**Design Parameters Effects on the Required Steel Areas of Two-Way Slabs**

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**Abstract**

*The present research shows clearly the effects of yield strength of steel, compressive strength of concrete, thickness of slab and the bar diameter used on the areas (As) of steel required in edge-supported two-way slabs (short and long direction). The analysis was done by computer program constructed by author utilizing the facility of coefficient method to analysis the two-way slabs. A survey analysis and design had been done for many cases of study considering the shear forces effect and deflection limits according to the (ACI 318M-2008) requirements. The results of analysis and design are briefly discussed.* *Among conclusions the results show if (fc') increases from 15MPa to 35MPa the (As) will be decreased about (13%) if () and for any values of other variables,* *also it is not economical to use high strength of concrete when the term () between (0.5-1kN/mm) because the gain in areas of reinforcement will be very small.* *The study proved* *that* *when the thickness (t) increases (50%), the (As) decreases (28% to 32% according to the parameter), while; if the thickness of slab increases (100%), the (As) will decrease (62% to 64%).*

**الخلاصة**

هذا البحث يبين بوضوح تأثيرات مقاومة الخضوع لحديد التسليح، مقاومة الأنضغاط للخرسانة، سمك السقف وقطر حديد التسليح المستخدم على مساحات حديد التسليح *(As)* المطلوبة في السقوف ثنائية العمل وذات النهايات المستندة (في الأتجاه القصير والطويل). التحليلات تمت بأستخدام برنامج حاسوبي أنشأه المؤلف بالإستفادة من سهولة تحليل السقوف ثنائية العمل. مسح تحليلي وتصميمي تم لعدد من حالات الدراسة مع الأخذ بنظر الأعتبار تأثيرات قوى القص والحدود المسموحة للهطول وفقاً لمتطلبات معهد الخرسانة الأمريكي (ACI 318M-2008). نتائج التحليل والتصميم شرحت بأختصار. من بين الأستنتاجات أظهرت النتائج أن مقاومة أنضغاط الخرسانة *(fc')* اذا ازدادت من  *15MPa* الى *35MPa* فإن مساحات حديد التسليح *(As)* المطلوبة تنخفض بنسبة*(13%)*  اذا كان() ولاي قيم بالنسبة للمتغيرات الأخرى، كذلك فإنه ليس من الأقتصادي إستخدام خرسانة ذات مقاومة عالية اذا كان () ما بين (*0.5-1kN/mm*) لأن الربح في مساحات حديد التسليح يكون صغيراً جدا. الدراسة أثبتت انخفاض كمية الحديد المطلوبة *(As)* بنسبة تتراوح ما بين (*28%* الى *32%* وفقاًللمعامل) اذا ازداد سمك السقف (*t*) *(50%)*، بينما تنخفض كمية الحديد المطلوبة *(As)* بنسبة تتراوح ما بين (*62%* الى *64%*) اذا ازداد سمك السقف (*100%*).

**1- Introduction**

The investigation of design parameters effects on the required steel area is very important to show the affectivity of each parameter clearly that help the designers to achieve economical design and evaluate the deviation when occurs due to the following common practical reasons;

1. The differences between the theoretical and practical values of materials properties, according to the practical situation of Iraq usually (*fc'*) obtained in site less than the theoretical value which depends in design.
2. Bad performance or bad quality control leads to change the thickness of the slab, also some times, the size or movement of formwork leads to this problem.
3. Changing the type of steel bars, because the required size or type not available in site, produces another effective depths.

All these causes and others fixed the requirement to study these effects carefully.

 The continued use of method three is permissible under the current code provision that a slab system may be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if it is shown that the design strength at every section is at least equal to the required strength and the serviceability requirements are met [[ACI-Code, 2008], [Amer et al., 2008] and [Arthur and George, 1986]]. It is important to know that the type of analysis method does not change the results of this research, because it is concerned with computation of external moment, while the design parameters effects on internal moments (whatever the magnitude of external moment).

