# Chapter One

# Direct Design Method

## Flat slabs:

Concrete slabs may be carried directly by columns as shown in Fig.(1-1) without the use of beams or girders. Such slabs are described as flat plates and are commonly used where spans are not large and loads not particularly heavy.

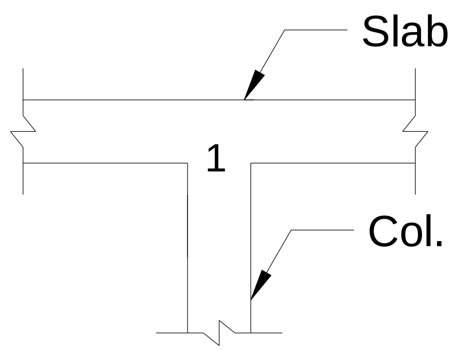



Fig.(1-1) Slab supported directly on column.

At point (1) there is more (-M) and shearing stress, and col. try to punch the slab.

Flat slab construction shown in Fig.(1-2) is also beamless but incorporates a thickened slab region (drop panels) and column capitals.

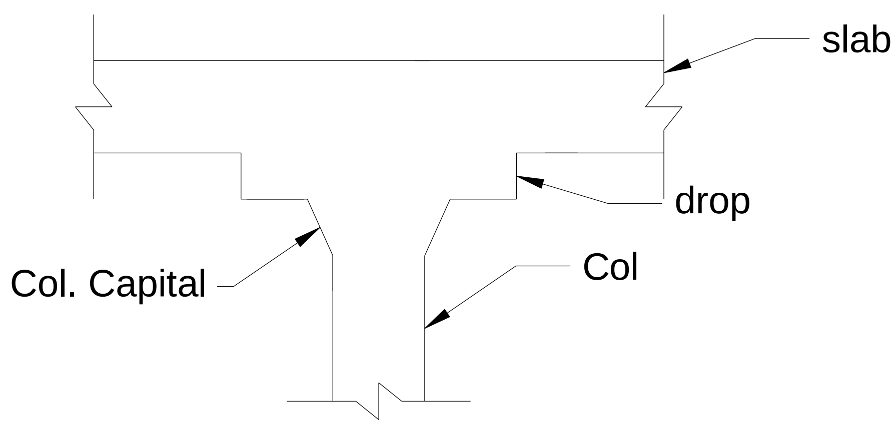


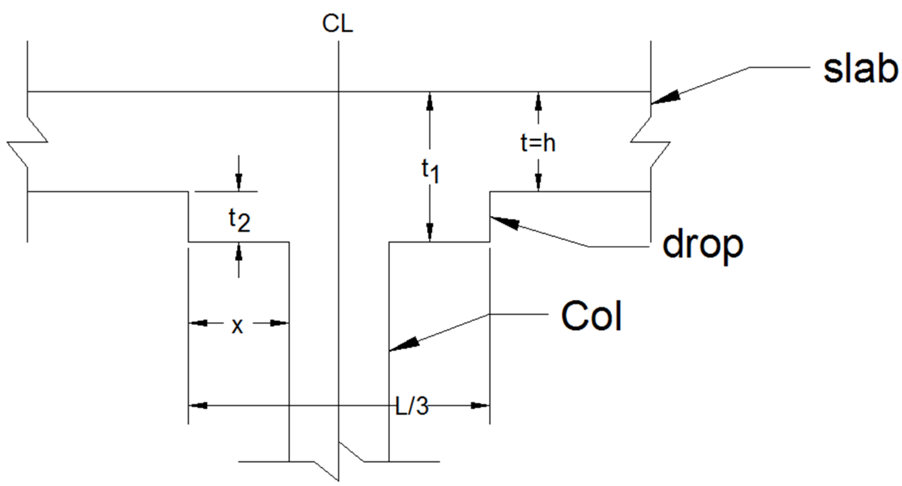

Fig.(1-2) Flat slab.

**Column capital:** An element at the end of the column to give a wider support for the floor slab

Column capital and drop panel are used to reduce:

1- Stresses due to shear.
2- Negative bending around the columns.

Size of drop panel shall be in accordance with the following ACI-code (8.2.4 –ACI-2014). Drop panel shall be extended in each direction from center line of support a distance not less than one-sixth the span length measured from center to center of supports in that direction.




