a prime fuzzy submodule of N-module M.

Let A(ra) ≥ t ,for some r ∈ N , so (ra)t ⊆ A , implies that rtat ⊆ A. (since A is a prime fuzzy submodule of N-module M) . Since λN(0) = 1

Let r ∈ N and a ∈ M ,then λN(ra) =1 = λN(r) or λN(ra) =1= λN(a) . Let r ∈ N and a ∉ M ,then λN(ra) =1 = λN(r) or λN(ra) =0= λN(a) . Let r ∉ N and a ∈ M ,then λN(ra) =0 = λN(r) or λN(ra) =1= λN(a) .Let r ∉ N and a ∉ M ,then λN(ra) =0 = λN(r) or λN(ra) =0= λN(a) .

Hence λN is a prime fuzzy submodule of N-module M.Suppose that λN is a prime fuzzy submodule of N-module M, we must prove that A is a prime fuzzy submodule of N-module M .Let ra ∈ At for some r ∈ N , a ∈ Xt . Then, λN(ra) =1 then λN(r)=1 or λN(a)=1 (since λN is a prime fuzzy submodule of N-module M).Thus A(ra) =1 ≥ t implies that A(ra) ≥ t , so (ra)t ⊆ A , implies that rtat ⊆ A. Hence A is a prime fuzzy submodule of N-module M.

**PROPOSITION 2 .9 :**

Let M = N and A be a fuzzy ideal of N . Then A is a prime fuzzy submodule of N-module M if and only if A is a prime fuzzy ideal of N .

**Proof:**

Let A be a prime fuzzy submodule of N-module M , to prove A is a prime fuzzy ideal of N.

Let A be a prime fuzzy submodule of N-module M , then A is a fuzzy ideal of N by **[Zadehi M .M .and Mashinchi M. , 1992**].Let arbt ⊆ A . Suppose that ar ⊄ A , then bt ⊆ (A:X) since A is a prime fuzzy submodule of N-module M .

Thus bt λN ⊆ A and (btλN )(b) ≤ A(b) , b ∈N since b= 1.b = b.1 , and (btλN)(w) .Hence (btλN )(w) = t implies that t ≤ A(b) , bt ⊆ A . Hence A is a prime fuzzy ideal of N .

Conversely, Let A is a prime fuzzy ideal of N , to prove A is a prime fuzzy submodule of N-module M . Let A is a prime fuzzy ideal of N , by [30] A is a fuzzy submodule of N-module M .

Let rtas ⊆ A . Suppose that as ⊄ A , then rt ⊆ A. Hence rt λN ⊆ A since for all w ∈ M =N .

(rtλN )(w) .If w = rc , then (rtλN )(w) = t , but A(w) = A(rc) ≥ A(r) ≥ t .Hence (rtλN)(w) ≤ A(w) .If (rtλN )(w) = 0 , then 0= (rtλN)(w) ≤ A(w) . Thus rtλN ⊆ A implies that rt ⊆ (A: λN) .Hence A is a prime fuzzy submodule of N-module M .

**PROPOSITION 2 .10 :**

Let A be a fuzzy submodule of a fuzzy module X of N-module M . Then A is a prime fuzzy submodule if and only if (A:X I) is a prime fuzzy submodule of X for every fuzzy ideal I of N .

**Proof:**

Let A be a prime fuzzy submodule , to prove (A:X I) is a prime fuzzy submodule of X for every fuzzy ideal I of N .

Let A be a prime fuzzy submodule of X , by proposition (1.23(1)) (A:X I) is a fuzzy submodule of X .To prove (A:X I) is a prime fuzzy submodule of X .

Let rtxs ⊆ (A: I) , then (rtxs) I ⊆ A, then (rtakxs) ⊆ A for all ak ⊆ I .

But A is a prime fuzzy submodule of X , so either xs ⊆ A or rtak ⊆ (A:X) .If xs ⊆ A , then xs ⊆ (A:I) by [30] .If rtak ⊆ (A:X) for all ak ⊆ I , then rtI ⊆ (A:X) , So rt IX ⊆ A .Hence rtX ⊆ (A:I) , thus rt ⊆ ((A:I):X) , so (A:X I) is a prime fuzzy submodule of X .

