University of Babylon College of Engineering Mechanical Engineering Dept.		Subject : Mathematics 4 Class: 2^{nd} Date: / / 2016

Formula Sheet Math Multi-Variable Calculus

1. length of a vector in Space $|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

2. 2 dimensional dot product $u \cdot v = u_1 v_1 + u_2 v_2$

3. 3 dimensional dot product $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$

4. Angle between two vectors $\cos \theta = \dfrac{u \cdot v}{|u|\,|v|}$

5. Cross product $u \times v = (u_2 v_3 - u_3 v_2)i - (u_1 v_3 - u_3 v_1)j + (u_1 v_2 - u_2 v_1)k$

6. parametric form equations of a line in space

$$x = x_1 + at$$

$$y = y_1 + bt$$

$$z = z_1 + ct$$

7. symmetric form of the equations of a line in space

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

8 Standard equation of a plane in Space

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

9. general form of the equation of a plane in Space $ax + by + cz + d = 0$

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10. **cylindrical to Cartesian (rectangular):**

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

11. **Cartesian (rectangular) to cylindrical**

$$r^2 = x^2 + y^2 \qquad \tan\theta = y/x \qquad z=z$$

Spherical coordinates:

- Spherical to Cartesian

$$x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi,$$

- Cartesian to Spherical

$$\rho = \sqrt{x^2+y^2+z^2}, \quad \tan\theta = y/x, \quad \cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}.$$

14. total differential: $dw = \frac{\delta w}{\delta x}dx + \frac{\delta w}{\delta y}dy + \frac{\delta w}{\delta z}dz + \frac{\delta w}{\delta u}du$

15 Chain rule one independent variable $\frac{dw}{dt} = \frac{\delta w}{\delta x}\frac{\delta x}{\delta t} + \frac{\delta w}{\delta y}\frac{\delta y}{\delta t}$

16. Chain rule two independent variables

$\frac{\delta w}{\delta s} = \frac{\delta w}{\delta x}\frac{\delta x}{\delta s} + \frac{\delta w}{\delta y}\frac{\delta y}{\delta s}$ **and** $\frac{\delta w}{\delta t} = \frac{\delta w}{\delta x}\frac{\delta x}{\delta t} + \frac{\delta w}{\delta y}\frac{\delta y}{\delta t}$

17. Chain rule Implicit differentiation

$\frac{dy}{dx} = \frac{-F_x(x,y)}{F_y(x,y)}$ $\qquad F_y(x,y) \neq 0$

18. Chain rule Implicit differentiation

$$\frac{dz}{dx} = \frac{-F_x(x,y,z)}{F_z(x,y,z)} \qquad \frac{dz}{dy} = \frac{-F_y(x,y,z)}{F_z(x,y,z)} \qquad F_z(x,y,z) \neq 0$$

19. Directional Derivative

For unit vector $u = \cos \theta\, i + \sin \theta\, j$

$$D_u f(x,y) = f_x(x,y) \cos \theta + f_y(x,y) \sin \theta$$

20. Gradient of f

$$\nabla f(x,y) = f_x(x,y) i + fy(x,y) j$$

21. Second Partials Test

f must have continuous second derivatives on an open region containing point (a,b) for which

$f_x(a,b) = 0 \quad f_y(a,b) = 0$

To test for extrema consider the quantity:

$$D = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

1. if $d > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a,b)
2. if $d > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a,b)
3. if $d < 0$ then $(a,b,f(a,b))$ is a saddle point.
4. the test is inconclusive if $d = 0$

22. Ellipse $x^2/a^2 + y^2/b^2 = 1$

23. Ellipsoid $x^2/a2 + y^2/b^2 + z^2/c^2 = 1$

24. **Hyperbola $x^2/a^2 - y^2/b^2 = 1$**

25. **Hyperboloid of one sheet $x^2/a^2 + y^2/b^2 - z^2/c^2 = 1$**

26. **Hyperboloid of two sheets $x^2/a^2 - y^2/b^2 - z^2/c^2 = 1$**

27. **Elliptic cone $x^2/a^2 + y^2/b^2 - z^2/c^2 = 0$**

28. **Elliptic Paraboloid $z = x^2/a^2 + y^2/b^2$**

29. **Hyperbolic paraboloid $z = y^2/b^2 - x^2/a^2$**

30. If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then the projection of $\mathbf{u}$ onto $\mathbf{v}$ is given by

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}.$$

31. The distance between a plane and a point Q (not in the plane) is

$$D = \|\text{proj}_{\mathbf{n}}\overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

where P is a point in the plane and $\mathbf{n}$ is normal to the plane.

32. The distance between a point Q and a line in space is given by

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

where $\mathbf{u}$ is a direction vector for the line and P is a point on the line.

33. **Using gradients to compute directional derivatives:** $D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$

THEOREM 13.18 Least Squares Regression Line

The **least squares regression line** for $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ is given by $f(x) = ax + b$, where

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \quad \text{and} \quad b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right).$$

35. LaGrange's Theorem

$$\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$$