Alternating voltage and current

The AC generator principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). Induced e.m.f is produced in it according to Faraday's law of electromagnetic induction. This e.m.f cause a current to flow if the conductor circuit is closed.

Hence, two basic essential parts of an electrical generator are:

a) Magnetic field.

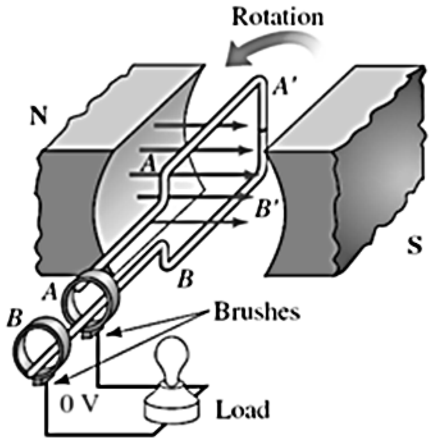
b) Conductor or conductors which can move as to cut the flux.

Generators are driven by a source of mechanical power, which is usually called the prime mover of the generator(steam turbine, diesel engine, or even an electric motor).

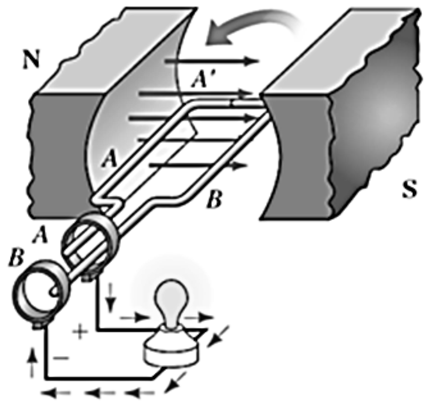
Simple loop generator

In figure is shown a single turn rectangular copper coil ($AA'BB'$) rotating about its own axis in a magnetic field provided by either permanent magnets or electromagnets. The two end of the coil are joined to two slip-rings which are insulated from each other and from the central shaft. Two collecting brushes (carbon or copper) press against the slip-rings. The rotating coil may be called (armature) and the magnets as (field magnets).

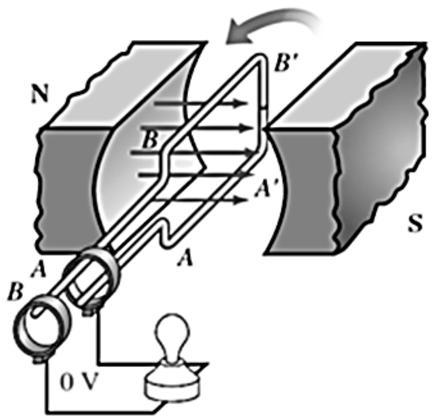
One way to generate an AC voltage is to rotate a coil of wire at constant angular velocity in a fixed magnetic field, slip rings and brushes connect the coil to the load. The magnitude of the resulting voltage is proportional to the rate at which flux lines are cut (faraday's law), and its polarity is dependent on the direction the coil sides move through the field.



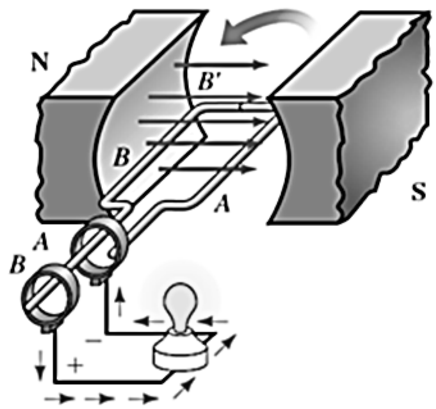
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



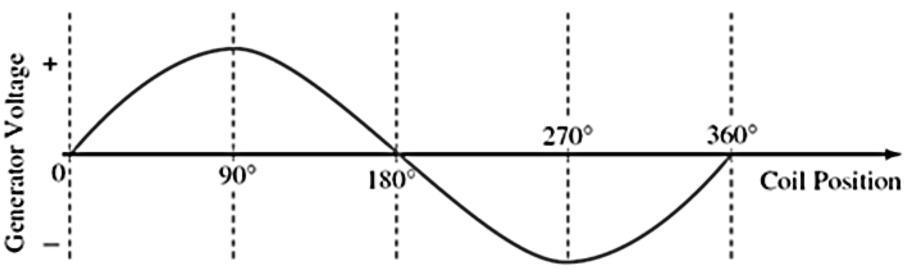
(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.



(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.



(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.



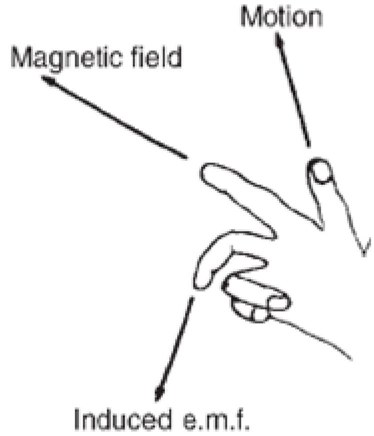
Coil voltage versus angular position.

The direction of an induced e.m.f can be predetermined by using Flemings Right-hand rule (often called the geneRator rule) .

First finger- Field

ThuMb – Motion

sEcond finger – E.m.f



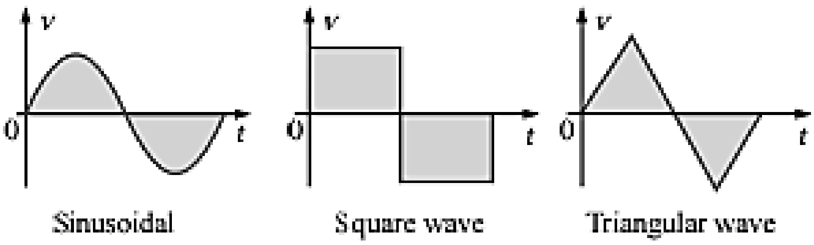
Since the rate of cutting flux varies with time, the resulting voltage will also vary with time.

. E.m.f. generated in one side of loop= $Blv \cdot \sin\phi$**, and total e.m.f. generated in loop=** $2 \times Blv \cdot \sin\phi$ **(volts), where**

(B): flux density in (teslas), (l**): length in (meters), (**v**): the conductor velocity, is measured in meters per second.**

Electronic signal generators (Function generators):

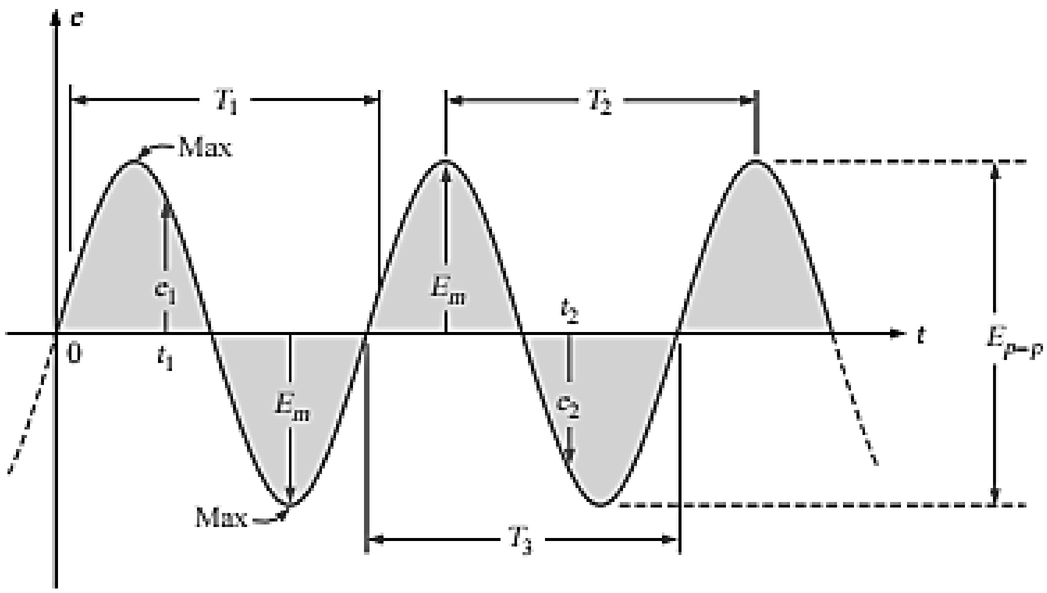
AC waveforms may also be created electronically using signal generators. The signal generators can produce a variety of variable frequency waveforms, including sinusoidal, square wave, triangular, and so on.



The wide range of theorems and methods introduced for DC networks will also be applied to sinusoidal AC systems.

A sinusoidal is a signal that has the form of the *sine* or *cosine* function

The sinusoidal waveform as shown in figure.



Definitions:

Waveform: The path traced by a quantity, such as the voltage in Figure, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform *at any instant* of time; denoted by lowercase letters (e_1, e_2).

Peak amplitude: The *maximum value* of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform as shown in figure, *the average value is zero volts*.

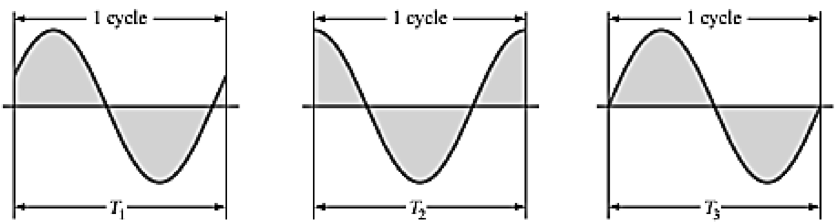
<u>Peak value:</u> The maximum instantaneous value of a function as measured from the *zero-volt* level. For the waveform as shown in figure.

<u>Peak-to-peak value</u>: Denoted by E_{p-p} or V_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks. Similarly, peak-to peak currents are denoted as I_{P-P}.

<u>Periodic waveform:</u> A waveform that continually repeats itself after the same time interval. The waveform as shown in figure, is a periodic waveform.

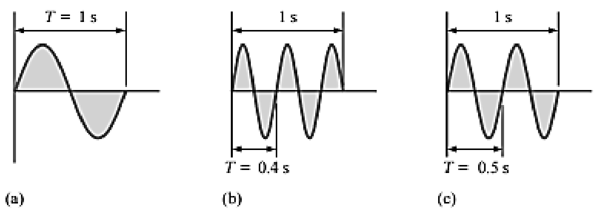
<u>Period (T):</u> The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Figure), as long as successive similar points of the periodic waveform are used in determining,(T). it is the inverse of frequency.

<u>Cycle:</u> The portion of a waveform contained in one period of time. The cycles within T1, T2, and T3 of figure.



Defining the cycle and period of a sinusoidal waveform.

<u>Frequency (f</u>): The number of cycles that occur in 1 s. The unit of measure for frequency is the hertz (Hz),



Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

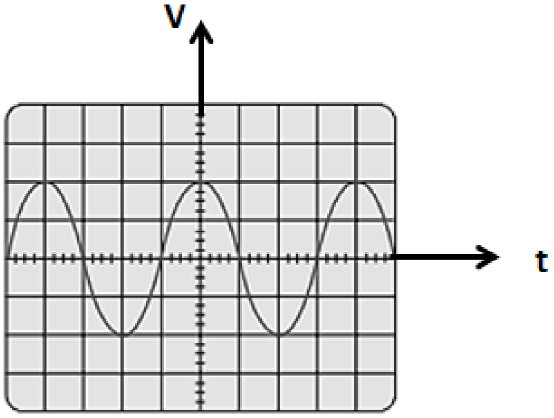
Since the frequency is inversely related to the period.

$$f = \frac{1}{T}$$

<u>Example:</u> The oscilloscope is an instrument that will display alternating waveforms as shown. Determine the period, frequency, and peak value of the waveform.

Vertical sensitivity =0.1V/div

Horizontal sensitivity =50µ/div



Sol/

One cycle spans 4 divisions. The period is therefore

$$\mathbf{T = 4div\left(50\,\frac{\mu s}{div}\right) = 200\mu s}$$

and the frequency is

$$\mathbf{f = \frac{1}{T} = \frac{1}{200 \times 10^{-6}} = 5kHz}$$

The vertical height above the horizontal axis encompasses 2 divisions.

Therefore,

$$\boldsymbol{V_m = 2div \times \left(\frac{0.1V}{div}\right) = 0.2V}$$

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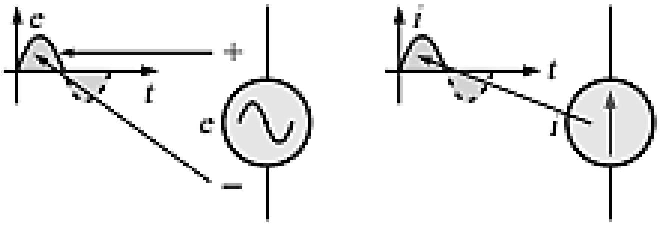
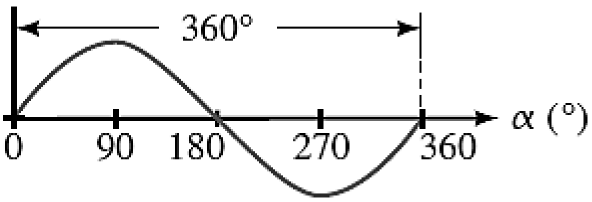
Defined Polarities and Direction: The polarity and current direction will be for an instant of time in the positive portion of the sinusoidal waveform. This is shown in Figure below with the symbols for the sinusoidal AC voltage and current.



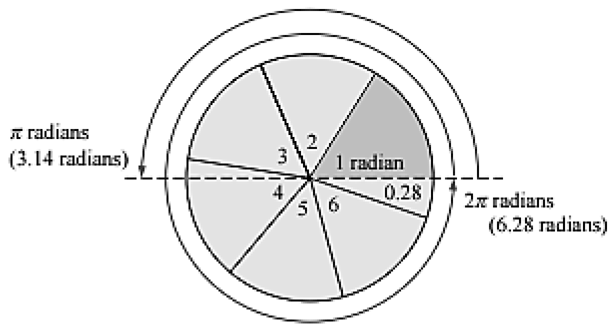
THE SINE WAVE: The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.

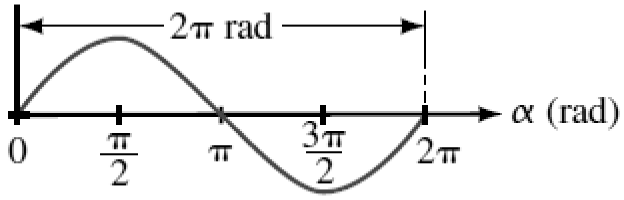
The unit of measurement for the horizontal axis scaled is the degree360^0. A second unit of measurement frequently used is the radian (rad)2π. A third unit of measurement used time , however, if scaled in terms of period T.

$$2\pi(\text{rad}) = 360^0(degree)$$

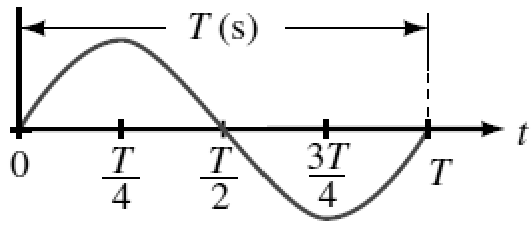


(a) Degrees





(b) Radians



(c) Period

The velocity with which the radius vector rotates about the center، called the <u>angular velocity</u>, can be determined from the following equation.

$$\text{Angular velocity}(\omega) = \frac{distance\ (\alpha)(degree\ or\ radians)}{Time\ (t)(seconds)}$$

In Figure below (i), the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform. The radians subtended in this time interval are 2π. Substituting, we have,

$\omega = \frac{2\pi}{T}$ $\qquad$ $\omega = 2\pi f$ $\qquad$ **(rad/s)**

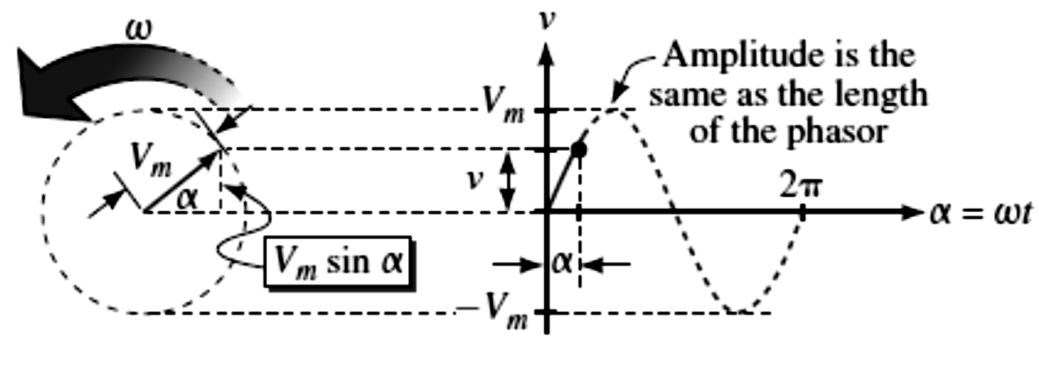
$$\alpha_{radians} = \alpha_{degrees} \times \frac{\pi}{180^0}$$

$$\alpha_{degree} = \alpha_{radians} \times \frac{180^0}{\pi}$$

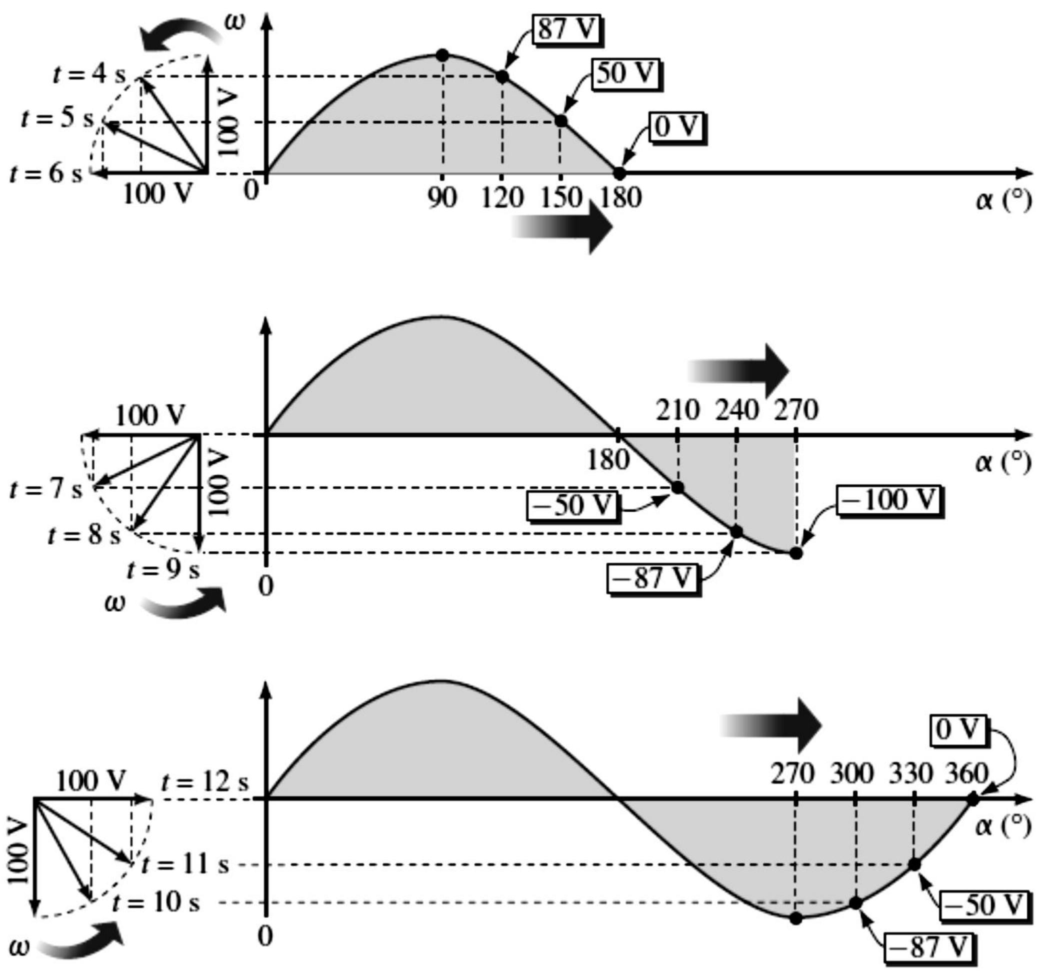
(Normally angular velocity is expressed in radian per second instead of degrees per second. We will make this change shortly).

