Design of Cylindrical Shells

Cylindrical shells are used in petrochemical industries. They are also used in heat exchangers of the shell and tube type. These vessels are easy to fabricate and install and economical to maintain. The design procedures in pressure vessel codes for cylindrical shells are mostly based on linear elastic assumption, occasionally allowing for limited inelastic behavior over a localized region. **The shell thickness is the major design parameter and is usually controlled by internal pressure and sometimes by external pressure which can produce buckling.** Applied loads are also important in controlling thickness and so are the **discontinuity and thermal stresses.** The basic thicknesses of cylindrical shells are based on simplified stress analysis and allowable stress for the material of construction.

**Thin-shell equations**

\[
\sigma_\theta = \frac{pR}{t} \quad (1)
\]

\[
\sigma_t = \frac{pR}{2t} \quad (2)
\]
Fig.(1) Thin cylindrical Shell

Thick-shell equations

The following stress can be induced in thick shell cylinder (see figure 2)

\[ \sigma_{\text{rad}} = \frac{A}{r^2} + 2B \]  

(3)

\[ \sigma_{\text{hoop}} = -\frac{A}{r^2} + 2B \]  

(4)

Fig.(2) Thick Cylindrical Shell

The constants A and B are determined from the following boundary conditions:
\[ \sigma_{rad} = -p \text{ at } r = R_i \]  \hfill (5)  
\[ \sigma_{rad} = 0 \text{ at } r = R_o \]  \hfill (6)

Substituting equations (5) and (6) into (3) and (4) to get

\[
A = -\frac{R_i^2 R_o^2 p}{(R_o^2 - R_i^2)}
\]

and

\[
B = \frac{R_i^2 p}{2(R_o^2 - R_i^2)}
\]

Define \[ m = \frac{R_o}{R_i} \]

\[
\sigma_{rad} = \frac{p}{(m^2 - 1)} \left[ 1 - \frac{R_o^2}{r^2} \right]
\]  \hfill (7)

\[
\sigma_{hoop} = \frac{p}{(m^2 - 1)} \left[ 1 + \frac{R_o^2}{r^2} \right]
\]  \hfill (8)

Figure (3) shows the pressure distribution in radial and Hoop directions.

Fig.(3) Pressure distribution in thick shell cylinder
The longitudinal stress, $\sigma_{\text{long}}$, can be determined as:

$$\sigma_{\text{long}} = \frac{p R_i^2}{(R_o^2 - R_i^2)} = \frac{p}{(m^2 - 1)} \quad (9)$$

It should be noted however that the solutions indicated by Equations (7), (8), and (9) are valid for regions remote from discontinuities.

**Approximate Equations:**

For a moderately thick shell employing thin-shell theory and using the mean radius $R_m$ we get the expression of the hoop stress, $\sigma_{\text{hoop}}$, as

$$\sigma_{\text{hoop}} = \frac{p R_m}{t} = \frac{p(R_i + t/2)}{t} \quad (10)$$

Equating the hoop stress, $\sigma_{\text{hoop}}$, to the code-allowable design stress, $S_m$,

To get

$$t = \frac{p R_i}{S_m - 0.5p} \quad (11)$$

Rewriting Equation (11) in terms of the outside radius, $R_o$, to get

$$\sigma_{\text{hoop}} = \frac{p R_m}{t} = \frac{p(R_o - t/2)}{t} \quad (12)$$

**Hence**

$$t = \frac{p R_o}{S_m + 0.5p} \quad (13)$$

The equations in the ASME Boiler and Pressure Vessel Code are based on equating the maximum membrane stress to the allowable stress corrected for weld joint efficiency. The allowable stress, $S_m$, is replaced by the term $SE$ (S=Allowable stress and E is the joint efficiency).

**Discontinuity stresses in pressure vessels**

Let us take the special case of discontinuity at a juncture between a cylindrical vessel and a hemispherical head subjected to internal pressure.
p. For simplicity let us assume the spherical head and the cylindrical shell are of the same thickness. If the mean radius and the thickness of the shell are denoted by $R_m$ and $t$ respectively, then the hoop and the longitudinal stresses in the cylindrical shell are given by:

$$
\sigma_{\text{hoop}}^c = \frac{pR_m}{t}
$$

$$
\sigma_{\text{long}}^c = \frac{pR_m}{2t}
$$

The hoop and the longitudinal stresses in the spherical shell are given by:

$$
\sigma_{\text{hoop}}^s = \frac{pR_m}{2t}
$$

$$
\sigma_{\text{long}}^s = \frac{pR_m}{2t}
$$

The radial growth or dilation of the cylindrical shell under internal pressure $p$ is given by