**2- Scope of Analysis**

The analysis done for many common cases, is summarized as below:

1. The length of slab (L) changed from 5m to 10m, and the step of increment was 1m (i.e. six lengths considered).
2. The width of slab (W) changed from 0.5L to L, and the step of increment was 0.05L (i.e. eleven widths considered for each length).
3. Cases, all cases considered for each dimension of slab, LW, (nine casesfor each slab, [Arthur and George, 1986] and [Syal and Goel, 2004]).

1. Four commonly group loading considered [Amer et al., 2008], [Syal and Goel, 2004], [Arthur et al., 2004] and [Mekdam, 1985]]as follows:
2. Wds = 2.0 kN/m2 and WL = 5.0 kN/m2, floor.
3. Wds = 2.0 kN/m2 and WL = 3.0 kN/m2, floor.
4. Wds = 2.0 kN/m2 and WL = 2.0 kN/m2, floor.
5. Wds = 3.6 kN/m2 and WL = 2.0 kN/m2, roof.

Thus the performed cases were (6 × 11 × 9 × 4 = 2376), in these cases the value of *fc' =21MPa and fy = 400MPa* because these values represent the common materials properties in Iraqi sites. Also, additional cases of changing the design parameters investigated.

**3- Governing Equation**

The general strength equation for slabs is [ACI-Code, 2008];

 …………(1)

when solving this equation for area of steel (As) gives:

 …………(2)

 ∅ = 0.9 , if , (ACI-Code, 9.3.2.2) …………(3)

 …………(4)

 …………(5)

All of survey analyses were done (illustrated previously), proved that the capacity reduction factor (∅) always equal to 0.9, i.e., thus Eq.1 will be rewritten as:

 …………(6)

As shown above the area of steel affected by many variables. To appear the effects of these variables, each variable will be discussed individually.

**4- Effect of Yield Strength of Steel (fy) on the Required Steel Area (As)**

For a certain external ultimate moment, effective depth and compressive strength of concrete, the relationship between area of steel and yield strength of steel will be:

 …………(7)

Where, C1 = constant value depends on, *Mu, d,* *fc', b=1* meter.

As shown in Eq.7, when (***fy***) increases, (***As***) decreases but the product of (***As***) and (***fy***) is still constant, Fig.(1) shows this relation but the magnitude of (***fy***) equals to (400 MPa)[[1]](#footnote-1)◊ selected to be the reference value which gives (***As***) 100%. Hence the effect of (***fy***) on the required steel area is unrelated with other variables.

***Yield strength of steel*** (***fy***) ***MPa***

***%(As)/(As when fy=400MPa)***

**Fig.(1):** Effect of yield strength on required steel area.

**5- Effect of Compressive Strength of Concrete (fc') on the Required Steel Area (As)**

Equation (6) can be written as:

 …………(8)

The relation between (***fc'***) and (***As***) is not direct and simple as between (***fy***) and (***As***), i.e., the effect of (***fc'***) on (***As***) related by the magnitude of (/), (as well as the direct effect of (***fc'***) and (***As***)). Therefore, by the computer program presented by the author, the effect of (***fc'***) on the required steel area investigated for all possible value of (/), as illustrated in Fig.(2).

The area of required steel (***As***) when ***fc'***[[2]](#footnote-2)+=21 MPa adopted as a reference value for all values of(/). As shown when ***fc'*** > 21 MPa the required area decreased (smaller than 100%) and vice verse, the units of / are (kN/mm).