<u>Introduction to Phasors</u>

A phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities. To get at the idea, consider the line of length V_m shown in Figure (It is the phasor.). The vertical projection of this line (indicated in dotted line) is $V_m sin\alpha$. Now, assume that the phasor rotates at angular velocity of ωrad/s in the counterclockwise direction.



(a) Phasor (b) Resulting sine wave



<u>Note that:</u> If the phasor has a length of V_m the waveform represents voltage. If the phasor has a length of I_m it represents current. Note carefully: phasor apply only to sinusoidal waveforms.

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT:

The basic mathematical format for the sinusoidal waveform is

$$\mathbf{A_m \sin(\alpha)}$$

Where:

A_m: Is the peak value of the waveform

α : Is the unit of measure for the horizontal axis.

$\alpha = \omega t$

For electrical quantities such as current and voltage, the general format is

$i(t) = I_m \sin(\omega t) = I_m \sin(\alpha)$

$\upsilon(t) = V_m \sin(\omega t) = V_m \sin(\alpha)$

<u>PHASE RELATIONS:</u> If a sine wave does not pass through zero at t=0 as in figure, it has a *phase shift*. The waveform is shifted to the right or left of 0°, the expression becomes.

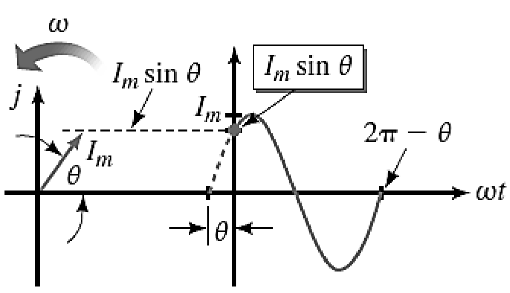
$A_m \sin(\omega t \pm \theta)$

where ϴ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive going(increasing with time) slope <u>before</u> 0°, as shown in Figure, the expression is

$A_m \sin(\omega t + \theta)$

$i(t) = I_m \sin(\omega t + \theta)$

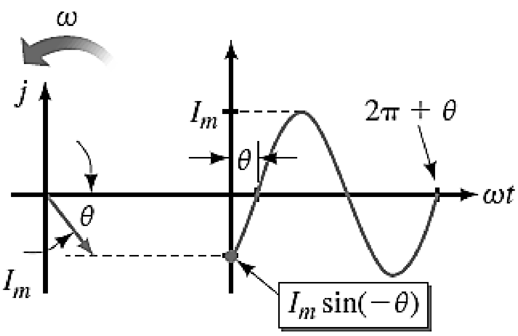


(a) $i = I_m \sin(\omega t + \theta)$

If the waveform passes through the horizontal axis with a positive-going slope <u>after</u> 0°, as shown in Figure, the expression is

$$A_m \sin(\omega t - \theta)$$

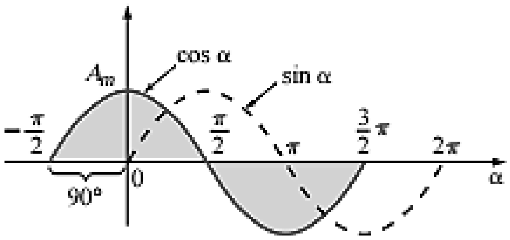
$$i(t) = I_m sin(\omega t - \theta)$$



(b) $i = I_m \sin(\omega t - \theta)$

And at $\omega t = \alpha = 0^0$ the magnitude is $A_m\, sin(-\theta)$, which, by a trigonometric identity, is $-A_m\, sin\, \theta$.

Sometimes voltage and currents are expressed in terms of cos(wt) rather than sin(wt). a cosine wave is a sine wave shifted by 90 degree, or alternatively, a sine wave is cosine wave shifted by 90 degree.

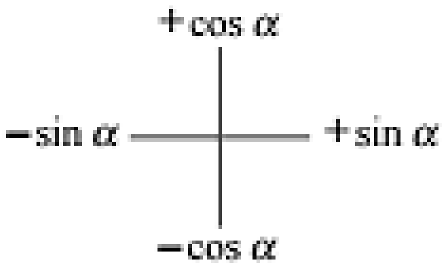


$$\sin(\omega t + 90) = \sin\left(\omega t + \frac{\pi}{2}\right) = cos(\omega t)$$

$$\cos(\omega t - 90) = \cos\left(\omega t - \frac{\pi}{2}\right) = \sin(\omega t)$$

<u>Note that</u>: (Phase difference) refers to the angular displacement between different waveforms of the same frequency. The phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the same slope.

The geometric relationship between various forms of the sine and cosine functions can be derived from Figure below.



$$\begin{aligned} \cos\alpha &= \sin(\alpha + 90°) \\ \sin\alpha &= \cos(\alpha - 90°) \\ -\sin\alpha &= \sin(\alpha \pm 180°) \\ -\cos\alpha &= \sin(\alpha + 270°) = \sin(\alpha - 90°) \\ &\text{etc.} \end{aligned}$$

In addition, one should be aware that

$$\begin{aligned} \sin(-\alpha) &= -\sin\alpha \\ \cos(-\alpha) &= \cos\alpha \end{aligned}$$

The <u>*phase relationship*</u> between two waveforms indicates which one <u>leads</u> or <u>lags</u>, and by how many *degrees or radians*.

Phase difference refers to the angular displacement between different waveforms of the *same frequency*. If the angular displacement is 0^0, the waveforms are said to be <u>*in phase*</u>, otherwise, they are <u>*out of phase*</u>.

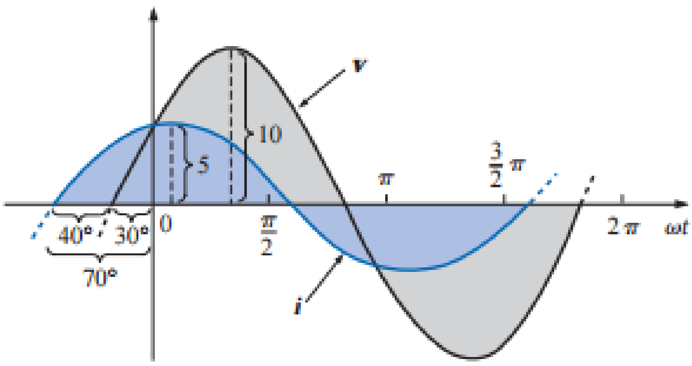
<u>Example:</u> What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a) $v(t) = 10\,sin(\omega t + 30^0)$

$i(t) = 5\,sin(\omega t + 70^0)$

Sol/

i leads v by 40°



b) $i(t) = -2cos\,(\omega t - 60^0)$

$v(t) = 3sin\,(\omega t - 150^0)$

Sol/ If you multiply the waveform by -1, you get the inverted waveform. Note that the phasor is the same as original phasor except that it is rotated by 180 degree.

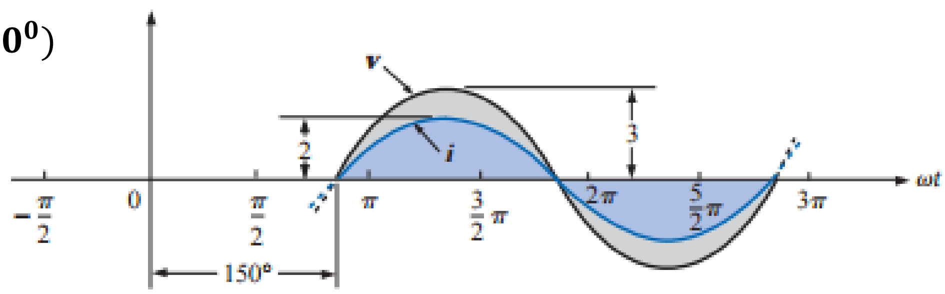
$$i(t) = -2cos(\omega t - 60^0) = 2cos(\omega t - 60^0 - 180^0)$$

$$= 2\cos(\omega t - 240^0)$$

$$\cos(\alpha) = \sin(\alpha + 90^0)$$

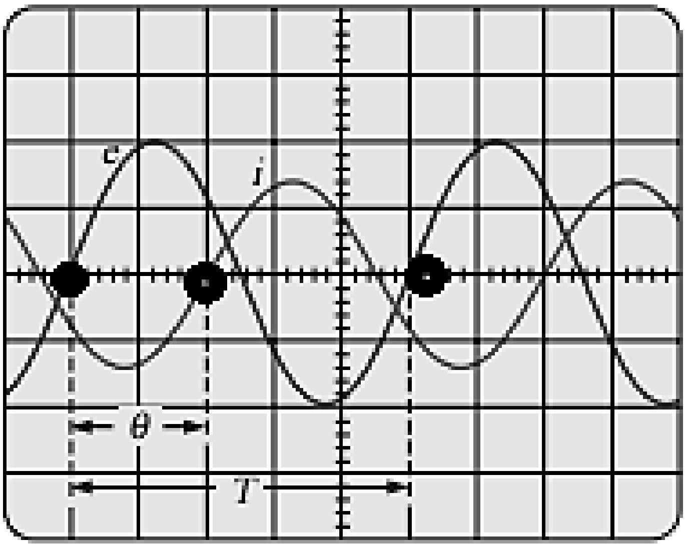
$$i(t) = 2sin(\omega t - 150^0)$$

<u>V and I in phase</u>



Phase Measurements (using a dual-trace oscilloscope):

Using an oscilloscope to measure phase angles, the equation for determining the phase angle can be introduced using Figure.



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

$$\frac{360^0}{T\ (no. of\ div.)} = \frac{\theta}{phase\ shift(no. of\ div.)}$$

$$\boxed{\theta = \frac{\text{phase shift (no. of div.)}}{T\text{(no. of div.)}} \times 360^\circ}$$

Substituting into Eq. will result in

$$\theta = \frac{(2div)}{(5div)} \times 360^0 = 144^0$$

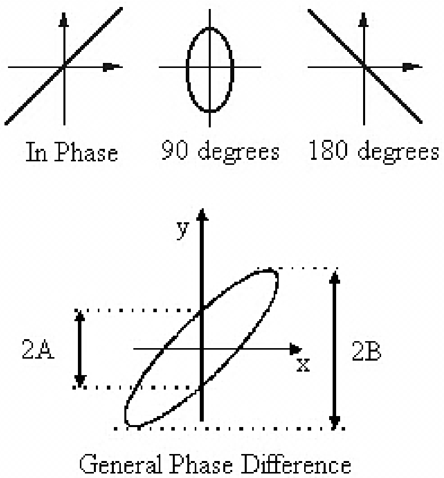
(e) leads (I) by 144^0

> If you have trouble determining which waveform leads and which lags when you are solving a problem, make a quick sketch of their phasors, and answer will be apparent.

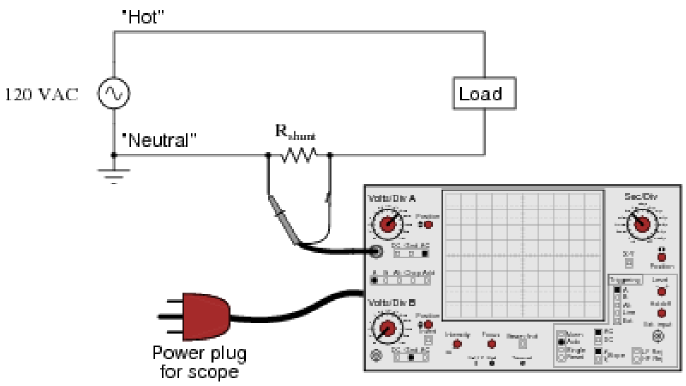
The Lissajous pattern:

The XY display format can be used to measure the phase relationship between two or more synchronous signals. This measurement technique involves inputting one signal into the vertical system as usual and then another signal into the horizontal system- called an XY measurement because both the X and the Y axis are tracing voltages. The waveform that results from this arrangement is called a Lissajous pattern.

XY mode is selected by turning opening the DISPLAY menu and changing *Format* from "YT" to "XY". In this mode the scope shows a single trace generated by plotting the channel 1 voltage on the horizontal axis against channel 2 on the vertical axis. The VOLTS/DIV and position controls for channels 1 and 2 still work in exactly the same way, except the channel 1 controls now determine the horizontal axis scale and position.



$$\sin \Theta = \frac{A}{B}$$



Oscilloscopes measure voltage. Current can be measurement by oscilloscopes:

1) By a current-to voltage converter. One type of converter is clip-on device, known as a current gun (it work only with AC).

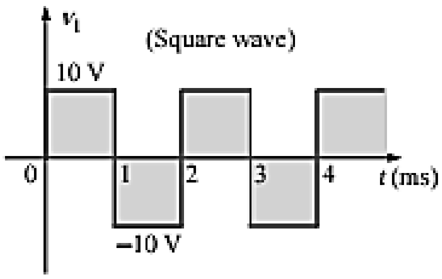
2) By put a small resistor in the current path, measure voltage across it with the oscilloscope, then use ohm's law to determine the current.

AVERAGE VALUE:

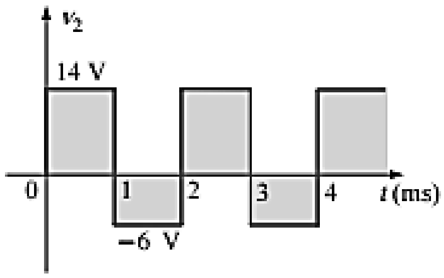
The average value of any current or voltage is the value are also called D.C meter, because DC meters indicate average values rather than instantaneous values. In other words, over a complete cycle, the average value is the equivalent D.C value. In the analysis of electronic circuits to be considered, both D.C and A.C sources of voltage will be applied to the same network. It will then be necessary to know or determine the D.C (or average value) and AC components of the voltage or current in various parts of the system. Equation can be extended to include any variable quantity, such as current or voltage, if we let G denote the average value, as follows:

$$G(average\ value) = \frac{algebraic\ sum\ of\ areas}{length\ of\ curve}$$

Example: Find the average values of the following waveforms over one full cycle:



(a)



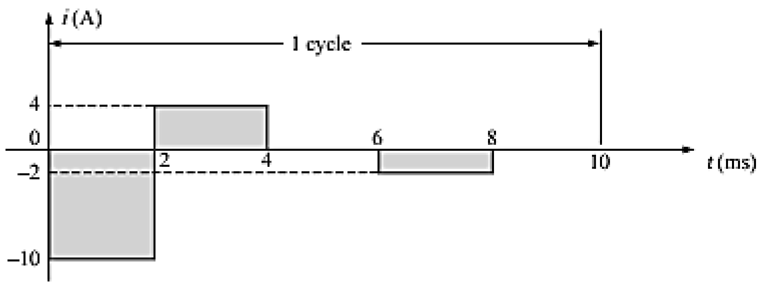
(b)

Sol/(a)

By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts

(b)

$$G = \frac{(14\ V)(1\ ms) + (-6\ V)(1\ ms)}{2\ ms} = 4\ V$$



(c)

Sol/(c)

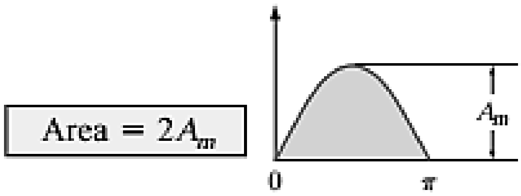
$$G = \frac{(-10\ V)(2\ ms) + (4\ V)(2\ ms) + (-2\ V)(2\ ms)}{10\ ms} = -1.6\ V$$

<u>Sine wave Averages:</u> The average of *<u>half</u>* a sine wave, however, is not zero. The area under the positive pulse of a <u>sine wave</u> using integration, we have

$$Area = \int_0^{\pi} A_m \sin\alpha\, d\alpha$$

$$= V_m[\cos(0) - \cos(\pi)]$$

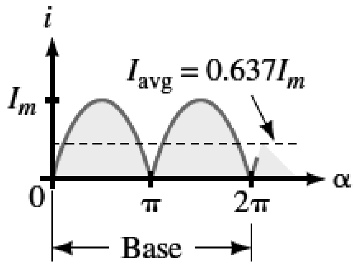
$$Area = 2A_m$$



The average value of the positive (or negative) region of half a sine wave pulse is:

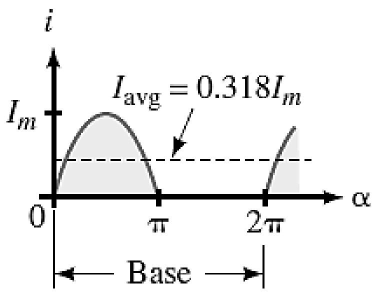
Full-wave average

$$I_{avg.} = \frac{2I_m}{\pi} = 0.637\ I_m$$



Half-wave average,

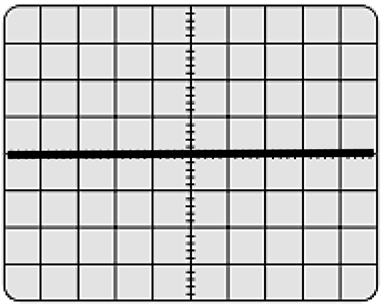
$$I_{avg.} = \frac{I_m}{\pi} = 0.318\ I_m$$



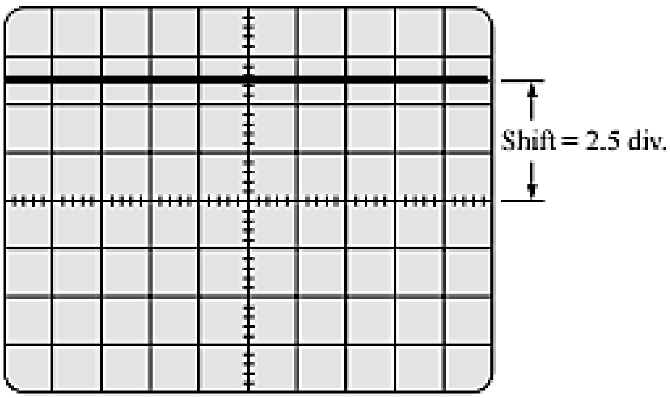
Instrumentation:

The D.C level or average value of any waveform can be found using a digital, analog multimeter or an oscilloscope. Oscilloscopes are limited to voltage levels using the sequence of steps listed below:

1) First choose GND from the DC-GND-AC option list associated with each vertical channel. Set the resulting line in the middle of the vertical axis on the horizontal axis, as shown in Figure.