Conversely , let (A:X I) is a prime fuzzy submodule of X for every fuzzy ideal I of N , to prove A is a prime fuzzy submodule .Suppose that (A:X I) is a prime fuzzy submodule of X for all fuzzy ideal I of N . Let I = λN , where λN(a) = 1 , for all a ∈ N .

A ⊆ (A: λN) by [30] , and let xt ⊆ (A: λN) , implies that xtλN ⊆ A , then

(xtλN )(w) ≤ A(w) ,for all w ∈ M .

Consequently , (xtλN )(x) = t ≤ A(x) , implies that xt ⊆ A , hence (A: λN ) ⊆ A . Thus A = (A: λN) .Therefore, A is a prime fuzzy submodule of X .

**LEMMA 2 .11 :**

Let A be a fuzzy submodule of N-module M and let x ∈N . Then A(x) = sup{ t | x ∈ At} , t ∈(0,A(0)] .

**Proof:**

Since At = { x ∈N |A(x) ≥ t}. Let x∈N . Then for each t ∈ (0,1] , A(x) ≥ t ⇔ x ∈ At  . Therefore , A(x) is the least upper bound of the set {t| x∈At} . Thus A(x) = sup{t| x ∈At} , t ∈(0,A(0)] .

**PROPOSITION 2. 12 :**

Let f : M → M**׳** be an epimorphism of modules of near-ring N , then :

1. A : M → [0,1] be an f-invariant prime fuzzy submodule of N-module M . Then f (A) is a prime fuzzy submodule of N-module M ׳ .

2. B : M**׳**→ [0,1] a prime fuzzy submodule of N-module M **׳**, then f -1(B) is a prime fuzzy submodule of N-module M .

**Proof:**

1. Since f (A) is a fuzzy submodule of N-module M ׳ by [27] To prove f (A) is a prime fuzzy submodule of N-module M ׳ .

Suppose rtyk ⊆ f(A) for a fuzzy singleton rt of N and yk ⊆ Mˊ , then (ry) λ ⊆ f(A) where λ = min { t, k} implies that f(A) (ry) ≥ λ .But f is an epimorphism , there exists a ∈ M such that y= f(a) . Hence f(A)(rf(a)) ≥ λ . Thus f(A) (rf(a)) ≥ λ implies that [ f-1 f(A)](ra) ≥ λ .A(ra) ≥ λ .implies that (ra)λ ⊆ A , thus rtak ⊆ A .

On the other hand yk ⊆ Mˊ implies that Mˊ(y) ≥ k. Hence Mˊ (f(a))≥ k , so M(a) ≥ k that mean ak ⊆ M . Thus rtak ⊆ A , ak ⊆ X . But A is a prime fuzzy submodule of X , hence either ak ⊆ A or rt ⊆ (A:X) .If ak ⊆ A , then A(a) ≥ k implies that yk ⊆ f(A) since (f(A))(y) = f(A)f(a) = (f-1 f(A))(a) = A(a) since A is f-invariant .Thus f(A)(y) ≥ k implies that yk ⊆ f(A) .

If rt ⊆ (A:X) , then rt X ⊆ A , hence f(rt X) ⊆ f(A) , but f(rt X) = rt Xˊ , implies that (f(rt X))(y) = sup { (rt X)(a) | f(a) = y}

= sup = 

= (rt Xˊ)f(a) = rt Xˊ(y).Hence f(rt X) = rt Xˊ, implies that rt Xˊ ⊆ f(A) , thus rt ⊆ (f(A):Xˊ), then f (A) is a prime fuzzy submodule of N-module M ׳ .