***Compressive strength of concrete*** (***fc'***) ***MPa***

***%(As)/(As when fc'=21MPa)***

**Fig.(2):** Effect of (***fc'***) on the required steel area (***As***).

**6- Effect of Thickness of Slab (*t*) on the Required Steel Area (*As*)**

 The relation between the thickness of slab and the effective depth is:

 , for short direction …………(9)

 , for long direction ..………(10)

If the diameter of bar db=12 mm[[3]](#footnote-3)♦, cover = 20 mm, thus ds = t-26, dl = t-38, and Eq.(8) becomes:

 ..………(11)

 ..………(12)

Here, there is a difference between the effect of thickness on the area of required steel in short direction (*Asw*) than on the area of required steel in long direction (*Asl*) because the appearance of diameter bar (*db*). The effect of thickness on the area of steel is also affected by (/) as well as the (*db*), Fig.(3) and Fig.(4) show the effects of (*t*) on (*As*w) and (*Asl*) respectively for all possible values of (/), when (*db*=12 mm) is chosen. The difference between effect of (*t*) on (*As*w) and (*Asl*) is about (3%) as the results of investigation shows (for the same value of/). If *db*=16 mm the difference reach (5%). The required steel area when the thickness is equal to (100 mm) adopted as a reference value. As evident in Fig.(3) and Fig.(4), when (t) increases, the required area will decrease (decreases about 70% when *t* increases from 100 mm to 250 mm).

It is important to note that when the thickness of slab changes to be smaller than required in design (for example from 20cm to 15cm Figs.(3and 4)), it is still applicable but the serviceability requirements may be not satisfied.

***%(Asw)/(Asw when t= 100 mm)***

/

***(Thickness of slab, mm)***

**Fig.(3): Effect of thickness (t) on required steel area in short direction (Asw).**

***(Thickness of slab, mm)***

***%(Asl)/(Asl when t= 100 mm)***

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**Fig.(4):** Effect of thickness (t) on required steel area in long direction (Asl).

**7- Effect of Bar Diameter on the Required Steel Area (As)**

The bar diameter effect is more complex than others, because it is effected by thickness of slab, compressive strength of concrete and the ultimate moment. Therefore, a survey analysis had been done by the presented computer program and the results re-arranged in Table (1). The required steel area equal one unit whenever diameter bar is equal to (8 mm), when the bar diameter increases (for the same thickness of slab), the required steel area will increase because the effective depth of slab decreases. Because the effective depth in long direction is smaller than that one in short direction the additional steel area in long direction (range) when bar diameter increases to be larger than in short direction as shown in Table (1).This effect changes rapidly when the thickness of slab changes (i.e. more sensitive when changing the slab thickness), therefore, the analysis done for each thickness as shown in Table (1). When the thickness of slab increases, the effect of changing diameter bar decreases because the changing in effective depth due to changing in diameter of bar will be smaller, for example: for long direction; when t =10 cm, and using db=12 mm the area of steel needed equals (1.085-1.150) multiplied by the area required if db=8 mm [i.e. (8.5-15)% additional area must be provided when used db=12 mm], while when t = 20 cm, and using db=12 mm other than db=8 mm, the range of additional steel area in long direction is about (3.0-6.0)% as shown in Table (1).

**Table (1):** Effect Ranges of Changing Diameter of Bar on the Area of Reinforcement Required in Two-Way Slabs.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *db*(mm) | 8 | 10 | 12 | 16 |
| **t = 100** mm | ***Asw*** | 1 | 1.010-1.022 | 1.021-1.045 | 1.045-1.100 |
| ***Asl*** | 1 | 1.035-1.070 | 1.085-1.150 | 1.200-1.350 |
| **t = 125** mm | ***Asw*** | 1 | 1.005-1.017 | 1.015-1.035 | 1.030-1.080 |
| ***Asl*** | 1 | 1.020-1.060 | 1.050-1.110 | 1.135-1.250 |
| **t = 150** mm | ***Asw*** | 1 | 1.004-1.015 | 1.012-1.028 | 1.028-1.060 |
| ***Asl*** | 1 | 1.015-1.045 | 1.043-1.095 | 1.100-1.200 |
| **t = 175**  mm | ***Asw*** | 1 | 1.003-1.013 | 1.009-1.024 | 1.023-1.055 |
| ***Asl*** | 1 | 1.012-1.035 | 1.020-1.075 | 1.065-1.160 |
| **t = 200** mm | ***Asw*** | 1 | 1.003-1.011 | 1.008-1.021 | 1.020-1.045 |
| ***Asl*** | 1 | 1.013-1.030 | 1.030-1.060 | 1.068-1.140 |