2) Apply the oscilloscope probe to the voltage to be measured (if not already connected), and switch to the DC option. If a DC voltage is present, the horizontal line will shift up or down, as demonstrated in Figure. Multiplying the shift by the vertical sensitivity will result in the D.C voltage. An upward shift is a positive voltage (higher potential at the red or positive lead of the oscilloscope), while a downward shift is a negative voltage (lower potential at the red or positive lead of the oscilloscope).



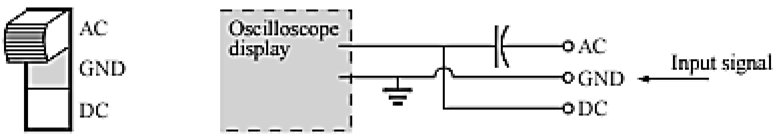
Vertical sensitivity = 50 mV/div.

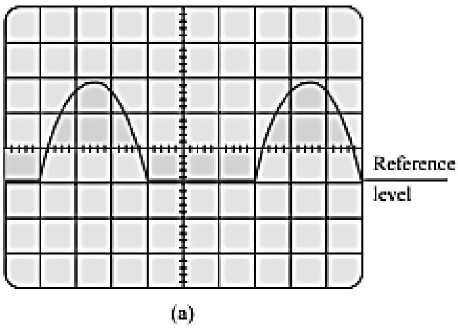
In general,

$$V_{D.C} = (\text{vertical shift in div.}) \times \left(\text{vertical sensitivity in } \frac{V}{div.}\right)$$

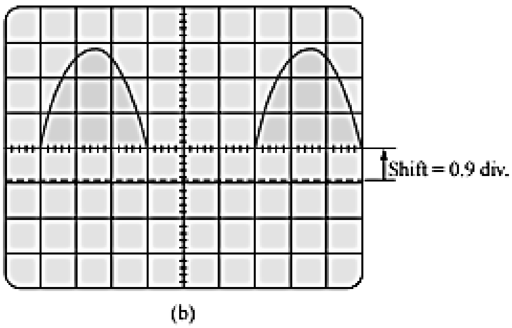
$$V_{D.C} = (2.5\ div.)\left(50\ \frac{mv}{div.}\right) = 125\ mv$$

- ☒ **The oscilloscope can also be used to measure the DC or average level of any waveform using the following sequence:**
- ☒ **The Switch to AC (all DC components of the signal to which the probe is connected will be <u>blocked</u> from entering the oscilloscope only the alternating, or changing, components will be displayed).**
- ☒ **The switch to DC (to permit both the D.C and the A.C components of the waveform to enter the oscilloscope).**





A.C Switch

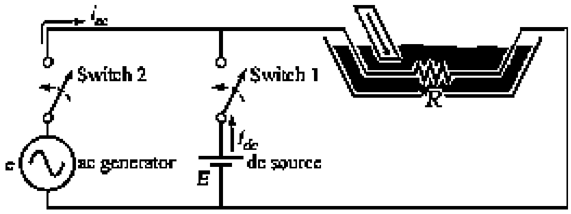


D.C Switch

$$V_{av.} = V_{D.C} = (0.9\ div.)\left(5\ \frac{V}{div}\right) = 4.5\ V$$

EFFECTIVE (rms) VALUES:

A fixed relationship between AC and DC voltages and currents can be derived from the experimental setup shown in Figure. A resistor in a water bath is connected by switches to a D.C and an A.C supply. If switch 1 is closed, a D.C current I, determined by the resistance R and battery voltage E, will be established through the resistor R. The temperature reached by the water is determined by the D.C power dissipated in the form of heat by the resistor.



If switch 2 is closed and switch 1 left open, the A.C current through the resistor will have a peak value of I_m. The temperature reached by the water is now determined by the A.C power dissipated in the form of heat by the resistor. The A.C input is varied until the temperature is the same as that reached with the D.C input. When this is accomplished, the average electrical power delivered to the resistor R by the AC source is the same as that delivered by the D.C source.

The power delivered by the A.C supply at any instant of time is:

$$p(t) = (i_{AC})^2 R = (I_m sin\omega t)^2 R = (I_m^2 sin^2 \omega t) R$$

$$P(t) = I_m^2 \left[\frac{1}{2}(1 - cos\, 2\omega t)\right] R\,, \qquad p(t) = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} cos\, 2\omega t$$

$$p_{avg.} = average\ of\ p(t) = \frac{I_m^2 R}{2}$$

The average power delivered by the A.C source is just the first term, since the average value of a cosine wave is *zero* even though the wave may have twice the frequency of the original input current waveform. Equating the average power delivered by the AC generator to that delivered by the DC source,

$$P_{av.(A.C)} = P_{D.C}$$

$$\frac{I_m^2 R}{2} = I^2 R$$

$$I_{eff.} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

The equivalent DC value is called the effective value of the sinusoidal quantity. In summary,

$$I_{eq(D.C)} = I_{eff(rms)} = 0.707\, I_m$$

$$I_m = \sqrt{2} I_{rms} = 1.414\, I_{rms}, \rightarrow \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

And

$$E_{eq(D.C)} = E_{eff(rms)} = 0.707\, E_m$$

$$E_m = \sqrt{2} E_{rms} = 1.414\, E_{rms}$$

As you can see, effective values for sinusoidal waveforms depend only on magnitude.

The effective value (root-mean-square value, rms) of any quantity plotted as a function of time can be found by using the following equation derived from the experiment just described.

$$I_{rms} = \sqrt{\frac{Area(i^2(t))}{T}}$$

To compute effective values using this equation, do the following:

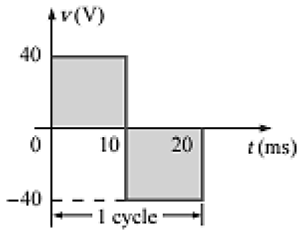
<u>Step 1:</u> Square the current (or voltage) curve.

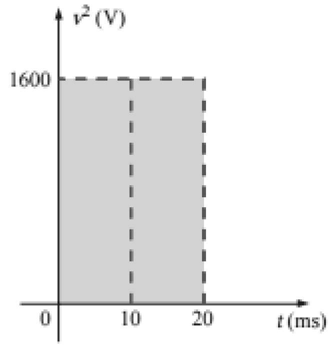
<u>Step 2:</u> Find the area under the squared curve.

<u>Step 3:</u> Divide the area by the length of the curve.

<u>Step 4:</u> Find the square root of the value from step 3.

Example: Determine the average and rms values of the square





Sol/ By inspection, the average value is zero.

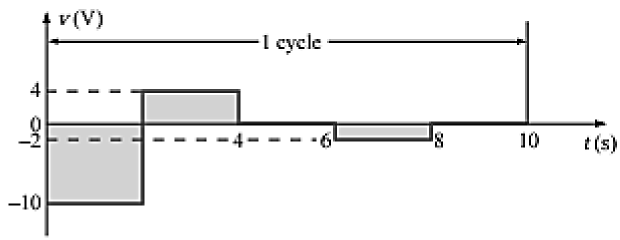
$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{32{,}000 \times 10^{-3}}{20 \times 10^{-3}}} = \sqrt{1600}$$

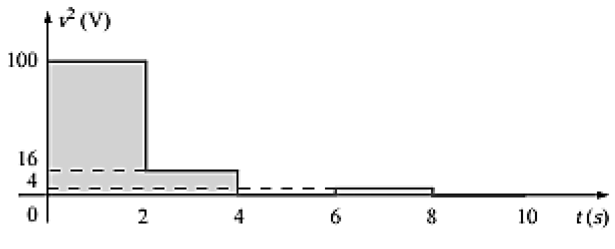
$$\mathbf{V_{rms} = 40\ V}$$

****

Example: Calculate the rms value of the voltage of Figure.



Sol/

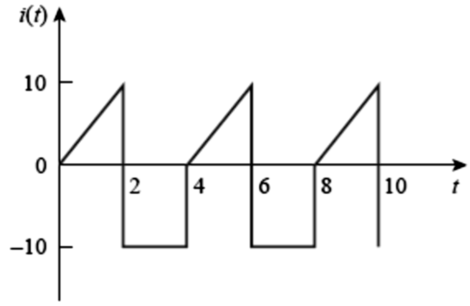


$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$

$$= \mathbf{4.899\ V}$$

****

Example: **Determine the rms value of the current waveform in Figure shown.**



Sol/

The period of the waveform is T = 4. Over a period, we can write the current waveform as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{2 - 0} = 5$$

$$y = mx + b$$

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 \, dt}$$

$$I_{rms} = \sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 \, dt + \int_2^4 (-10)^2 \, dt\right]}$$

$$I_{rms} = \sqrt{\frac{1}{4} \left[25 \frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4} \left(\frac{200}{3} + 200\right)} = 8.165\ A$$

****

Superimpose AC and DC:

Sometime AC and DC are used in the same circuit, since we know that the average of a sine wave is zero, the average value of the combined waveform will be DC component, E

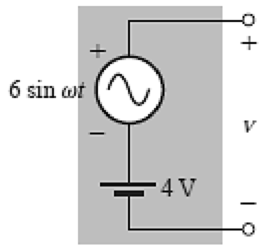
$$v(t) = V_{DC} + V_m sin(\omega t)$$

Example:

a)Sketch the input resulting from the combination of sources in Figure.

b)Determine average, peak, and minimum voltage?

c) Determine the rms value of the input?



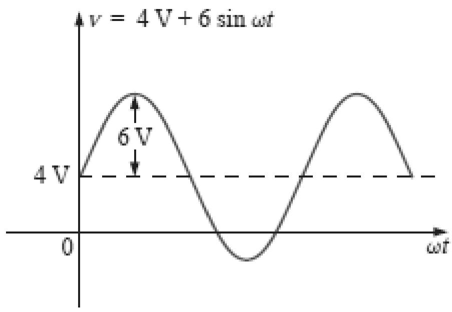
Sol/

$$v(t) = 4 + 6sin\ \omega t$$

$$v_{avg.} = 4V$$

The peak voltage is 4+6=10V, while the minimum voltage is 4-6=-2V.

$$V_{rms} = \sqrt[2]{(V_{DC})^2 + (V_{eff.})^2}$$



$$V_{rms} = \sqrt{(V_{DC})^2 + \frac{(V_m)^2}{2}} = \sqrt{(4\ \ )^2 + \frac{(6)^2}{2}}$$

$$V_{rms} = 5.831V$$

The rms value of a waveform having both DC and AC components is not simply the sum of the effective values of each.

Capacitors and Inductors

Introduction: So far we have limited our study to resistive circuits. In this lecture, we shall introduce two new and important passive linear circuit elements: the *capacitor* and the *inductor*. Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy، which can be retrieved at a later time. For this reason, capacitors and inductors are called storage elements

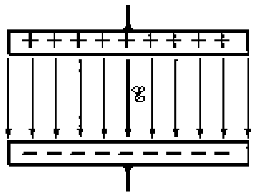
Capacitors:

- ☒ A capacitor is a passive element designed to store energy in its electric field.
- ☒ A capacitor consists of two conducting plates separated by an insulator (or dielectric).
- ☒ A capacitor is an open circuit to D.C.
- ☒ When a voltage source is connected to the capacitor, the source deposits a positive charge + q on one plate and a negative charge -q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage so that.

$$Q = CV$$

$$\boxed{C = \frac{Q}{V}}$$

C = farads (F)
Q = coulombs (C)
V = volts (V)

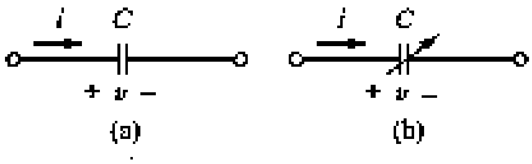


where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F). Although the capacitance of capacitor is the ratio of Q per plate to applied voltage V, it does not depend on Q or V it depends on the physical dimensions of capacitor. The capacitance is given by

$$C = \frac{\varepsilon A}{d}$$

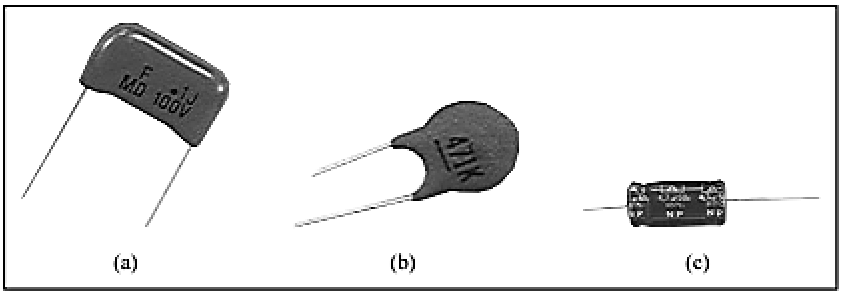
Where A is the surface area of each plate, d is the distance between the plates, and ε is the permittivity of the dielectric material between the plate.

TYPES OF CAPACITORS: Like resistors, all capacitors can be included under either of two general headings: fixed or variable.



Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.

Fixed Capacitors: Many types of fixed capacitors are available today. Some of the most common are the mica, ceramic, electrolytic, tantalum, and polyester- film capacitors.



Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor. Courtesy of Tech America.

Variable Capacitors:

The most common of the variable-type capacitors is shown as figure. The capacitance of the trimmer capacitor is changed by turning the screw, which will vary the distance between the plates.

Relationship between current-voltage:

To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Equation

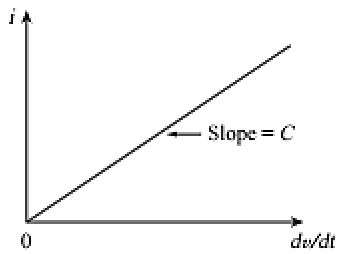
$$q=cv \quad ....(1)$$

Since $\mathbf{i} = \frac{\mathbf{dq}}{\mathbf{dt}}$

differentiating both sides of Eq. (1) gives

$$i = c\frac{dv}{dt} \quad \dots \dots ..(2)$$

We will assume linear capacitors in this lecture



Relationship between voltage-current:

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (2). We get

$$v = \frac{1}{c}\int_{t_o}^{t} idt + v(t_0)$$

Where $v(t_o) = \frac{q(t_o)}{c}$ is the voltage across the capacitor at time t_o . Equation shows that capacitor voltage depends on the past history of the capacitor current. Hence, the capacitor has memory—a property that is often exploited.

The instantaneous power delivered to the capacitor is

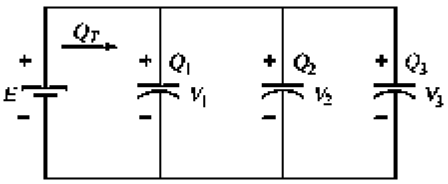
$$p = vi = Cv\frac{dv}{dt}$$

Series and Parallel Capacitors:

We know from resistive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor.

Parallel circuit

For capacitors in parallel, as shown in Figure, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:



$$Q_T = Q_1 + Q_2 + Q_3$$

$$Q = CV$$

$$C_{eq.}E = C_1V_1 + C_2V_2 + C_3V_3$$

$$E = V_1 = V_2 = V_3$$

We now obtain $C_{eq.}$ Of N capacitors

$$\boxed{C_{eq.} = C_1 + C_2 + C_3 + \cdots \ldots . C_N}$$

which is similar to the manner in which the total resistance of a series circuit is found.