2- Since f -1(B) is a fuzzy submodule of N-module M׳ by [27] To prove f -1(B) is a prime fuzzy submodule of N-module M .Suppose rtak ⊆ f -1(B) for a fuzzy singleton rt of N and ak ⊆ M , then (ra) λ ⊆ f -1(B) where λ = min { t, k} implies that f -1(B) (ra) ≥ λ . Then B(f(ra)) ≥ λ implies that B(rf(a)) ≥ λ . Hence (rf(a))λ ⊆ B . But B is a prime fuzzy submodule of N-module Mˊ , so either (f(a))k ⊆ B or rt ⊆ (B:Xˊ) .If (f(a))k ⊆ B, then B(f(a)) ≥ k implies that f -1(B) (a) ≥ k , hence ak ⊆ f -1(B) .If rt ⊆ (B:Xˊ) , then rt Xˊ ⊆ B , hence rt ⊆( f -1(B):X) .

(rt X) (w) = 

= = (rt Xˊ)f(w) ≤ B(f(w)) = f -1(B) (w) .

Hence rt X ⊆ f -1(B), implies that rt ⊆ (f -1(B) :X) .Thusf -1(B) is a prime fuzzy submodule of N-module M .

**PROPOSITION 2. 13 :**

Let A and B be two prime fuzzy submodules of N-module M. Then (A∩B) is a prime fuzzy submodule of N-module M.

**Proof:**

Let A and B be two prime fuzzy submodules of N-module M. To prove A∩B is a prime fuzzy submodule of N-module M.

A∩ B is a fuzzy submodule of N-module M by proposition (1.6) .

To prove A∩B is a prime fuzzy submodule of N-module M .

Let rt at ⊆ A for fuzzy singleton rt of N and at ⊆ X , then either rt ⊆ (A:NX) or at ⊆ A .

Let rt at ⊆ B for fuzzy singleton rt of N and at ⊆ X , then either rt ⊆ (B:NX) or at ⊆ B .

Thus if rt at ⊆ (A∩B) for fuzzy singleton rt of N and at ⊆ X , then either rt ⊆ (A:NX) and rt ⊆ (B:NX) implies that rt ⊆ ((A∩B) :NX).or at ⊆ A and at ⊆ B implies that at ⊆ (A∩B)

either rt ⊆ ((A∩B):NX) or at ⊆ (A∩B) .Then A∩B is a prime fuzzy submodule of N-module M.

**S.3 STRONGLY PRIME FUZZY SUBMODULES :**

In this section , we introduce and study the concept of the strongly prime fuzzy submodule. We give some basic properties and theorems of this concept .The idea of fuzzifying the concept of strongly prime submodule was appeared in [Kash F. , 1982] as the following :-

**DEFINITION 3.1 :**

Let A be a fuzzy submodule of N-module M .Then A is called **a strongly prime fuzzy submodule of N-module M** if A(rx) =A(x) where r ∈ N , r ≠ 0 and x ∈ M .

**THEOREM 3 .2 :**

Let A be a fuzzy submodule of N-module M. Then A is a strongly prime fuzzy submodule if and only if At is a strongly prime submodule of N-module M , for all t ∈ [0,A(0) ] .

**Proof:**

Suppose A is a strongly prime fuzzy submodule of N-module M. Let t∈ [0, A(0) ], to prove that At is a strongly prime submodule of N-module M. Let rx ∈ At where r ∈ N , r ≠ 0 and x ∈ M , then A(rx) ≥ t . But A is a strongly prime fuzzy submodule of N-module M, then A(rx) =A(x) ≥ t .Thus A(x) ≥ t . Hence x ∈ At .Therefore , At is a strongly prime submodule of N-module M for all t ∈ [0,A(0) ].

Conversely , suppose At is a strongly prime submodule of M for all t∈ [0, A(0) ] , to prove that A is a strongly prime fuzzy submodule of N-module M.Let r ∈ N , r ≠ 0 and x ∈ M , suppose A(rx) = t , then rx ∈ At and rx ∉ Ak for all k > tby lemma (2.12).But At is a strongly prime submodule of M implies that x ∈ At and x ∉ Ak for all k > t ,then A(x) = t. That mean A(rx) =A(x) = t by lemma (2.12) .Therefore , A is a strongly prime fuzzy submodule of N-module M .