**8- Deflection Control**

The [ACI-Code, 2008], section 9.5.3, specifies a minimum slab thickness in two-way slabs to control deflection, this specification, as illustrated in references [[Arthur and George, 1986], [Arthur et al., 2004] and [Hassoun, 1985]], depends on properties of beams around the slab, thus not depended here. Alternative to the use of minimum slab thickness equations, the deflection at the center of a slab panel can be calculated, and results compared against limitations of ACI-Code, Table 9.5 (b), as explained in [Arthur and George, 1986]. The last technique adopted in this research with the critical case of occupation considered, which assumed the slab supporting or attached to nonstructural elements likely to be damaged by large deflections.

**9- Illustration Example**

Design a two-way slab has clear dimensions (10m length and 10m width), the superimposed dead load Wds = 2 kN/m2, and live load Wl = 5 kN/m2. The slab supporting to nonstructural elements is likely to be damaged by large deflections. = 400MPa, = 21MPa and the diameter of bars (available in site) db = 12mm. The slab location represents case six. (Note: the dimensions and loads of this example represent the critical among all studied cases).

**Solution**

*Compute the ultimate load and effective depths: (assume t=230mm);*

 Wu = 1.2 (2 + 24.5 0.23) +1.6 5 thus; Wu = 9.162 + 8 = 17.162 kN/m2

ds = t – cover – db/2 = 230 – 20 – 12/2 = 204 mm

dl = ds – db = 204 – 12 = 192 mm.

*Determine the coefficients:* Case (6), m = 10/10 = 1 thus from tables 8.3 to 8.6 respectively [Arthur and George, 1986], the coefficients are as follows: = 0.071, = 0.0, = 0.033, = 0.027, = 0.035, = 0.032 and the ratio of load in short directions = 0.71.

*Compute the moments:*

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*Calculate the ratios of reinforcements*; by solving the Eq.1 gives:

 , (at first assume, i.e. assume all ratios of reinforcements smaller than ), thus: = 0.0090528, = 0.0040720,= 0.0 and= 0.0039687, now compute which equal to , [ACI-Code, 2008], hence 0.0142242 larger than all ratios, **thus always**. The areas of reinforcements will be:

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Compute the minimum area of reinforcement must be provided for temperature and shrinkage, ,

*Check shear requirements*; the ratio of load in short direction represents the critical and equals to 71%, thus the maximum reaction along the long direction is equal to 17.162 0.71 10 0.5 = 60.925 kN, thus the shear at the critical section *Vud* = 60.925 – 17.162 0.71 0.204 = 58.44 kN, , which, ∴ o.k..

*Check the deflection requirements*; the deflection due to sustain dead load can be computed as (computed for long and short directions then take the average):

 , where; = the positive dead load moment of long or short direction, taken here as the gross moment of inertia because the amount of cracking at service load is usually not extensive [Arthur and George, 1986],

,

,

=21538MPa,/12=1013916667,thus

The deflection due to live load can be computed as (computed for long and short directions then take the average):

, = the positive live load moment of long or short direction,

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 Total deflection DLT= × T + × 1.5 + 7.19 = 17.03mm

 The limit of deflection DefL = , [ACI-Code, 2008], thus , is larger than the total deflection, hence ACI Code requirements for deflection will be satisfied.