Series circuit:

For capacitors in series, the charge is the same on each capacitor

$$Q_T = Q_1 = Q_2 = Q_3$$

Applying Kirchhoff's voltage law around the closed loop gives

$$E = V_1 + V_2 + V_3$$

$$v = \frac{Q}{C}$$

$$\frac{Q}{C_{eq.}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is:

$$C_{eq.} = \frac{C_1 \times C_2}{C_1 + C_2}$$

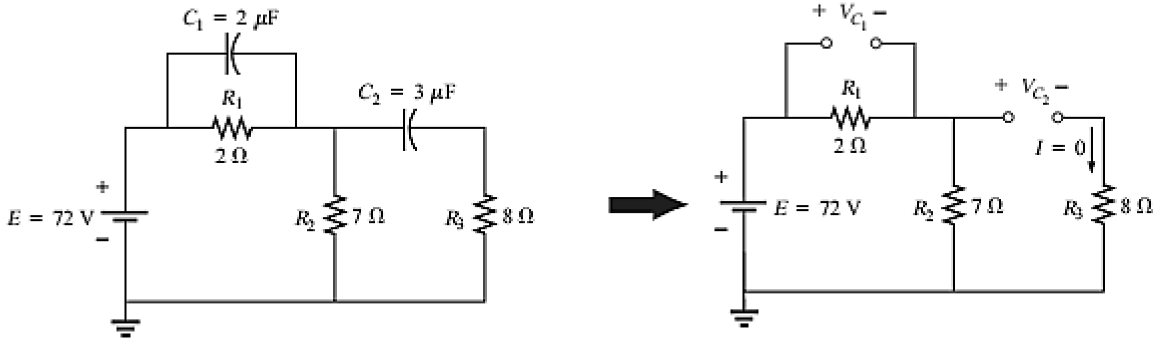
The voltage across each capacitor can be found by first recognizing that

$$Q_T = Q_1$$

$$C_{eq.}E = C_1 V_1$$

$$V_1 = C_{eq.}\frac{E}{C_1}$$

<u>Example:</u> Find the voltage across and charge on each capacitor of the network



Sol/

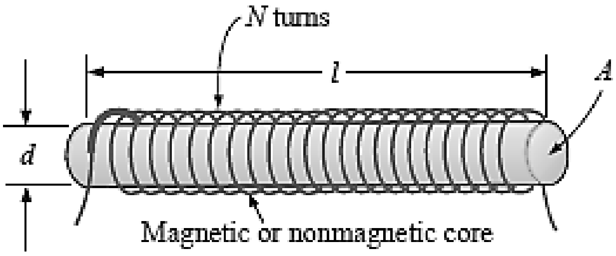
$$V_{C1} = 72\frac{2}{2+7} = 16\,V \quad , \qquad V_{C2} = 72\frac{7}{2+7} = 56\,V$$

$$Q_1 = C_1 V_{C1} = 32\mu C \qquad , Q_2 = C_2 V_{C2} = 168\mu C$$

****

Inductor:

An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems. They are used in power supplies, transformers, radios، TVs, radars, and electric motors.



An inductor consists of a coil of conducting wire.

If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current. Using the passive sign convention,

$$v = L\frac{di}{dt}$$

where L is the constant of proportionality called the inductance of the inductor. The unit of inductance is the henry (H).

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

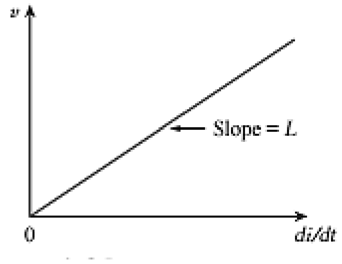
The inductance of an inductor depends on its physical dimension and construction. Formulas for calculating the inductance of inductors of different shapes are derived from electromagnetic theory and can be found in standard electrical engineering handbooks. the inductor,

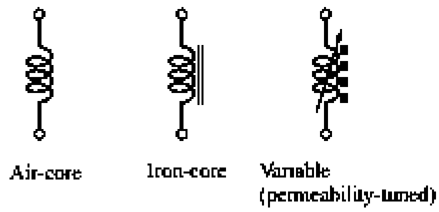
$$L = \frac{N^2 \mu A}{\ell}$$

where N is the number of turns, ℓ length of magnetic circuit , A is area of cross-section of magnetic circuit through which flux is passing , and μ is the permeability of the core.

The voltage-current relationship:

Inductance is independent of current. Such an inductor is known as a linear inductor .We will assume linear inductors in this lecture unless stated otherwise





Inductor symbols.

The current-voltage relationship is obtained

$$di = \frac{1}{L} v\, dt$$

$$i = \frac{1}{L}\int_{t_o}^{t} v(t) + i(t_o)$$

where $i(t_o)$is the total current for $-\infty < t < t_o$ and $i(-\infty) = 0$. The idea of making $i(-\infty) = 0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.

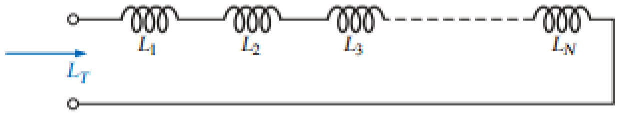
An inductor acts like a short circuit to D.C.

INDUCTORS IN SERIES AND PARALLEL

Inductors, like resistors and capacitors, can be placed in series or parallel. Increasing levels of inductance can be obtained by placing inductors in series, while decreasing levels can be obtained by placing inductors in parallel.

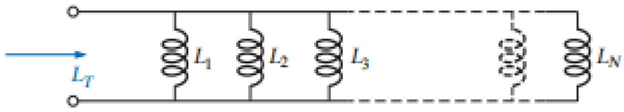
For inductors in series, the total inductance is found in the same manner as the total resistance of resistors in series

$$L_{eq.} = L_1 + L_2 + L_3 \dots \dots \dots \dots . . L_N$$



<u>For inductors in parallel</u>, the total inductance is found in the same manner as the total resistance of resistors in parallel,

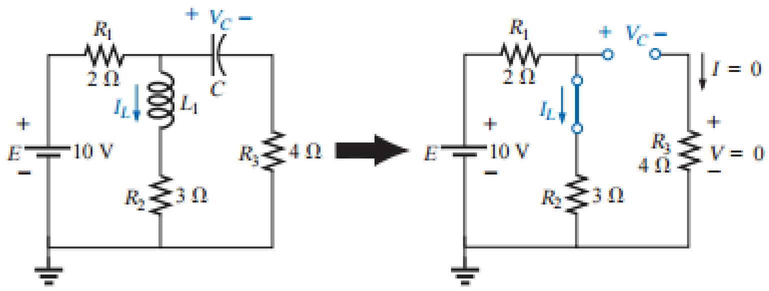
$$\frac{1}{L_{eq.}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots\ldots\ldots\ldots.. \frac{1}{L_N}$$



For two inductors in parallel,

$$L_{eq.} = \frac{L_1 L_2}{L_1 + L_2}$$

<u>Example:</u> Find the current I_L and the voltage V_C for the network



Sol/

$$I_L = \frac{E}{R_1 + R_2} = \frac{10\text{ V}}{5\ \Omega} = \mathbf{2\text{ A}}$$

$$V_C = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(10\text{ V})}{3\ \Omega + 2\ \Omega} = \mathbf{6\text{ V}}$$

The Basic Elements and Phasors

INTRODUCTION:

Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions.

> **A phasor is a complex number that represents the amplitude and phase of a sinusoid.(phasor apply only to sinusoidal waveforms)**

Complex number review:

- **A complex number z can be written in rectangular form as**

$$z = x + jy$$

Where $j = \sqrt{-1}$

x is the real part of z; y is the imaginary part of z.(in circuit theory, j is used to denoted the imaging component rather than i to avoid confusion with current i).

- **The complex number z can also be written in polar or exponential form as**

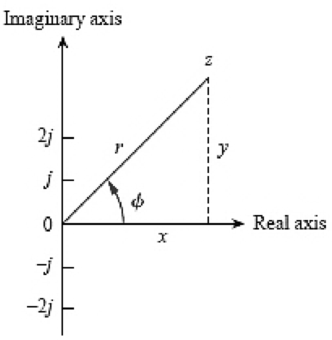
$$z = r\angle\emptyset = re^{j\emptyset}$$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$z = x + jy$	Rectangular form
$z = r\angle\phi$	Polar form
$z = re^{j\phi}$	Exponential form

The relationship between the rectangular form and the polar, if given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2} \qquad \varphi = \tan^{-1}\frac{y}{x}$$



On the other hand, if we know r and ϕ

we can obtain x and y as

$x = r\cos\varphi,\ \ y = r\sin\varphi$

Thus, z may be written as

$z = x + jy = r\angle\emptyset = r(cos\emptyset + jsin\emptyset)$

***Addition* and *subtraction* of complex numbers are better performed in *rectangular form*. *multiplication* and *division* are better done in *polar form*. Given the complex numbers**

$$Z_1 = x_1 + jy_1 = r_1 \angle\varphi_1, \qquad Z_2 = x_2 + jy_2 = r_2 \angle\varphi_2$$

Addition: $z_1 + z_2 = (x_1 + x_2) + j\,(y_1 + y_2)$

Subtraction: $z_1 - z_2 = (x_1 - x_2) + j\,(y_1 - y_2)$

Multiplication: $z_1 z_2 = r_1 r_2 \angle(\varphi 1 + \varphi 2)$

Division: $\frac{z1}{z2} = \frac{r1}{r2} \angle(\varphi 1 - \varphi 2)$

Reciprocal: $\frac{1}{z} = \frac{1}{r} \angle - \varphi 1$

Square Root: $\sqrt{z} = \sqrt{r}\angle(\frac{\varphi}{2})$

Complex Conjugate: $z^* = x - jy = r\angle - \varphi = r\,e^{-j\varphi}$

The idea of phasor representation is based on *Euler's* identity.
In general,

$e^{\pm j\varphi} = \cos\varphi \pm j\sin\varphi$, $\cos\varphi = Re(e^{j\varphi})$, $\sin\varphi = Im(e^{j\varphi})$

<u>Power of j:</u> power of j are frequently required in calculations. Here are some useful powers

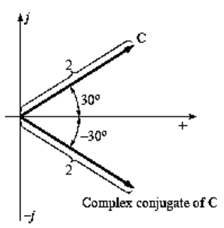
$$j^2 = (\sqrt{-1})(\sqrt{-1}) = -1 , \quad j^4 = 1 , \; j^3 = -j , \quad \frac{1}{j} = -j$$

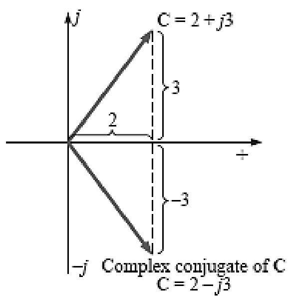
<u>Complex Conjugate:</u> The conjugate or complex conjugate of a complex number (denoted by an asterisk *) can be found by simply *changing the sign* of the *imaginary part* in the rectangular form or by using the *negative* of the angle of the polar form. For example, the conjugate of

$c = 2 + j3$

Is $= 2 - j3$

The conjugate of, $c = 20\angle 30^0$





Is $20\angle -30^0$

Complex numbers in A.C analysis:

The sinusoidal, thus, $e(t) = 200sin(\omega t - 40^0)$ can be represented by its phasor equivalent $e = 200\angle -40^0$.

Summing A.C voltage and currents:

Sinusoidal quantities must sometimes be added or subtracted, here, for example,

$e_1 = 10sin\omega t$ **thus,** $e_1 = 10\angle 0^0$

$e_2 = 15sin(\omega t + 60^0)$ **thus ,** $e_2 = 15\angle 60^0$

The sum of e_1 and e_2 can be found

$v = e_1 + e_2 = 10\angle 0^0 + 15\angle 60^0 = 21.8\, V\angle 36.6^0$

$v(t) = 21.8sin(\omega t + 36.6^0)$

Note that:

1. **To this point, we have used peak values such as V_m and I_m to represent the magnitudes of phasor voltages and currents. In practice, however, *r.m.s values are used instead.***
2. **Quantities expressed as time functions are said to be in the time domain, while quantities expressed as phasors are said to be in the phasor (or frequency domain).**
3. **It should be noted that sine wave and phasors are not the same thing. Sinusoidal voltage and currents are real- they are the actual quantities that you measure with meters and waveforms you see on oscilloscope. Phasor, on the other hand, are mathematical abstractions that we use to help visualize relationships and solve problems.**
4. **Quantities expressed as time function are said to be in the time domain, while quantities expressed as phasors are said to be in the phasor (or frequency) domain.**
5. **The subscripts eff and r.m.s are not used in practice. One the concept is familiar, we drop them.**

Example: if $i_1(t) = 14.14 sin(\omega t - 55^0)A$ and

$i_2(t) = 4 sin(\omega t + 15^0)A$

Determine their sum I work with r.m.s value,

Sol/

$I_1 = (0.707)(14.14)\angle -55^0 = 10\angle -55^0$,
$I_2 = (0.707)(4)\angle 15^0 = 2.828\angle 15^0$

$I = I_1 + I_2 = 11.3\ A\angle -41.4^0$

In time domain

$i(t) = \sqrt{2}(11.3) sin(\omega t - 41.4^0)$

Application of complex numbers to series A.C. circuits

The impedance concept:

In practice, we represent circuit elements by their impedance, and determine magnitude and phasor relationships in one step. Before we do it, however, we need to learn how to represent circuit elements as impedance.

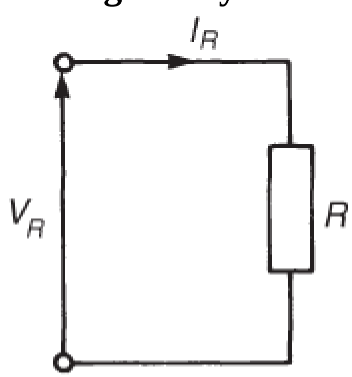
The equation is sometimes referred to ohm's law for A.C circuit

$Z = \frac{V}{I}\angle\emptyset$ **(ohms)**

Where V and I are the r.m.s magnitudes of V and I respectively, and ϕ is the angle between them.

Pure resistance: **The current I_R is in phase with the applied voltage V_R as shown in the phasor. The impedance Z of the circuit is given by**

Taken voltage as reference $V\angle 0^0$,



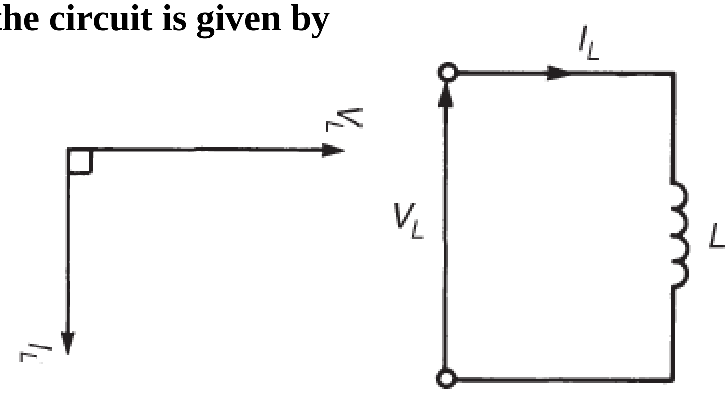
$$Z = \frac{V\angle 0^0}{I\angle 0^0} = R$$

Pure inductance:

In an A.C. circuit containing pure inductance L only, the current I_L lags the applied voltage V_L by 90° as shown in the phasor diagram. The impedance Z_L of the circuit is given by

$$Z_L = \frac{V\angle 0^0}{I\angle -90^0} = X_L\angle 90^0 = jX_L$$

$$X_L = 2\pi f L = \omega L \quad \text{(ohms)}$$

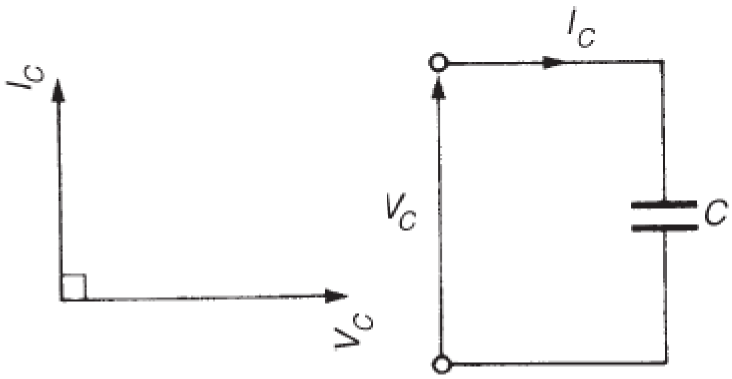


Pure capacitance:

In an A.C. circuit containing pure capacitance, the current I_C leads the applied voltage V_C by 90° as shown in the phasor diagram. The impedance Z_c of the circuit is given by

$$Z_C = \frac{V\angle 0^0}{I\angle 90^0} = X_C\angle -90^0 = -jX_C$$

$$X_C = \frac{1}{2\pi f c} = \frac{1}{\omega c}$$

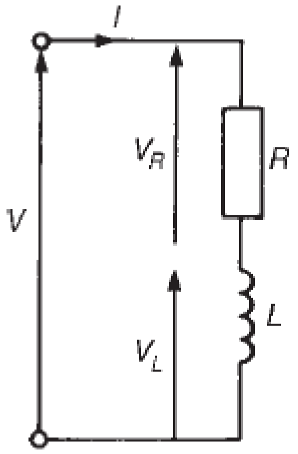


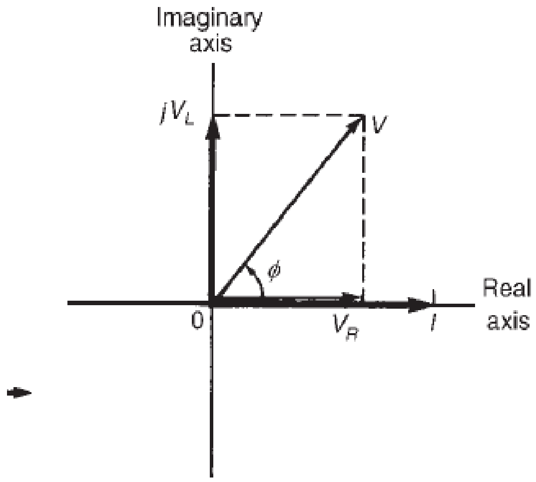
Note that:

- ✓ Although Z is a complex number, it is not a phasor since it does not represent a sinusoidal varying quantity.
- ✓ The phasor analysis applies only when frequency is constant, it applies in manipulating two or more sinusoidal signal only if they are of the same frequency.