**REMARK 3 .3 :**

Note that the converse of proposition (3.3) is not true in general , for example : Let N be the Z-module Z⊕Z2 : ,X is not strongly prime fuzzy submodule of N-module M and proposition(3.2) but A is a strongly prime fuzzy submodule of N-module M by [9] and proposition(3.2) .

**PROPOSITION 3 .4 :**

The intersection of two strongly prime fuzzy submodules of N-module M is strongly prime fuzzy submodule of N-module M .

**Proof:**

Let A and B be two strongly prime fuzzy submodules of N-module M. To prove A∩B is a strongly prime fuzzy submodule of N-module M.A∩ B is a fuzzy submodule of N-module M by proposition (1.6) . A∩B is a prime fuzzy submodule of N-module M by proposition (2.14).

Let r ∈ N , r ≠ 0 and x ∈ M , (A∩B) (rx) = min {A(rx) , B(rx) } .If A(rx) =A(x) and B(rx) =B(x) , then(A∩B) (rx) = min {A(x) , B(x) }= (A∩B) (x) .

Then A∩B is a strongly prime fuzzy submodule of N-module M.

**PROPOSITION 3 .5 :**

Let I be a submodule of an N-module M . Then I is a strongly prime submodule of N-module M if and only if λI is a strongly prime fuzzy submodule of N-module M.

**Proof:**

Suppose that I is a strongly prime submodule of N-module M, we must prove that λI is a strongly prime fuzzy submodule of N-module M. Let r ∈ N , x ∈ M, if rx ∈ I , then x ∈ I (since I is a strongly prime submodule of N-module M). Hence λI(rx) =1 = λI(x) . If rx ∉ I , then x ∉ I ( since I is a strongly prime ) .Hence λI(rx) =0 = λI(x) . Therefore , λI(rx) = λI(x) for all r ∈N and x ∈ M.Hence λI is a strongly prime fuzzy submodule of N-module M.

Conversely , let λI is a strongly prime fuzzy submodule of N-module M, we must prove that I is a strongly prime submodule of N-module M. Let r ∈ N , x ∈ M such that rx ∈ I , then λI(rx) = 1 . But λI(rx) = λI(x) (since λI is a strongly prime fuzzy submodule of N-module M). Then λI(rx) = 1 implies that x ∈ I .. Hence A is a strongly prime fuzzy submodule of N-module M .

**PROPOSITION 3 .6 :**

Let A be a fuzzy submodule of N-module M and I is submodule of N-module M . Then I is a strongly prime submodule of N-module M if and only if AI is a strongly prime fuzzy submodule of N-module M .

**Proof:**

Suppose that I is a strongly prime submodule of N-module M , to prove AI is a strongly prime fuzzy submodule of N-module M. We have to show that At and As are strongly prime submodules of M, for all t, s ∈[0,A(0)] . Let r ∈ N , r ≠ 0 and x ∈ M , rx ∈ At , then A(rx) ≥ t . But by lemma (2.12) A(rx) = sup{t| xy ∈At }= t, thus A(rx) = t implies that rx ∈I . Thus x ∈I (since I is a strongly prime submodule of M, by [27] .

Therefore , A(x) = t , implies that x∈ At .Hence At is a strongly prime submodule of M.

Similarly, As is a strongly prime submodule of M. Hence, AI is a strongly prime fuzzy submodule of N-module M by theorem (3.2) .

Conversely, suppose that AI is a strongly prime fuzzy submodule of N-module M , to prove I is a strongly prime submodule of N-module M. Let r ∈ N , r ≠ 0 and x ∈ M, rx ∈ N, then A(rx) = t implies that rx ∈ At .But A is a strongly prime fuzzy submodule of M, then At is a strongly prime submodule of M for all t ∈ [0,A(0) ] by theorem (3.2) .Thus x ∈ At implies that A(x) ≥ t and according by Lemma (2.12) , A(x) = t .Thus x ∈ I. Hence I is a strongly prime submodule of N-module M.