1. ***Effect of changing from 400MPa to 480MPa ( i.e. multiplied by 1.2) :***

a-1- by Fig.(1), depending on the moments of original case, areas of steels required due to this changing = 0.833× areas of steels required when **=** 400MPa, thus

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a-2 by hand calculations: the moments do not change, thus applications of the equation of mentioned previously;

, and, sequent:

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**- *Effect of changing from 21MPa to 35MPa:***

b-1- by Fig.(2), take the same moments of original case =21MPa, compute the ratio of area of steel required with respect to that one when =21MPa:

 2.93 (kN/mm) for obtained ratio = 95.3 % gives, .953×= 1759.98 .

 1.40 (kN/mm) for obtained ratio = 98.2 % gives, .982×= 815.74 .

 1.40 (kN/mm) for obtained ratio = 98.2 % gives, .982×= 748.26 .

b-2- by hand calculations: the moments do not change, thus applications the equation of mentioned previously;

, and, sequent:

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**- *Effect of changing from 23cm to 20cm:***

c-1- by Figs.(3,4), take the same moments of original case t = 23 cm, compute the ratio of area of required steel with respect to that one when t = 10 cm and when t = 20 cm as follows:

for , by Fig.(3):

=5.80 (kN.m / MPa), leads to ratios =31% and 36% (at t=23cm, t=20cm respectively), thus (0.360/0.310)×= 2144.65 .

for , by Fig.(3):

=2.77 (kN.m / MPa), leads to ratios =32.5% and 37.5% (at t=23cm, t=20cm respectively), thus (0.375/0.327)×= 952.63 .

for , by Fig.(4):

=2.40 (kN.m / MPa), leads to ratios =33% and 38.5% (at t=23cm, t=20cm respectively), thus (0.383/0.332)×= 879.03 .

c-2- by hand calculations: here the moments change, therefore, they must be computed:

Wu = 1.2 (2 + 24.5 0.20) +1.6 5 thus; Wu = 8.28 + 8 = 16.28 kN/m2

ds = 200 – 20 – 12/2 = 174 mm, dl = ds – db = 174 – 12 = 162 mm.

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, and, sequent:

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**- *Effect of changing from 12mm to 16mm (let t =20 cm as c-2 above):***

d-1- from Table (1) , t = 20 cm the range for long direction equals (1.030-1.060) and (1.068-1.140) for db = 12mm and db = 16 mm respectively with respect to the results of db = 8mm. Thus when changing db from 12mm to 16mm, the range will be 1.068/1.030 to 1.140/1.060 or (1.036-1.075), i.e. **3.6%** to **7.5%** additional area required in long direction to complete the defect of changing diameter bar. In short direction the range will be (1.020/1.008) to (1.045/1.021), i.e. **1.2%** to **2.4%**

d-2- by hand calculations: the moments do not change, but the effective depths changed: ds = 200 – 20 – 16/2 = 172 mm, dl = ds – db = 172 – 16 = 156 mm. Now, computs the ratios of reinforcement:

, and, sequent:

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. Thus the increments:

( for between 1.2% to 2.4%, ∴ o.k..

( for between 1.2% to 2.4%, ∴ o.k..

( for between 3.6% to 7.5%, ∴ o.k..

**10- Conclusions**

Based on the results obtained from several test examples done the following conclusions can be drawn. These may be summarized as follows:

1. The yield strength of steel () has direct effect on the area of reinforcement required (***As***) (not related with other variables). For example if the () increase 50%. (i.e. multiplied by 1.5), the required area of reinforcement in two directions will be divided by 1.5.

1. The very important result is that, the effect of compressive strength of concrete (***fc'***) on the (***As***) depending on the magnitude of the term *()*. If *()* equals about (3.5 kN/mm) and the (***fc'***) increases from (15 MPa) to (35 MPa) the (***As***) will be decreased about (13%) for any values of other variables, while if *()* equals about (0.5 kN/mm), the (***As***) will be decreased about (1%). Hence, it is not economical to use high strength of concrete when the term *()* between (0.5-1 kN/mm) because the gain in reinforcement will be very small, unless higher durability required.

1. When the thickness of slab (***t***) increases (50%) the (***As)*** decreases (28% to 32% according to). While if (***t***) increases (100%), the (***As***) decreases (62% to 64%).