R–L series circuit:

In an AC circuit containing resistance R and inductance L in series, the applied voltage V is the phasor sum of V_R and V_L as shown in the phasor diagram. The current I lags the applied voltage V by an angle lying between 0° and 90°—the actual value depending on the values of V_R and V_L, which depend on the values of R and L.

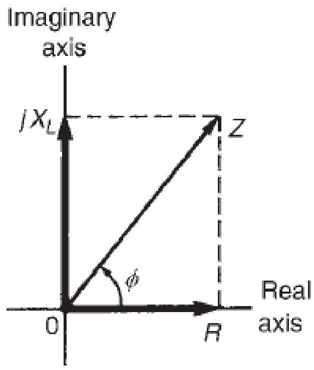




$$V = V_R + V_L$$

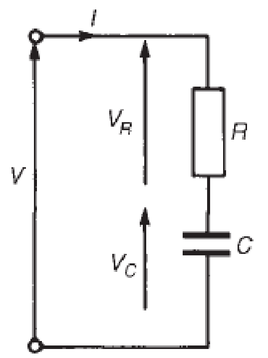
In complex form, the impedance Z is given by

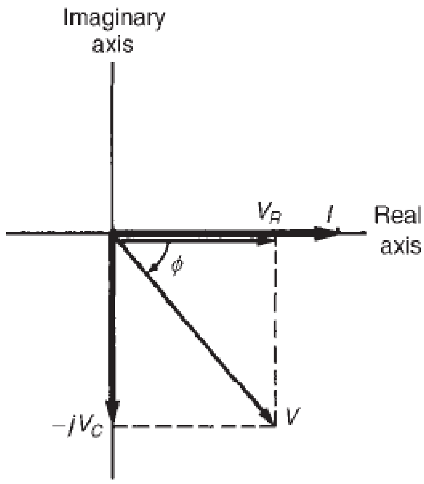
$$Z = R + jX_L$$



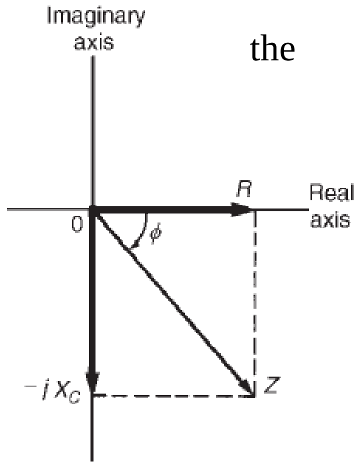
R–C series circuit:

In an A.C circuit containing resistance R and capacitance C in series, the applied voltage V is the phasor sum of V_R and V_C as shown in the phasor diagram. The current I leads the applied voltage V by an angle lying between 0° and 90°—the actual value depending on the values of V_R and V_C, which depend on the values of R and C.





the

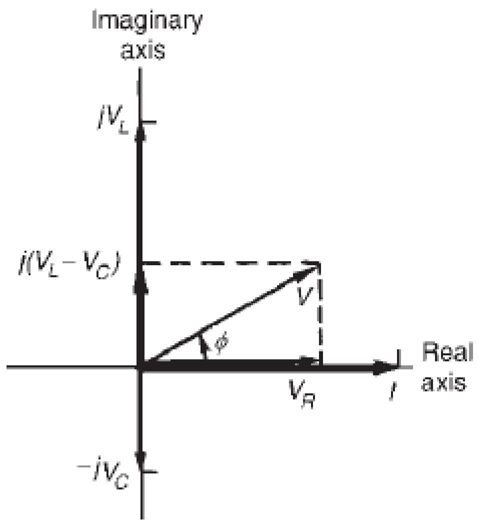


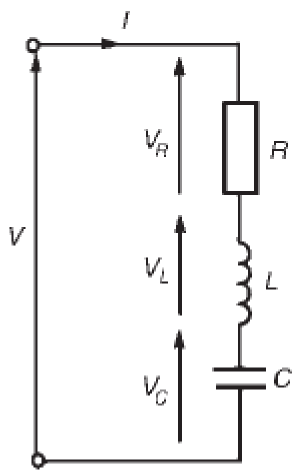
In complex form, the impedance Z is given by

$$Z = R - jX_C$$

R–L–C series circuit:

In an A.C circuit containing resistance R, inductance L and capacitance C in series, the applied voltage V is the phasor sum of V_R, V_L and V_C as shown in the phasor diagram (where the condition $V_L > V_C$ is shown).





It may be seen that in complex form the supply voltage V is given by:

$$V = V_R + j(V_L - V_C)$$

In complex form، impedance

$$Z = R + j(X_L - X_C)$$

<u>General series circuit:</u>

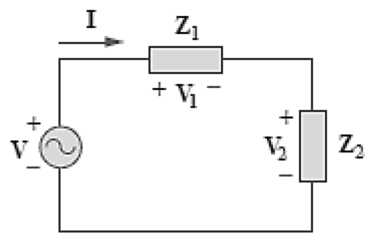
In an A.C circuit containing several impedances connected in series, say، Z_1, Z_2, Z_3, . . ., Z_n, then the total equivalent impedance Z_T is given by

$$Z_T = Z_1 + Z_2 + Z_3 \dots \dots . Z_n$$

KVL (The phasor sum of voltage drops and voltage rises around a closed loop is equal to zero).

$$V_T = V_1 + V_2 + V_3 \dots . . V_n$$

If n=2, the current through the impedances



$$I = \frac{V}{Z_1 + Z_2}$$

Since ,

$$V_1 = Z_1 I \quad and \quad V_2 = Z_2 I$$

$$V_1 = V\frac{Z_1}{Z_1 + Z_2}$$

$$V_2 = V\frac{Z_2}{Z_1 + Z_2}$$

which is the voltage-division relationship.

Example:

A 240 V, 50 Hz voltage is applied across a series circuit comprising a coil of resistance 12 ohms and inductance 0.10 H، and 120 µF capacitor. Determine the current flowing in the circuit.

Sol/

Inductive reactance,

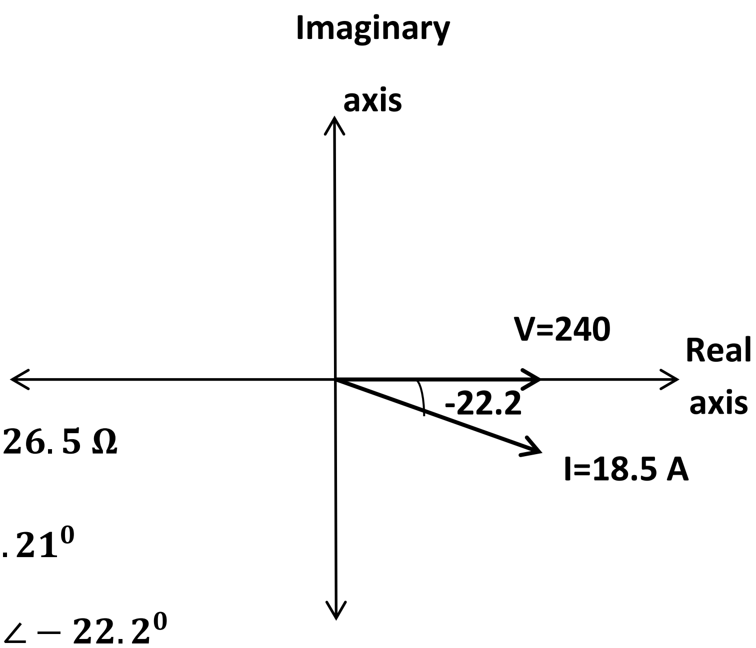
$X_L = 2\pi(50)(0.1) = 31.4\ \Omega$

Capacitive reactance,

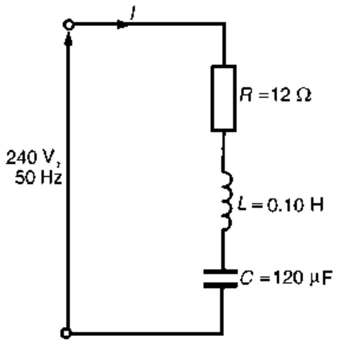
$X_C = \dfrac{1}{2\pi(50)(120\times10^{-6})} = 26.5\ \Omega$

$z = 12 + j4.9 = 12.96\ \Omega\angle 22.21^0$

Current flowing , $I = \frac{V}{Z} = 18.5\angle -22.2^0$



i.e., the current flowing is 18.5 A, lagging the voltage by 22.2°.



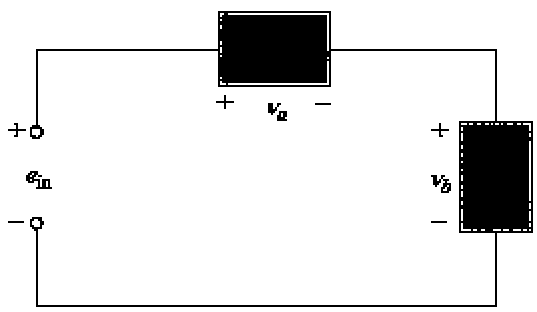
Example: **Find the input voltage of the circuit of Figure, if**

$v_a(t) = 50sin(377t + 30^o)$

$v_b(t) = 30sin(377t + 60^o)$

Sol/ Using KVL

$E_{in} = V_a + V_b$



Converting from the time to the phasor domain yields

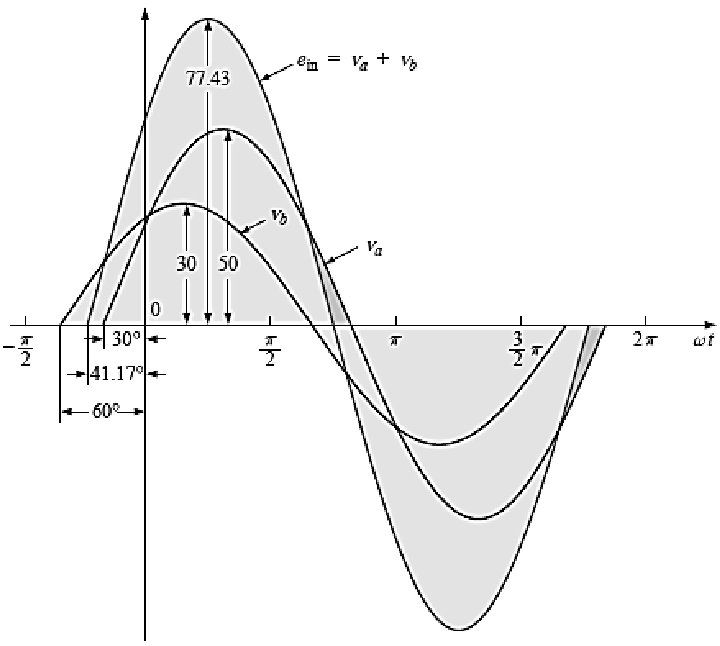
$$v_a(t) = 50sin(377t + 30^o) \rightarrow V_a = 35.35\angle 30^0$$

$$v_b(t) = 30sin(377t + 60^o) \rightarrow V_b = 21.21\angle 60^0$$

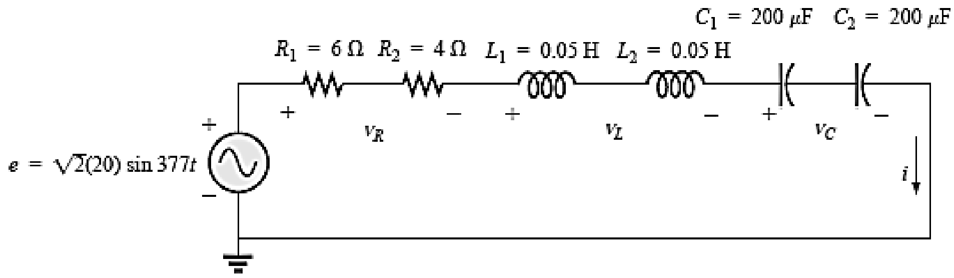
Then

$$E_{in} = (41.22 + j36.05)V = 54.76\angle 41.17^0 V$$

$$e_{in(t)} = 77.43sin(377t + 41.17^0)$$



Example: **For the circuit shown**



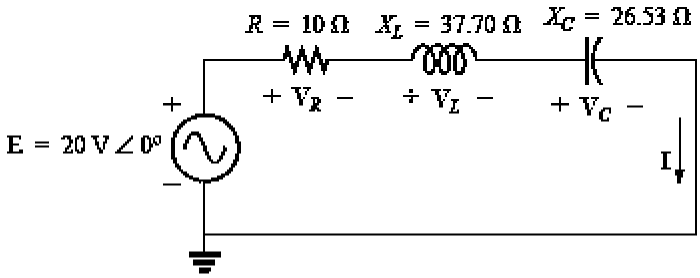
a) **Calculate I, V_R, V_L, and V_C in phasor form.**
b) **Draw the phasor diagram. Obtain the phasor sum of V_R, V_L, and V_C.**
c) **Find V_R and V_C using the voltage divider rule**

Sol/

a) $R_T = 6+4=10\Omega$

$L_T = 0.05+0.05=0.1H$

$$C_T = \frac{200}{2} = 100\mu F$$

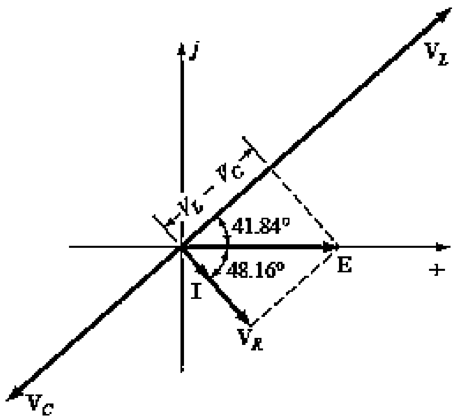


$$X_L = \omega L = 37.7\Omega$$

$$X_C = \frac{1}{\omega C} = 26.53\Omega$$

$$Z_T = 10 + j37.7 - j26.53 = 10 + j11.17 = 15\angle 48.16^o$$

$$I = \frac{E}{Z_T} = 1.33\angle{-48.16^0}A$$



$$V_R = IZ_R = 13.3\angle{-48.16^0}V$$

$$V_L = IZ_L = 50.14\angle 41.84^0 V$$

$$V_C = IZ_C = 35.28\angle{-138.16^0}V$$

c)

$$V_R = E\frac{Z_R}{Z_T} = (20\angle 0)\frac{(10\angle 0^0)}{(15\angle 48.16^0)} = 13.3\angle{-48.16^0}\text{V}$$

$$V_C = E\frac{Z_C}{Z_T} = (20\angle 0)\frac{(26.50\angle{-90^0})}{(15\angle 48.16^0)} = 35.37\angle{-138.16^0}V$$

Application of complex numbers to parallel A.C. networks

Introduction:

As with series circuits, parallel networks may be analyzed by using phasor diagrams. However, with parallel networks containing more than two branches this can become very complicated. It is with parallel A.C. network analysis in particular that the full benefit of using complex numbers may be appreciated.

Admittance، conductance and susceptance

Admittance is defined as the current I flowing in an AC circuit divided by the supply voltage V (i.e. it is the reciprocal of impedance Z). The symbol for admittance is Y. Thus

$$Y = \frac{I}{V} = \frac{1}{Z}$$

The unit of admittance is the Siemen, S.

an admittance may be resolved into two parts—the real part being called the conductance G, and the imaginary part being called the susceptance B—and expressed in complex form. Thus admittance

$$Y = G \pm jB$$

a) pure resistance, then

$$Z = R \;\; and \;\; Y = \frac{1}{Z} = \frac{1}{R} = G$$

b) pure capacitance, then

$$Z = -jX_C \; and \; Y = \frac{1}{Z} = \frac{1}{jX_C} = jB_C$$

Thus a positive sign is associated with capacitive susceptance, B_C

c) pure inductance, then

$$Z = jX_L \;\; and \;\; Y = \frac{1}{Z} = \frac{1}{jX_L} = -jB_L$$

Thus a negative sign is associated with inductive susceptance, B_L

d) resistance and inductance in series, then

$$Z = R + jX_L \ \ and \ \ Y = \frac{1}{Z} = \frac{1}{R + jX_L} = \frac{(R - jX_L)}{R^2 + X_L^2}$$

i.e $$Y = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2} \quad or \quad Y = \frac{R}{|Z|^2} - j\frac{X_L}{|Z|^2}$$

Thus conductance, $G = \frac{R}{|Z|^2}$ and inductive susceptance, $B_L = -\frac{X_L}{|Z|^2}$

e) resistance and capacitance in series, then

$$Z = R - jX_C \ \ and \ \ Y = \frac{1}{R - jX_C} = \frac{(R + jX_L)}{R^2 + X_L^2}$$

i.e $$Y = \frac{R}{R^2+X_c^2} + j\frac{X_L}{R^2+X_c^2} \quad or \quad Y = \frac{R}{|Z|^2} + j\frac{X_c}{|Z|^2}$$

(Note that in a capacitive circuit, the imaginary term of the impedance, X_C, is negative, whereas the imaginary term of the admittance, B_C, is positive).

f) resistance and inductance in parallel, then

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_L} = \frac{R + jX_L}{(R)(jX_L)}$$

$$Z = \frac{(R)(jX_L)}{R + jX_L}$$

$$Y = \frac{1}{Z} = \frac{R + jX_L}{jRX_L} = \frac{R}{jRX_L} + \frac{jX_L}{jRX_L}$$

$$Y = \frac{1}{R} - \frac{j}{X_L}$$

Thus conductance, $G = \frac{1}{R}$, and inductive susceptance, $B_L = -\frac{1}{X_L}$

g) **resistance and capacitance in parallel, then**

$$Z = \frac{(R)(-jX_C)}{R - jX_C}$$

And,

$$Y = \frac{1}{Z} = \frac{R - jX_C}{-jRX_C} = \frac{R}{-jRX_C} - \frac{jX_C}{-jRX_C}$$

i.e

$$Y = \frac{1}{R} + \frac{j}{X_C}$$

Thus conductance, $G = \frac{1}{R}$ and capacitive susceptance, $B_C = \frac{1}{X_C}$

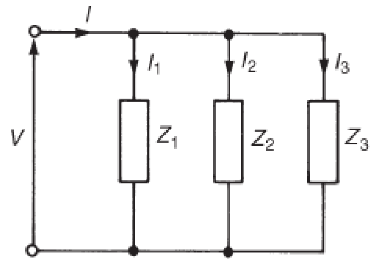
The conclusions that may be drawn from Sections (d) to (g) above are:

(i) That a series circuit is more easily represented by an impedance،

(ii) That a parallel circuit is often more easily represented by an admittance especially when more than two parallel impedances are involved.