**REMARK 3 .7 :**

Note that the converse of proposition (3.8) is not true in general , for example :

Let N = Z , M = Q⊕Q , I = Q⊕ <0> ; I is a strongly prime of N-module M .

A: Q⊕ Q →[0,1] such that : It is easily seen that A is a fuzzy submodule of N-module M.. A is a strongly prime fuzzy submodule of N-module M.by proposition (3.7) . But A is not singular fuzzy submodule of Q⊕Q .

**PROPOSITION 3.8 :**

Let N be a torsion integral domain and M be a torsion N-module . If A is a strongly prime fuzzy submodule of N-module M.such that A(0)=1 , then A is the singular fuzzy submodule of N-module M. .

**Proof:**

Let x ∈ M x ≠ 0, then rx =0 ,for some r∈ N ,r ≠ 0 . A(rx )= A(0) =1 . But A(rx) = A(x) for all x ∈ M because A is a strongly prime fuzzy submodule of N-module M. Hence A(x)=1, for all x∈ M .Therefore , A is the singular fuzzy submodule of N-module M.

**THEOREM 3.9 :**

Let N be an integral domain and M be a torsion N-module. If A is a divisible torsion-free fuzzy submodule of N-module M..Then At is a strongly prime submodule of N-module M., for all t ∈(0,A(0)] .

**Proof:**

Since A is a divisible torsion-free fuzzy submodule of N-module M.. Then At is a divisible torsion-free submodule of M., for all t ∈(0,A(0)] by proposition(1.24) and proposition(1.25) .Let x∈ M, r ∈ N and r ≠ 0 such that rx ∈ At .Since At is divisible , then there exists z ∈ At such that rx =rz and hence r(x-z) =0 . But At is torsion –free and r≠0 . Then x-z =0 which implies that x = z and hence x ∈ At . Therefore , At is strongly prime submodule of M by definition (3.1) .

**PROPOSITION 3.10 :**

Let f : M→ M**׳** be an epimorphism of a modules of N ,if A and B are strongly prime fuzzy submodules of M and M׳ respectively **,** then :

1. f (A) is a strongly prime fuzzy submodule of M׳ provided that A is f-invariant .

2. f -1(B) is a strongly prime fuzzy submodule of M .

**Proof:**

1- Since f(A) is a fuzzy submodule of M׳ by proposition (1.18(1)) and f(A) is a prime fuzzy submodule of M׳ by proposition (2.13(1)) .Then we must prove that f(A)(ry) = f(A)(y), for all y ∈ M׳ , r ∈ N , r≠ 0 . f(A)(ry) = f(A)(rf(x)) , since y ∈ M׳ , then there exists x ∈ M =such that y = f(x) and f is an epimorphism . = f(A)(f(rx)) ,[since f is an homomorphism] .

= f-1 (f(A))((rx)) ,by definition of the inverse image .

= A(rx) , by proposition (1.2, (2)) .

= A(x) , ( since A is a strongly prime fuzzy submodule)

= f-1 (f(A))(( x)) , by proposition (1.2, (2))

= f(A)(f(x)) ,by definition of the inverse image .

= f(A)(y) , for all y ∈ M׳ , r ∈ N , r≠ 0.Thus f(A) is a strongly prime fuzzy submodule of N-module M׳ .

2- Since f -1(B) is a fuzzy submodule of M׳ by proposition (1.18(2)) and f -1(B) is a prime fuzzy submodule of M׳ by proposition (2.13(2))Then we must prove that f -1(B)(ra) = f -1(B)(a),

= (B)(rf (a)) ,[since f is an homomorphism] .

= (B)(f (a)) , ( since B is a strongly prime fuzzy submodule)

= f -1(B) (a), by definition of the inverse image , for all a ∈M, r ∈ N, r ≠ 0.

Thus f-1(B) is a strongly prime fuzzy submodule of N-module M .

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