$$
\delta_r^c = \frac{pR_m^2}{2Et} (2 - \nu)
$$

That of the spherical region is given by

$$
\delta_r^s = \frac{pR_m^2}{2Et} (1 - \nu)
$$

If the spherical and the cylindrical portions were separated, the difference in the radial growth would be

$$
\delta_r = \delta_r^c - \delta_r^s = \frac{pR_m^2}{2Et}
$$

In the actual vessel the hemispherical head and the cylindrical shell are kept in place by shear force, $V$ and moment $M$ per unit circumference. These discontinuity forces produce local bending stresses in the adjacent
portions of the vessel. The deflection and the slope induced at the edges of the cylindrical and spherical portions by the force $V$ are equal. The continuity at the juncture will be satisfied if $M$ equals zero and $V$ is such that it produces a deflection of $\delta/2$.

\[
\frac{\delta}{2} = \frac{2V\beta R_m^2}{Et}
\]

\[
V = \frac{p}{8\beta}
\]

Where

\[
\beta = \sqrt{\frac{3(1-\nu^2)}{R_m^2 t^2}}
\]

**Design of heads and covers**

Heads are one of the important parts in pressure vessels and refer to the parts of the vessel that confine the shell from below, above, and the sides. The ends of the vessels are closed by means of heads before putting them into operation. The heads are normally made from the same material as the shell and may be welded to the shell itself. They also may be integral with the shell in forged or cast construction. The head geometrical design is dependent on the geometry of the shell as well as other design parameters such as operating temperature and pressure. The heads may be of various types such as:

- Flanged
- Ellipsoidal
- Torispherical
- Hemispherical
- Conical
- Toriconical
The different types of heads are shown in figure (4). The geometry of the head is selected based on the function as well as on economic considerations, and methods of forming and space requirements. The elliptical and torispherical heads are most commonly used. The carbon steel hemispherical heads are not so economical because of the high manufacturing costs associated with them. They are thinner than the cylindrical shell to which they are attached, and require a smooth transition between the two to avoid stress concentration effects. The thickness values of the elliptical and torispherical heads are typically the same as the cylindrical shell sections to which they are attached. Conical and toriconical heads are used in hoppers and towers.
Fig. (4) Different types of heads

**Hemispherical heads under internal pressure**

The force due to internal pressure is resisted by the membrane stress in the shell (see Figure 5). Because of the geometrical symmetry, the stresses are the same in all directions, and are denoted by $S$. 
Fig.(5) Hemispherical end

Hence

\[ P \pi R^2 = 2 \pi RS_t \]

And

\[ S = \frac{PR}{2t} \] \hspace{1cm} (14)

**ASME equation for hemispherical heads**

The design thickness of a hemispherical head is given by

\[ t = \frac{PR}{25E - 0.2P} \] \hspace{1cm} (15)

where \( R \) is the inside radius, \( S \) is the allowable shear, and \( E \) is the joint efficiency.

**Ex:**

A hemispherical head having an inside radius of 380 mm is subjected to an internal pressure of 28 MPa. The allowable stress is 160 MPa. What is the required thickness using the shell theory and the ASME equation (assume joint efficiency, \( E = 1 \))?
Thin-shell theory

\[ S = \frac{PR}{2t} \]

Or

\[ t = \frac{PR}{2S} = \frac{28 \times 380}{320} = 33.25 \text{ mm} \]

ASME equation

\[ t = \frac{28 \times 380}{2(160)(1 - 0.2(28))} = 33.8 \text{ mm} \]

**ASME design equation for ellipsoidal heads**

For an internal pressure \( P \), the thickness \( t \) of the ellipsoidal head is given by

\[ t = \frac{PDK}{2SE - 0.2P} \]  \hspace{1cm} (16)

where \( D = \) diameter of the shell to which the head is attached, \( E = \) joint efficiency, \( S = \) allowable stress, and \( K = \) stress intensity factor.

K is given by the following expression:

\[ K = \frac{1}{6} \left[ 2 + \left( \frac{a}{b} \right)^2 \right] \]  \hspace{1cm} (17)

where \( a \) and \( b \) are the semi-major and semi-minor axes of the ellipse.

**Reinforcement of openings**

The philosophy is based on providing additional material in the region of the opening by thickening the shell or adding a pad material. The additional material is deemed effective in carrying the higher loads. On most vessels, it is provided on the outside of the vessel.

The placement of this additional material is important. We note that at a distance of \( r = 2a \) for all three situations, namely a plate under tension, a cylindrical shell under internal pressure, and a spherical shell under internal pressure, the stresses die out sufficiently. So this distance is
generally taken at the boundary limit for the effective reinforcement to
the vessel surface. This is indicated in Figure(6).

Fig.(6) Circumferential and transversal reinforcement extents.