<u>Parallel A.C. networks:</u>

Figure shows a circuit diagram containing three impedances, Z_1، Z_2 and Z_3 connected in parallel. The potential difference across each impedance is the same, i.e. the supply voltage V.



The supply current, $I = I_1 + I_2 + I_3$

Thus ,$\frac{V}{Z_T} = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$ and $\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$

or total admittance, $Y_T = Y_1 + Y_2 + Y_3$

In general, for n impedances connected in parallel,

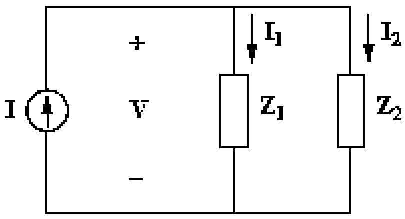
$$Y_T = Y_1 + Y_2 + Y_3 \dots \dots \dots Y_n$$

It is in parallel circuit analysis that the use of admittance has its greatest advantage.

KCL (the summation of current phasors entering and leaving a node is equal to zero).

If n=2, the equivalent impedance becomes

$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{Y_1+Y_2} = \frac{Z_1 Z_2}{Z_1+Z_2}$$



Since

$$V = IZ_{eq} = I_1 Z_1 = I_2 Z_2$$

The currents in the impedances are

$$I_1 = I\frac{Z_2}{Z_1 + Z_2}$$

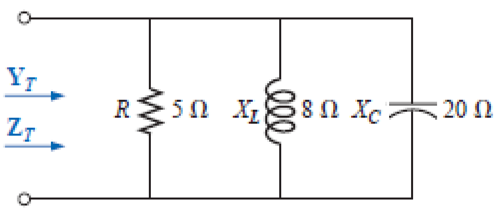
$$I_2 = I\frac{Z_1}{Z_1 + Z_2}$$

which is the current-division principle

****

<u>Example:</u> for the network of figure shown

a) Find the admittance of each parallel branch
b) Determine the input admittance.
c) Calculate the input impedance.
d) Draw the admittance diagram



SOL/

$$Y_R = G\angle 0 = \frac{1}{R}\angle 0 = 0.2S$$

$$Y_L = B_L\angle{-90^0} = \frac{1}{X_L}\angle{-90^0} = 0.125\,S\angle{-90^0} = -j0.125S$$

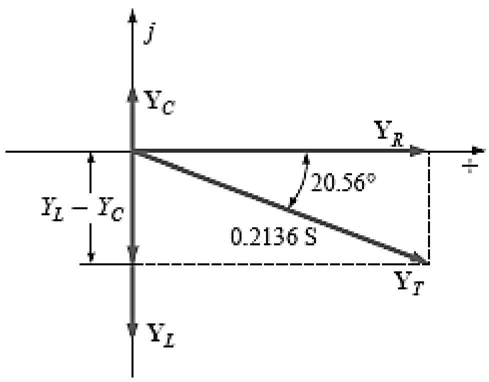
$$Y_C = B_C\angle 90^0 = \frac{1}{X_C}\angle 90^0 = 0.05\angle 90^0 = j0.05\ S$$

$$Y_T = Y_R + Y_L + Y_C = (0.2 - j0.075)S = 0.2136\ S\angle -20.56^0$$

$$Z_T = \frac{1}{(0.2136\angle -20.56^0)} = 4.68\ \Omega\angle 20.56^0 = (4.38 + \mathrm{j}1.644)\Omega$$

OR

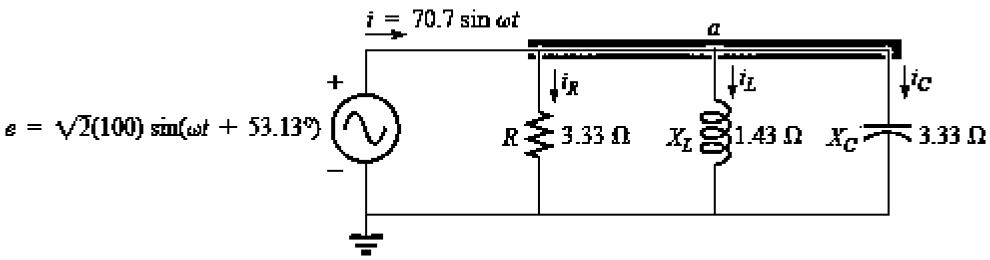
$$Z_T = \frac{Z_R Z_L Z_C}{Z_R Z_L + Z_L Z_C + Z_R Z_C} = (4.38 + \mathrm{j}1.644)\Omega$$

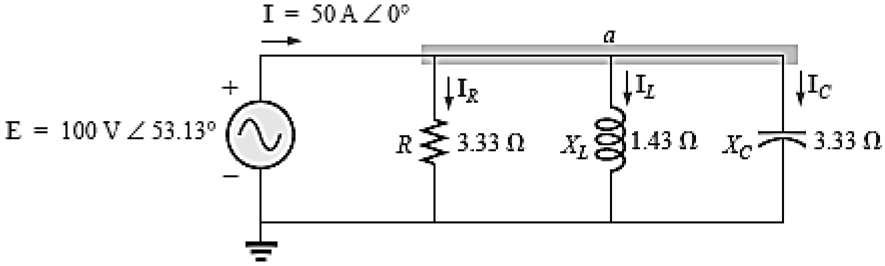


<u>Example:</u> for the network of figure shown

a) Find the admittance

b) Determine the I_R, I_L and I_C





sol/

$$Y_T = Y_R + Y_L + Y_C$$

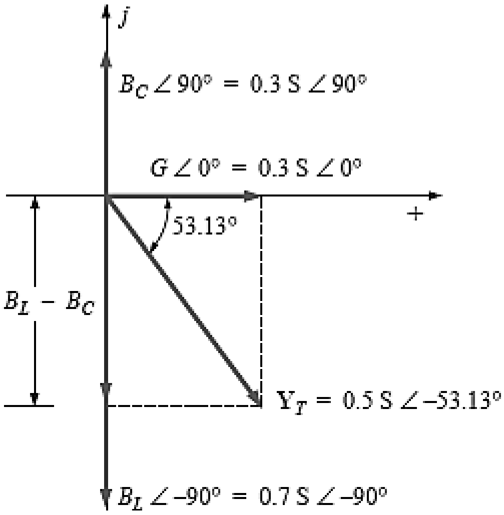
$$= \frac{1}{3.33}\angle 0^0 + \frac{1}{1.43}\angle -90^0 + \frac{1}{3.33}\angle 90^0$$

$$= (0.3 - j0.4)S = 0.5\ S\angle -53.13^0$$

$$Z_T = \frac{1}{Y_T} = 2\ \Omega\angle 53.13^0 = 1.2 + j1.6$$

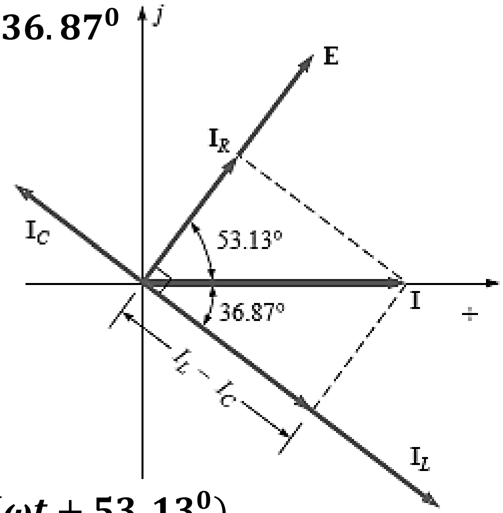
$$I = \frac{E}{Z_T} = E \times Y_T = 50\ A\angle 0^0$$

$$I_R = (E\angle\phi)(G\angle 0^0) = 30\ A\angle 53.13^0$$



$$I_L = (E\angle\phi)(B_L\angle -90^0) = 70\ A\angle -36.87^0$$

$$I_C = (E\angle\phi)(B_C\angle 90^0) = 30\ A\angle 143.13^0$$



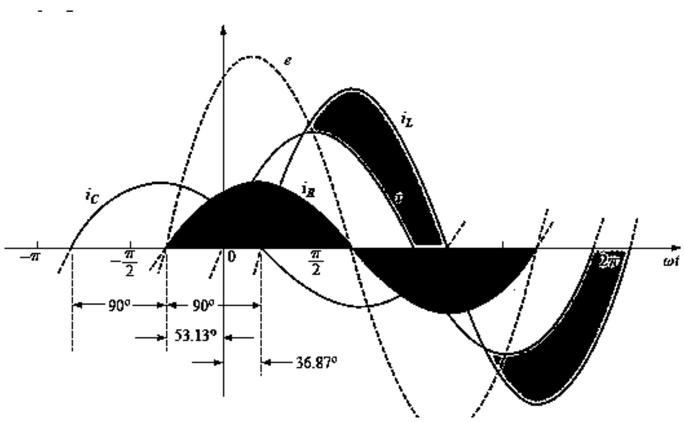
For time domain

$$i_t(t) = \sqrt{2}(50) sin\omega t = 70.70\ sin\omega t$$

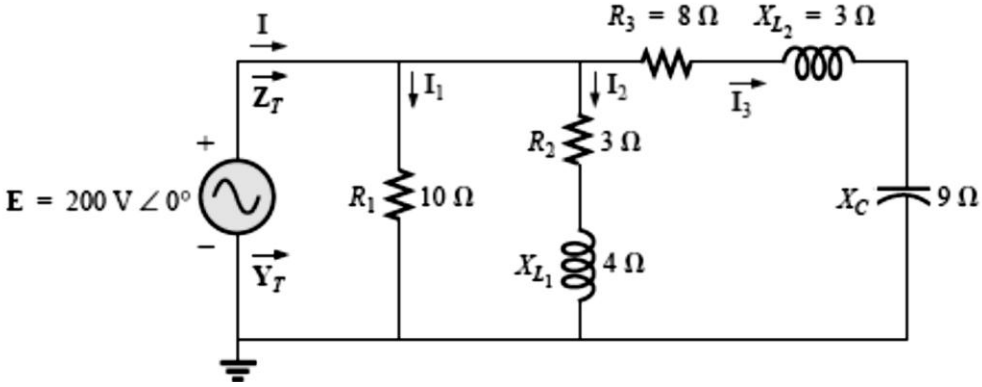
$$i_R(t) = \sqrt{2}(30) sin(\omega t + 53.13^0) = 42.42 sin(\omega t + 53.13^0)$$

$$i_L(t) = \sqrt{2}(70) sin(\omega t - 36.87^0) = 98.98 sin(\omega t - 36.87^0)$$

$$i_c(t) = \sqrt{2}(30) sin(\omega t + 143.13^0) = 42.42 sin(\omega t + 143.13^0)$$

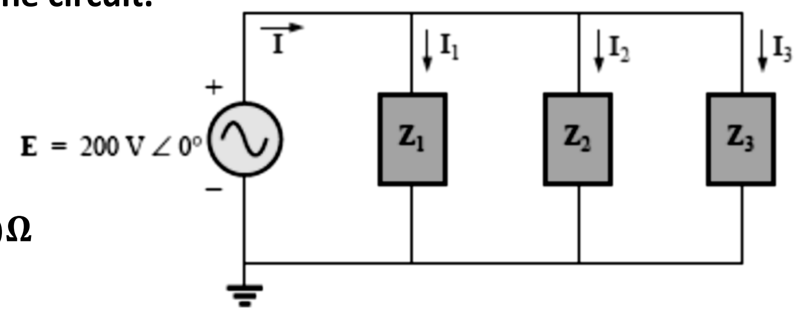


Example: For the parallel- series arrangement figure shown.



a) Compute I, **b) Find I_1, I_2 and I_3 ,**

C) Find total impedance of the circuit.



SOL/

$$Z_1 = R_1 = 10\Omega$$

$$Z_2 = R_2 + jX_{L1} = (3 + j4)\Omega$$

$$Z_3 = R_3 + jX_{L2} - jX_C = 8 + j3 - j9 = (8 - j6)\Omega$$

The total admittance is:

$$Y_T = Y_1 + Y_2 + Y_3 = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$Y_T = 0.1 + 0.12 - j0.16 + 0.08 + j0.06 = (0.3 - j0.1)S$$

a) $I = E \times Y_T = 63.2A\angle - 18.435^O$

b) $I_1 = \frac{E}{Z_1} = 20A\angle 0,$

$$I_2 = \frac{E}{Z_2} = 40A\angle{-53.13^O}, I_3 = \frac{E}{Z_3} = 20A\angle 36.87^O$$

c) $Z_T = \frac{1}{Y_T} = (3 + j1)\Omega$

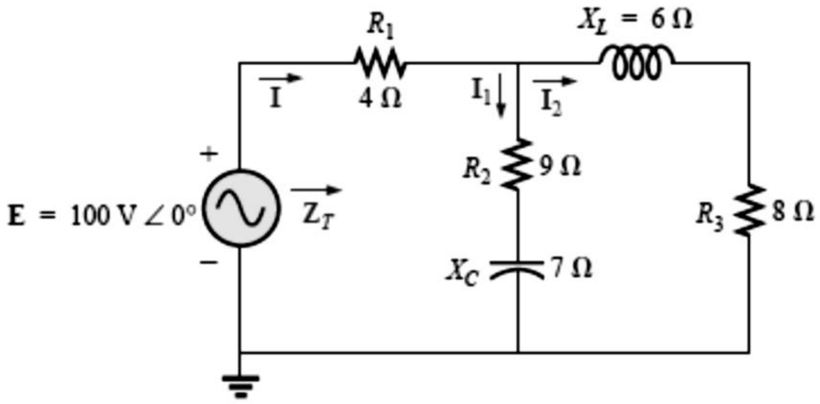
****

<u>Example:</u> For the network of figure

a) Calculate the total impedance Z_T

b) Compute I

C) Calculate I_1 and I_2

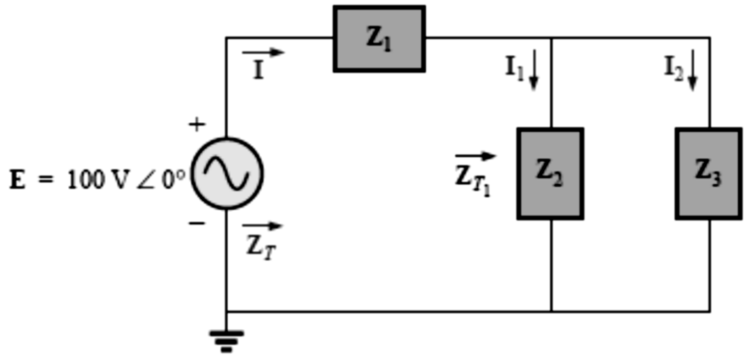


SOL/

a) $Z_1 = R_1 = 4\Omega$

$Z_2 = R_2 - jX_C = (9 - j7)\Omega$

$Z_3 = R_3 + jX_L = (8 + j6)\Omega$



$$Z_T = Z_1 + (Z_2 \parallel Z_3) = (10.68 + j0.28)\Omega$$

b) $$I = \frac{E}{Z_T} = 9.36A\angle{-1.5^O}$$

C) Current divider rule:

$$I_1 = I\frac{Z_3}{Z_2 + Z_3} = 5.5A\angle 38.72^O$$

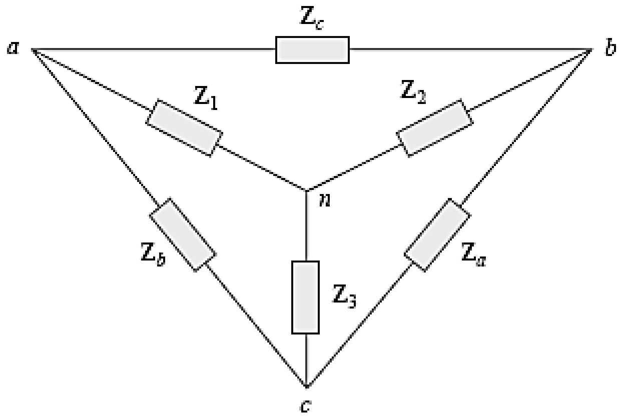
$$I_2 = I\frac{Z_2}{Z_2 + Z_3} = 6.27A\angle{-36^O}$$

OR applying KCL

$$I_2 = I - I_1 = 6.27A\angle{-36^O}$$

****

The delta-to-wye and wye-to-delta transformations that we applied to resistive circuits are also valid for impedances. With reference to figure shown, the conversion formulas are as follows.



<u>Y/Δ Conversion:</u>

$$Z_a = Z_2 + Z_3 + \frac{Z_2 Z_2}{Z_1}$$

$$Z_b = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}$$

$$Z_c = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

<u>Δ/Y Conversion :</u>

$$z_1 = \frac{z_b z_c}{z_a + z_b + z_c}$$

$$z_2 = \frac{z_c z_a}{z_a + z_b + z_c}$$

$$z_3 = \frac{z_a z_b}{z_a + z_b + z_c}$$

A delta or wye circuit is said to be *<u>balanced</u>* if it has equal impedances in all three branches.