1. For economical reasons, the designers must select the smallest bar diameter (as possible), to decrease the (***As***). For example if (***t*** = 17.5 cm) and the engineer used (db=16 mm) rather than (db=12 mm), then additional area of steel will be needed and as follows: for Asw (1.39%-3.03%)\* and Asl (4.41%-7.91%).
2. Though the results analysis depends on moments of two-way slabs, the curves of effects of properties and other design parameters (of short direction) can be used for one-way slabs because its strength equation coincide with Eq.1, but the serviceability requirement must be checked.

**Appendix 1: References**

ACI-Code (318M-08), 2008, “Building Code Requirement for Structural Concrete”, Provided by IHS under license with ACI.

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\* From Table (1): 1.023/1.009= 1.39% additional area required when using db=16 mm instead of db=12 mm.

***Appendix 2- Notation***

|  |  |
| --- | --- |
|  | = factor relating depth of equivalent rectangular compressive stress block to neutral axis depth. |
|  | = effective moment of inertia of the concrete cross section of unit width, mm4. |
|  | = 24.5 kN/m3. |
|  | = net tensile strain in extreme layer of longitudinal tension steel at nominal strength. |
|  | = ratio of negative reinforcement for long direction = ASnl/bdl. |
|  | = ratio of negative reinforcement for short direction = ASnw/bds. |
|  | = ratio of positive reinforcement for long direction = ASpl/bdl. |
|  | = ratio of positive reinforcement for short direction = ASpw/bds. |
|  | 2.55 \* \* / (8 \*) [ACI-Code, 2008]. |
| As | = area of reinforcement required in short and long direction, mm2/m. |
| Asl | = area of reinforcement required in long direction, mm2/m. |
| ASnl | = negative area of reinforcement required in long direction, mm2/m. |
| ASnw | = negative area of reinforcement required in short direction, mm2/m. |
| ASpl | = positive area of reinforcement required in long direction, mm2/m. |
| ASpw | = positive area of reinforcement required in short direction, mm2/m. |
| Asw | = area of reinforcement required in short direction, mm2/m. |
| b | = width of the section, usually taken (1000 mm) in slabs. |
| C | = case of the slab. |
| d | = effective depth of slab, m. |
| db | = diameter of bars used in site, mm. |
| DefL | = permissible deflection limit [ACI-Code, 2008], (Table 9.5.b), mm. |
| dl | = effective depth of slab in long direction, mm. |
| DLT | = computed deflection, mm. |
| ds | = effective depth of slab in short direction (mm). |
| Ec | = modulus elasticity of concrete, MPa. |
| fc' | = compressive strength of concrete (cylinder), MPa. |
| fy | = yield strength of steel, MPa. |
| L ,  | = length of the slab (clear), m. |
| Mnl | = negative moment of long direction, N.mm. |
| Mnw | = negative moment of short direction, N.mm. |
| Mpl | = positive moment of long direction, N.mm. |
| Mpw | = positive moment of short direction, N.mm. |
| Mu | = factored ultimate moment, MN.m. |
| t | = thickness of slab, mm. |
| T | = time dependent multiplier. |
| Vc | = nominal shear strength provided by concrete section, kN. |
| Vul | = shear force at the critical section of long direction, kN. |
| Vuw | = shear force at the critical section of short direction, kN. |
| W,  | = width of the slab (clear), m. |
| Wds | = dead load (superimposed), kN/m2. |
| Wl | = live load (superimposed), kN/m2. |
|  |  |

1. ◊ The available steel in Iraq having usually fy ≈ 400 MPa. [↑](#footnote-ref-1)
2. + The value of fc' depended in common designs in Iraq is equal to (21 MPa). [↑](#footnote-ref-2)
3. ♦ Usually 10 mm and 12 mm are used in slabs reinforcement. [↑](#footnote-ref-3)