When Δ-Y circuit balanced the equation become,

$$z_\Delta = 3 z_Y \quad or\ z_Y = \frac{1}{3} z_\Delta$$

When, $z_Y = z_1 = z_2 = z_3$ **, and** $z_\Delta = z_a = z_b = z_c$

Power in A.C. circuits

Alternating currents and voltages change their polarity during each cycle. It is not surprising therefore to find that power also pulsates with time. The product of voltage (v) and current (i) at any instant of time is called instantaneous power p(t), and is given by:

$$p(t) = v(t)i(t)$$

(a) Power to a purely resistance circuits

Let voltage $v(t) = V_m sin\omega t$ be applied to a circuit here, current is in phase with voltage assume $i(t) = I_m sin\omega t$, and the corresponding instantaneous power, P, is given by

$$p(t) = vi = (V_m sin\omega t)(I_m sin\omega t)$$

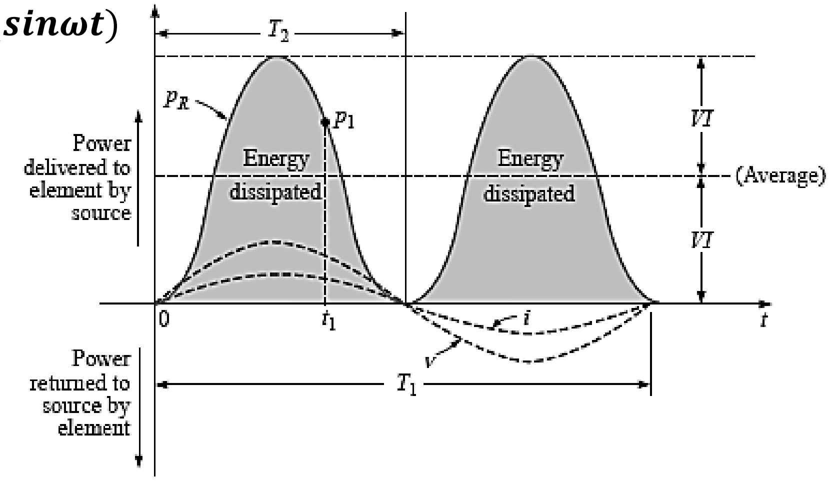
i.e

$$p(t) = V_m I_m sin^2 \omega t$$

Used the

trigonometric relationship

$$sin^2 \omega t = \frac{1}{2}(1 - cos2\omega t)$$



The power $p(t) = \frac{1}{2} V_m I_m (1 - cos2\omega t)$

The power has a frequency twice that of voltage and current. The power is always positive، having a maximum value of $V_m I_m$. The average or mean value of the power is $p = \frac{1}{2} V_m I_m$. Note that p is always positive (except when it is momentarily zero).

The r.m.s value of voltage $V_{r.m.s} = \frac{1}{\sqrt{2}} V_m = 0.707 V_m$, similarly, the r.m.s of current, $I_{r.m.s} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$, hence the average power p, developed in purely resistance is

$$P = \frac{1}{2} V_m I_m = VI \quad watt$$

Also, power

$$P = I^2 R \ , \qquad OR \qquad P = \frac{V^2}{R}$$

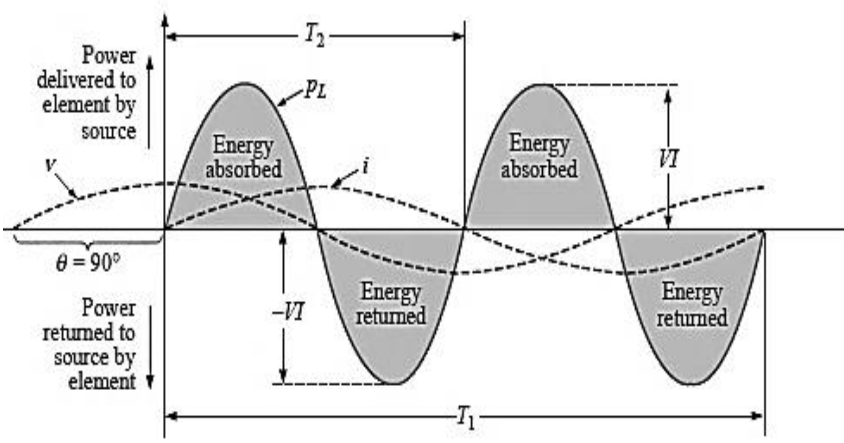
Where V and I are r.m.s values. Thus, the active power relationships for resistance circuits are the same for A.C as for D.C.

(b) Power to a purely inductive circuits

Let voltage $v(t) = V_m sin(\omega t + 90^0)$ be applied to a circuit containing pure inductance (theoretical case), the current is $i(t) = I_m sin\omega t$, and the corresponding instantaneous power, P, is given by

$$p(t) = V_m I_m sin\omega t \ sin(\omega t + 90^0)$$

$$p(t) = V_m I_m sin\omega t \ cos\omega t$$



$$2sin\omega t cos\omega t = sin2\omega t$$

Power, $p(t) = \frac{1}{2} V_m I_m sin2\omega t$

The frequency of power is twice that of voltage and current. For the power curve shown, the area above the horizontal axis is equal to area below, thus over a complete cycle the average power is zero. Also,

$$p = VIsin2\omega t$$

Where V and I are the magnitudes of r.m.s values of voltage and current respectively. The peak value of the curve VI is defined as the reactive power associated with a pure inductor and is given that symbol Q_L, the unit VAR(volt-amps-reactive), The Q is derived from the quadrature (90°) relationship between the various powers. In general, the reactive power associated with any circuit is defined to be

$$Q_L = VI \ sin\emptyset \qquad (volt - ampere\ reactive, VAR)$$

where ϕ is the phase angle between V and I.

For the inductor,

$$Q_L = VI \quad (VAR)$$

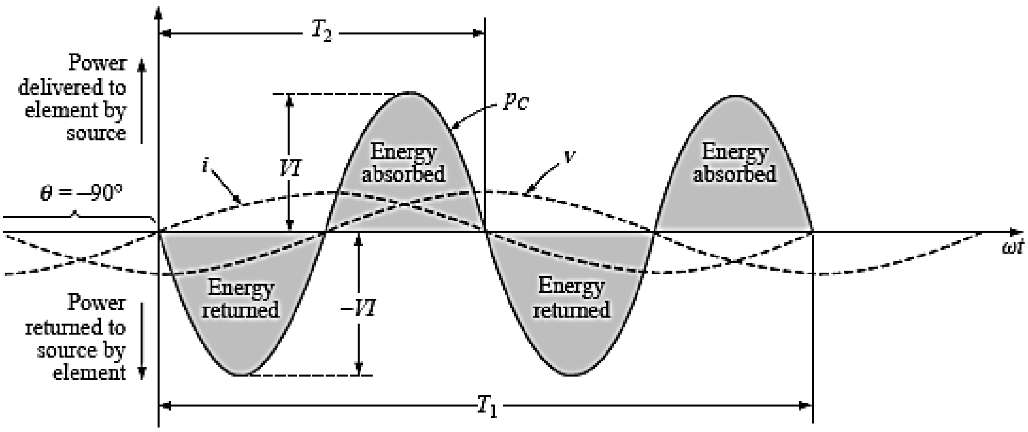
OR

$$Q_L = I^2 X_L = \frac{V^2}{X_L} \quad (VAR)$$

<u>Note that</u>: The VAR (like the watt) is a <u>*scalar quantity*</u> with magnitude only and <u>*No angle*</u>.

(C) Power to a purely capacitive circuits

Let a voltage $v(t) = V_m sin\,(\omega t - 90^0)$ be applied to circuit. The resulting current is $i(t) = V_m sin\omega t$



$$p(t) = vi = V_m I_m sin\omega t\, \sin(\omega t - 90^0)$$

Thus,

$$p(t) = -\frac{1}{2} V_m I_m sin2\omega t$$

The means that the average power to capacitance over full cycle is zero. There are no power losses associated with a pure capacitance. P_c=0 and the only power flowing in the circuit is reactive power.

Also

$$p(t) = -VIsin2\omega t$$

Where V and I are the magnitudes of the r.m.s values of voltage and current respectively. The reactive power

$$Q_C = I^2 X_C = VI = \frac{V^2}{X_C} \qquad (VAR)$$

(d) Power R-L or R-C circuits

Let a voltage $v(t) = v_m sin\omega t$ be applied to a circuit containing resistance and inductance or resistance and capacitance. Let the resulting current be $i(t) = I_m sin(\omega t + \emptyset)$, where phase angle ϕ will be positive for an R–C circuit and negative for an R–L circuit. The corresponding instantaneous power, p, is given by:

$$p(t) = i(t)v(t) = (V_m sin\,\omega t)(I_m sin(\omega t + \emptyset))$$

$$p(t) = V_m I_m sin\omega t\; sin(\omega t + \emptyset)$$

$$sinA\;\; sinB \;= -\frac{1}{2}[cos(A+B) - cos(A-B)]$$

$$p(t) = V_m I_m \left\{-\frac{1}{2}[cos(\omega t + \omega t + \emptyset) - cos(\omega t - (\omega t + \emptyset)]\right\}$$

$$p(t) = \frac{1}{2} V_m I_m [cos\emptyset - cos(2\omega t + \emptyset)]$$

Thus the average value of power

$$p = \frac{1}{2} V_m I_m cos\,\emptyset$$

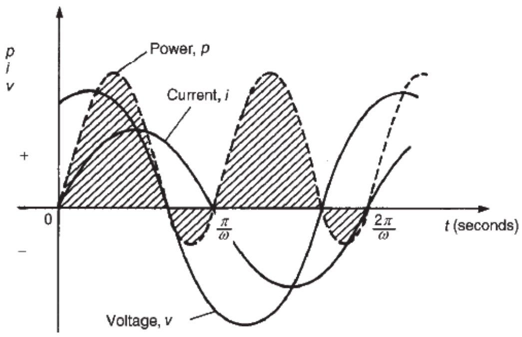
Also, in rms the average power

$$p = VIcos\emptyset$$

Since power is dissipated only in a pure resistance, the alternative equations for power,

$$P = I^2 R$$

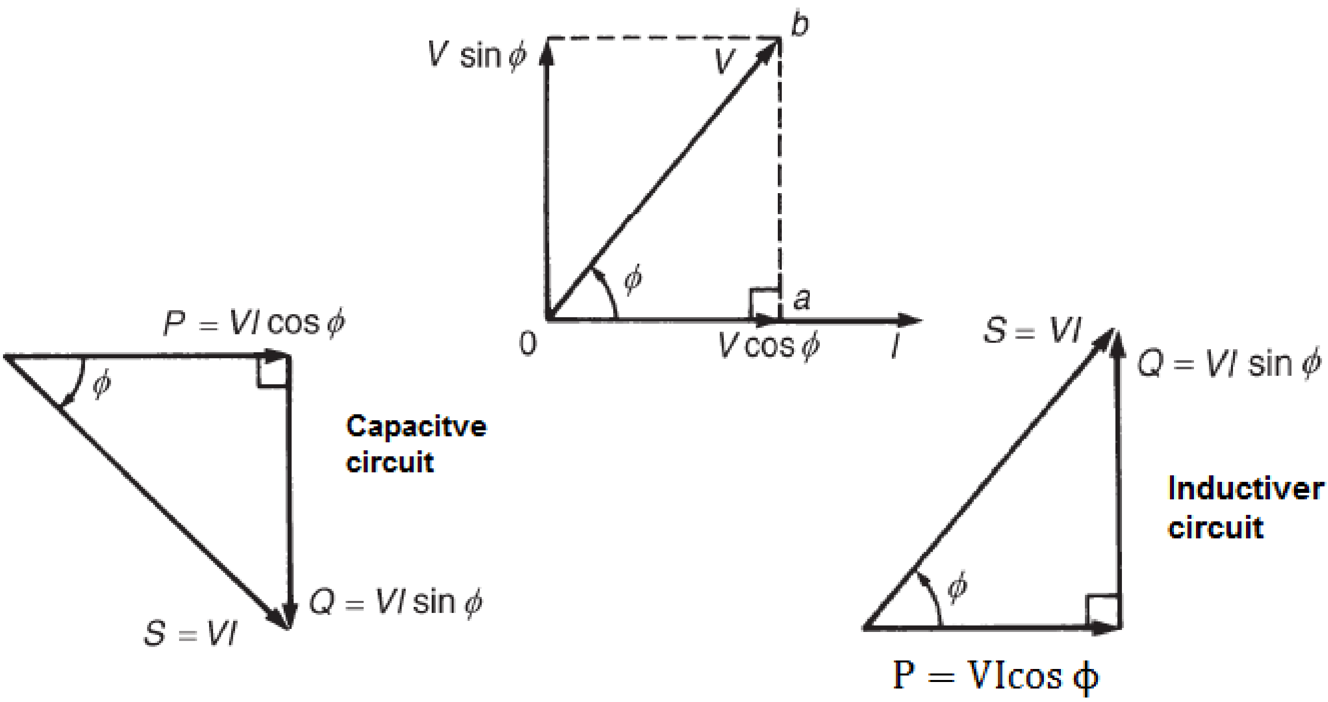
where I is the rms current flowing through the resistance



Power triangle and power factor:

A phasor diagram in which the current" I "lags the applied voltage V by angle ϕ (i.e., an inductive circuit) is shown in Figure. The horizontal component of V is Vcos ϕ, and the vertical component of V is Vsin ϕ. If each of the voltage phasors of triangle Oab is multiplied by I, is produced and is known as the 'power triangle'. Each side of the triangle represents a particular type of power:

$$\text{True or active power } P = VI\cos\phi \text{ watts (W)}$$
$$\text{Apparent power } S = VI \text{ voltamperes (VA)}$$
$$\text{Reactive power } Q = VI\sin\phi \text{ vars (var)}$$



<u>Complex power</u> : its important in power analysis because it contains all the information pertaining to the power absorbed by a given load. The three quantities average power(active power, real power), apparent power, and reactive power can be related in the vector domain by

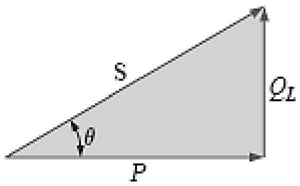
$S = P + Q$ With

$P = P\angle 0^0,\quad Q_L = Q_L\angle 90^0\ ,\ Q_C = Q_C\angle -90^0$

For an inductive load, the *phasor power* S, as it is often called, is defined by

$S = P + jQ_L$

as shown in Figure



Inductive load

The 90° shift in Q_L from P is the source of another term for reactive power: quadrature power. For a capacitive load, the *phasor power* S is defined by

$S = P - jQ_C$

$S = VI^* = I^2Z = \frac{V^2}{Z^*}$

> The power triangle is not a phasor diagram since quantities P,Q and S are mean values and not r.m.s values of sinusoidally varying quantity . Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity، its real part is real power P and its imaginary part is reactive power Q.

<u>Power factor is defined as:</u>

$$Power\ Factor = \frac{(Active\ power), p}{(Apparent\ power), s} = cos(\phi_V - \phi_I)$$

For sinusoidal voltage and currents

$$power\ factor = \frac{P}{S} = \frac{VIcos(\phi_V - \phi_I)}{VI} = cos\ \phi$$

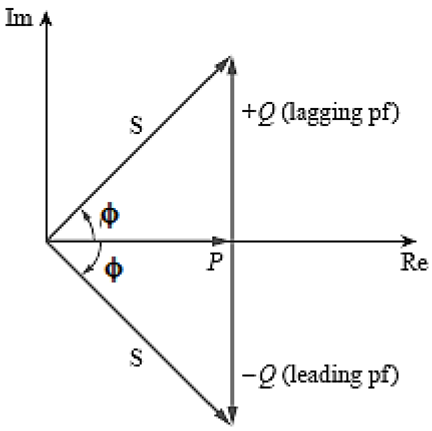
For the impedance triangle

$F._P = cos\emptyset = \frac{R}{Z}$ (The power factor cannot exceed one)

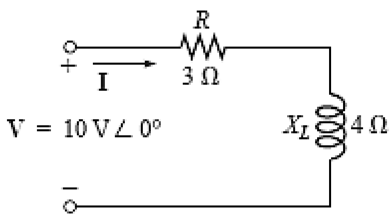
A circuit in which current lags voltage (i.e., an inductive circuit) is said to have a lagging power factor, and indicates a lagging reactive power Q.

A circuit in which current leads voltage (i.e., a capacitive circuit) is said to have a leading power factor, and indicates a leading reactive power Q. Note that:

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).



__Example:__ For the R-L circuit as shown in figure. Find (a) the complex and apparent power, (b) the power factor (c) the real and reactive power.



Sol/

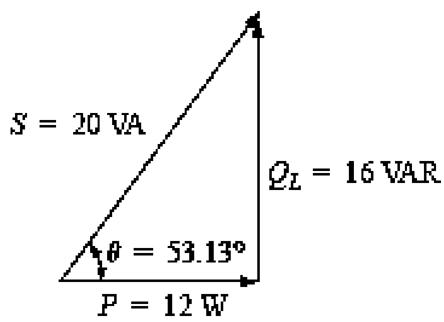
$$I = \frac{V}{Z_L} = \frac{10\angle 0^0}{3+j4} = 2A\angle{-53.13^0}$$

The real power

$$P = I^2R = (2)^2(3) = 12W$$

The reactive power

$$Q_L = I^2X_L = (2)^2(4) = 16VAR$$



Complex power apparent is

$S = P + jQ = 12 + j16 = 20A\angle 53.13^0$, Also,

$$S = VI^* = (10\angle 0^0)(2\angle 53.13^0) = 20A\angle 53.13^0 = 12 + j16$$

$$F_{.P} = cos\varphi = \frac{P}{S} = 0.6$$

Example: The voltage across a load is $v(t) = 60cos(\omega t - 10^0)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5cos(\omega t + 50^0)$A. Find: (a) the complex and apparent powers ،b) the real and reactive powers, and (c) the power factor and the load impedance.

SOL/

$$S = VI^* = \left(\frac{60}{\sqrt{2}}\angle - 10^0\right)\left(\frac{1.5}{\sqrt{2}}\angle{-50^0}\right) = 45\angle{-60^0}AV$$

The apparent power

$$S = |S| = 45VA$$

The complex power in rectangular form as

$$S = 45\angle{-60^0} = 22.5 - j38.97$$

Since $S = P + jQ$

P=22.5 W

Q=-38.97 VAR, $\phi = \phi_V - \phi_I = -60^0$

$$P_{.F} = cos(-60^0) = 0.5\ (leading)$$

$$Z = \frac{60\angle{-10^0}}{1.5\angle 50^0} = 40\angle{-60^0}\Omega = (20 - j34.64)\Omega$$

Which capacitive impedance

THE TOTAL P, Q, AND S: The total number of watts, volt-amperes reactive, and volt-amperes, and the power factor of any system can be found using the following procedure:

1) Find the real power and reactive power for each branch of the circuit.
2) The total real power of the system (P_T) is then the sum of the average power delivered to each branch.

3) The total reactive power (Q_T) is the difference between the reactive power of the inductive loads and that of the capacitive loads.

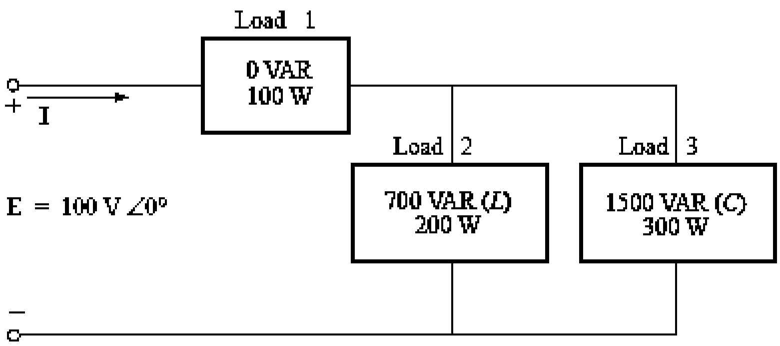
4) The total apparent power is $S_T = \sqrt{P_T^2 + Q_T^2}$

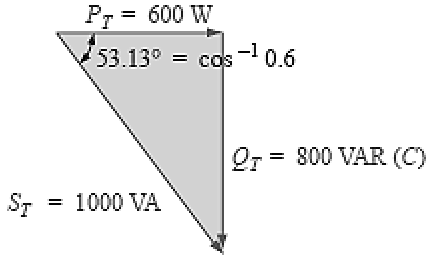
5) The total power factor is $P_{.F} = cos\varphi = \frac{P_T}{S_T}$

There are two important points in the above tabulation. First, the total apparent power must be determined from the total average and reactive powers and cannot be determined from the apparent powers of each branch. Second, and more important, it is not necessary to consider the series-parallel arrangement of branches. In other words, the total real, reactive, or apparent power is independent of whether the loads are in series, parallel, or series-parallel. The following examples will demonstrate the relative ease with which all of the quantities of interest can be found.

6) The total complex power in network is $S = S_1 + S_2 + \cdots S_N$

EXAMPLE : Find the total number of watts, volt-amperes reactive, and volt-amperes, and the power factor *F.p* of the network in Figure. Draw the power triangle and find the current in phasor form.





Sol/

The total power dissipated

P_T=100+200+300=600 watt

The total reactive power of network

Q_T=0+700-1500=-800VAR

$S_T = \sqrt[2]{(600)^2 + (800)^2} = 1000VA$

(Note that: The total apparent power $S_T \neq$ Sum of each branch)

$$F_P = \frac{P_T}{S_T} = 0.6\ Leading(c)$$
$$\boldsymbol{I} = \frac{\boldsymbol{S_T}}{\boldsymbol{V}} = 10A$$
$$\boldsymbol{I} = \mathbf{10}\boldsymbol{A}\angle\mathbf{+53.13^0}$$

The plus sign is associated with the phase angle since the circuit is predominantly capacitive.

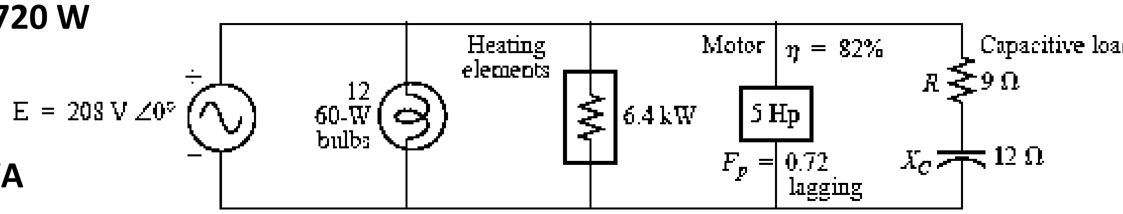
****

<u>EXAMPLE:</u> For the system of Figure

a) **Find the average power, apparent power, reactive power, and *Fp* for each branch.**
b) **Find the total number of watts, volt-amperes reactive, and voltamperes, and the power factor of the system. Sketch the power triangle.**
c) **Find the source current *I*.**

Sol/

1) **Bulbs**



P_1=12(60)=720 W

Q_1=0VAR

S_1=P_1=720VA

F.P_1=1

2) **Heating elements**

P_2=6.4KW

Q_2= 0VAR

S_2=P_2=6.4VA

F.P=1

3) **Motor**

$P_{O(Mech.)}$=746(5)=3730W

$$\eta = \frac{P_O}{P_i} \rightarrow P_{i(elec.)} = \frac{P_O}{\eta} = 4548.78W = P_3$$

$$F.P = 0.72\ Lagging$$

$$P_3 = S_3 cos\phi \quad \rightarrow S_3 = \frac{P_3}{cos\emptyset} = 6317.75VA$$

$$\emptyset = cos^{-1}(0.72) = 43.95^0$$

$$Q_3 = S_3 sin\emptyset = 4384.71VAR(L)$$

4) Capacitive load

$$I = \frac{E}{Z} = \frac{208\angle 0}{9 - j12} = 13.87A\angle 53.13^0$$

$$P_4 = I^2 R = 1731.39W$$

$$Q_4 = I^2 X_C = 2308.52VAR(C)$$

$$S_4 = \sqrt[2]{P_4^2 + Q_4^2} = 2885.65VA$$

$$P.F = \frac{P_4}{S_4} = 0.6\ Leading$$

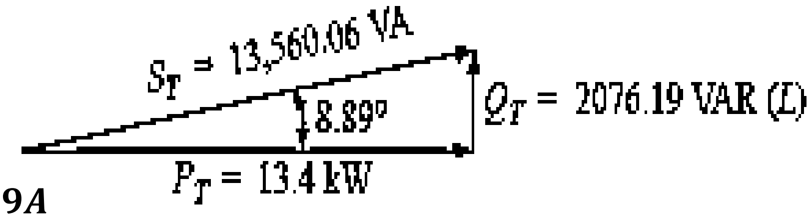
$$P_T = P_1 + P_2 + P_3 + P_4 = 1340017W$$

$$Q_T = Q_1 \pm Q_2 \pm Q_3 \pm Q_4 = 2076.19VAR\ (L)$$

$$S_T = \sqrt[2]{P_T^2 + Q_T^2} = 13560.06VA$$

$$F.P = \frac{P_T}{S_T} = 0.988\ lagging$$

$$\emptyset = cos^{-1}(0.988) = 8.89^0$$



$$S_T = E \times I \quad \rightarrow I_T = \frac{S_T}{E} = 65.19A$$

$$I = 65.19\angle -8.89^0 Lagging$$

E Leads I by 8.89^0

POWER-FACTOR CORRECTION:

Power factor correction is necessary for economic reason, it is the process of improving the power factor of a load by reducing the overall reactive power. For a particular active power supplied, a high power factor reduces the current flowing in a supply system and therefore reduces the cost of cables, transformers, switchgear and generators, because a load with poor power factor can draw excessive current.

One method of improving the power factor of an inductive load is to connect a bank of capacitors in parallel with the load. Capacitors are rated in reactive volt-amperes and the effect of the capacitors is to reduce the reactive power of the system without changing the active power. Most residential and industrial loads on a power system are inductive, i.e. they operate at a lagging power factor.

Another method of power factor improvement, besides the use of static capacitors, is by using synchronous motors; such machines can be made to operate at leading power factors

Power factor correction results in the apparent power S decreasing (from 0a to 0b in Figure) and thus the current decreasing, so that the power distribution system is used more efficiently.

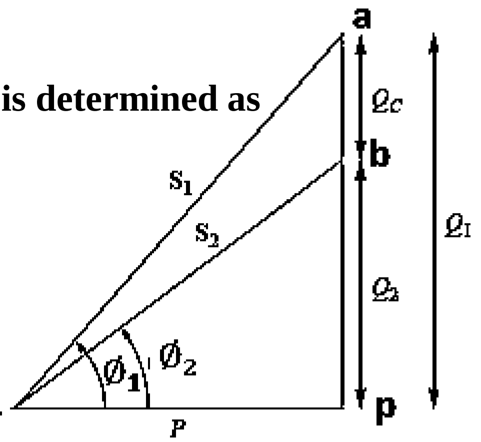
The reduction in the reactive power is caused by the shunt capacitor; that is

$$Q_C = Q_1 - Q_2 = P(tan\emptyset_1 - tan\emptyset_2)$$

$$Q_C = \frac{V_{rms}^2}{X_C} = \omega V_{rms}^2$$

The value of the required shunt capacitance *C* is determined as

$$\boxed{C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P(tan\emptyset_1 - tan\emptyset_2)}{\omega V_{rms}^2}}$$



Why electrical apparatus (generators, interconnecting wires, transformers etc) is rated in AV instead on watts?

EXAMPLE: **A 5-hp motor with a 0.6 lagging power factor and an efficiency of 92% is connected to a 208-V, 60-Hz supply.**

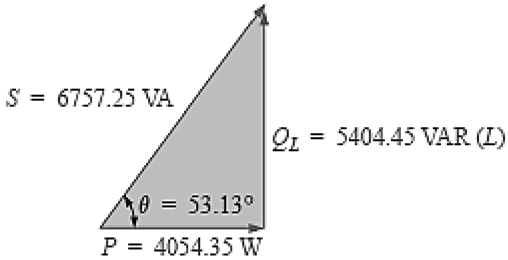
a) **Establish the power triangle for the load.**
b) **Determine the power-factor capacitor that must be placed in parallel with the load to raise the power factor to unity.**
c) **Determine the change in supply current from the uncompensated to the compensated system.**
d) **Find the network equivalent of the above, and verify the conclusions.**

Sol/(a)P_0=5(746)=3730W

$$P_i = \frac{P_0}{\eta} = 4054.35W\,, \qquad \phi = cos^{-1}(0.6) = 53.13^0$$

$$Q_L = P_i tan\phi = 5405.8VAR\ (L)$$

$$S = \sqrt[2]{P_i^2 + Q_C^2} = 6757.25VA$$



a) $Q_C = \frac{V^2}{X_C} \rightarrow \qquad X_C = \frac{208^2}{5405.8} = 8\Omega$

$$C = \frac{1}{2\pi f X_C} = 331.6\mu F$$

b) At 0.6 F.P

$$S = VI = 6757.25VA$$

$$I = \frac{S}{V} = 32.49A$$

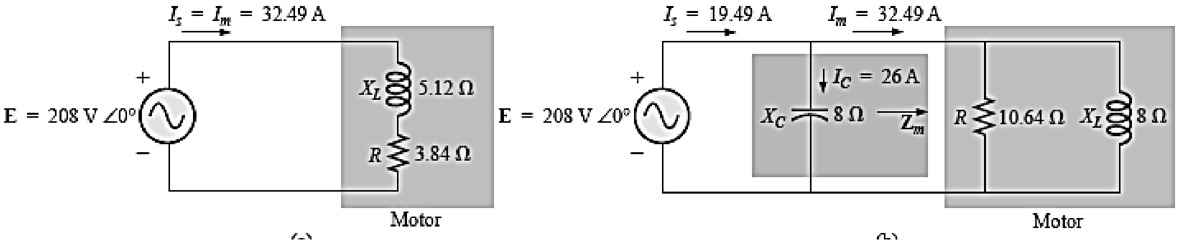
AT unity P.F

$$S = VI = 4054.3\,5VA \quad \rightarrow \quad I = \frac{S}{V} = 19.49A$$

producing a 40% reduction in supply current.

d) $\phi = cos^{-1}(0.6) = 53.13^0$

$$I = 32.49A\angle{-53.13^0} \qquad , \qquad Z = \frac{E}{I} = (3.84 + j5.12)\Omega$$

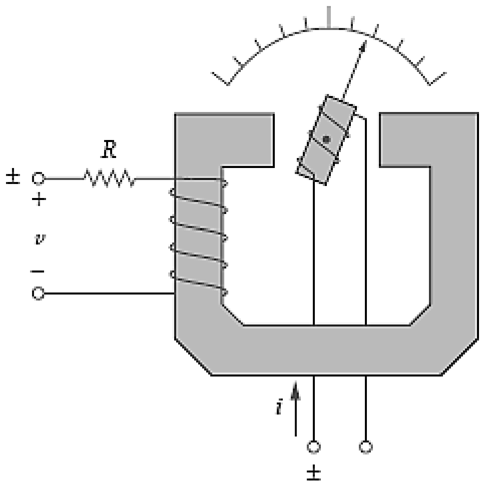


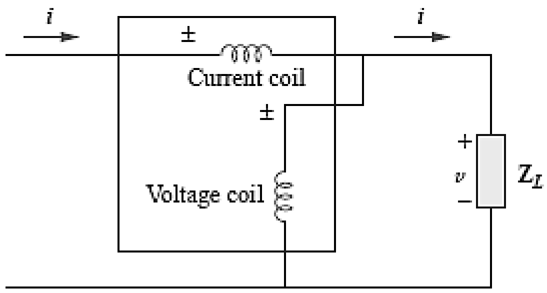
WATTMETERS AND POWER-FACTOR METERS

The digital display wattmeter of Figure employs a sophisticated electronic package to sense the voltage and current levels and, through the use of an analog-to-digital conversion unit, display the proper digits on the display.

The electrodynamometer (wattmeter) was introduced in along with its movement and terminal connections a wattmeter that consists essentially of two coils, the current coil and the voltage coil. A current coil with very low impedance (ideally zero) is connected in series with the load and responds to the load current. The voltage coil with very high impedance (ideally infinite) is connected in parallel with the load as shown in Figure and responds to the load voltage. The current coil acts like a short circuit because of its low impedance; the voltage coil behaves like an open circuit because of its high impedance. As a result, the presence of the wattmeter does not disturb the circuit or have an effect on the power measurement.

When the two coils are energized, the mechanical inertia of the moving system produces a deflection angle that is proportional to the





EFFECTIVE RESISTANCE:

Up to now, we have assume that resistance is constant, independent of frequency. However, this is not entirely true. For a number of reasons, the resistance of a circuit to AC is greater than its resistance to DC. The resistance of a conductor as determined by the equation R $R = \rho\left(\frac{\iota\ell}{A}\right)$ is often called the DC, ohmic, or geometric resistance. It is a constant quantity determined only by the material used and its physical dimensions. In AC circuits, the actual resistance of a conductor (called the effective resistance) differs from the DC resistance because of the varying currents and voltages that introduce effects not present in DC circuits. These effects include radiation losses, skin effect, eddy currents, and hysteresis losses. The first two effects apply to any network, while the latter two are concerned with the additional losses introduced by the presence of ferromagnetic materials in a changing magnetic field.

Experimental Procedure

The effective resistance of an AC circuit cannot be measured by the ratio *V/I* since this ratio is now the impedance of a circuit that may have both resistance and reactance. The effective resistance can be found, however, by using the power equation, $P=I^2R$

$$R_{eff} = \frac{P}{I^2}$$

A wattmeter and an ammeter are therefore necessary for measuring the effective resistance of an AC circuit.

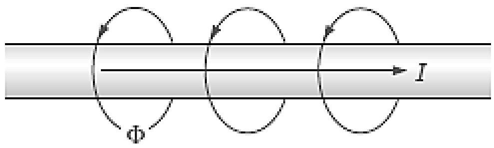
Radiation Losses

Let us now examine the various losses in greater detail. The radiation loss is the loss of energy in the form of electromagnetic waves during the transfer of energy from one element to another. This loss in energy. requires that the input power be larger to establish the same current *I*, causing *R* to increase as determined by $P=I^2R$. At a frequency of 60 Hz, the effects of radiation losses can be completely ignored. However, at radio frequencies, this is an important effect and may in fact become the main effect in an electromagnetic device such as an antenna. The resistance effect here is known as radiation resistance.

This resistance is much higher than simple DC resistance. For example, a TV transmitting antenna may have a resistance of a fraction of an ohm to DC but several hundred ohms effective resistance at its operating frequency.

Skin Effect

The amount of charge flowing in AC circuits changes with time, the magnetic field surrounding the moving charge (current) also changes. Recall also that a wire placed in a changing magnetic field will have an induced voltage across its terminals as determined by Faraday's law, $e = N\,(d\phi/dt)$. The higher the frequency of the changing flux as determined by an alternating current, the greater the induced voltage will be.



Demonstrating the skin effect on the effective resistance of a conductor.

These effects are more pronounced at the center of the conductor than at the surface because the center is linked by the changing flux inside the wire as well as that outside the wire. As the frequency of the applied signal increases, the flux linking the wire will change at a greater rate. An increase in frequency will therefore increase the counter-induced voltage at the center of the wire to the point where the current will, for all practical purposes, flow on the surface of the conductor. At 60 Hz, the skin effect is almost noticeable. However, at radio frequencies the skin effect is so pronounced that conductors are frequently made hollow because the center part is relatively ineffective (for this reason, hollow conductors are often used instead of solid wires).

Hysteresis and Eddy Current Losses

As mentioned earlier, hysteresis and eddy current losses will appear when a ferromagnetic material is placed in the region of a changing magnetic field. To describe eddy current losses in greater detail, we will consider the effects of an alternating current passing through a coil wrapped around a ferromagnetic core. As the alternating current passes through the coil, it will develop a changing magnetic flux ϕ linking both the coil and the core that will develop an induced voltage within the core as determined by Faraday's law. This induced voltage and the geometric resistance of the core $R_C = \rho(l/A)$ cause currents to be developed within the core, $i_{core} = (e_{ind} / R_C)$, called eddy currents. The currents flow in circular paths, as shown in Figure, changing direction with the applied ac potential. The eddy current losses are determined by

$$P_{eddy} = I_{eddy}^2 R_{core}$$

Eddy current losses can be reduced if the core is constructed of thin, laminated sheets of ferromagnetic material insulated from one another and aligned parallel to the magnetic flux. Such construction reduces the magnitude of the eddy currents by placing more resistance in their path.

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