The side of the drop panel shall be at least $(L/3)$,

Where

$L$: $(c.to.c)$

$t$: Thickness of slab

$t_1$:Thickness of slab with drop

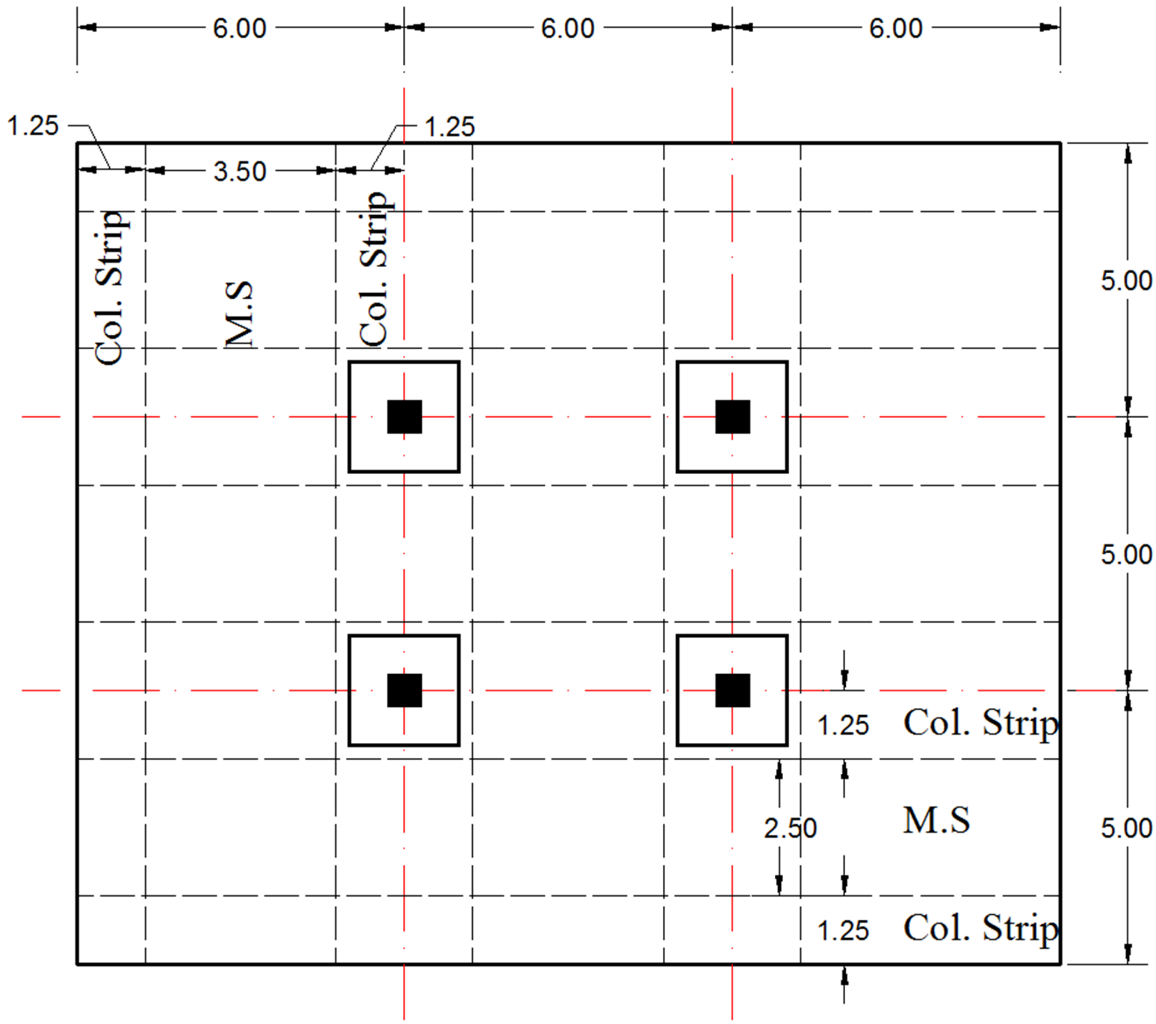
$t_2$:Thickness of drop

$t_2 \geq \frac{t}{4}$

## Bending moments in flat slab floors:

For purposes of design, a typical panel is divided into column strips and middle strips.

ACI-2014 (8.4.1.5) column strip is a design strip with a width on each side of a column centerline equal to $(0.25l_1)$ or $(0.25l_2)$, whichever is less. Column strip includes beams, if any.

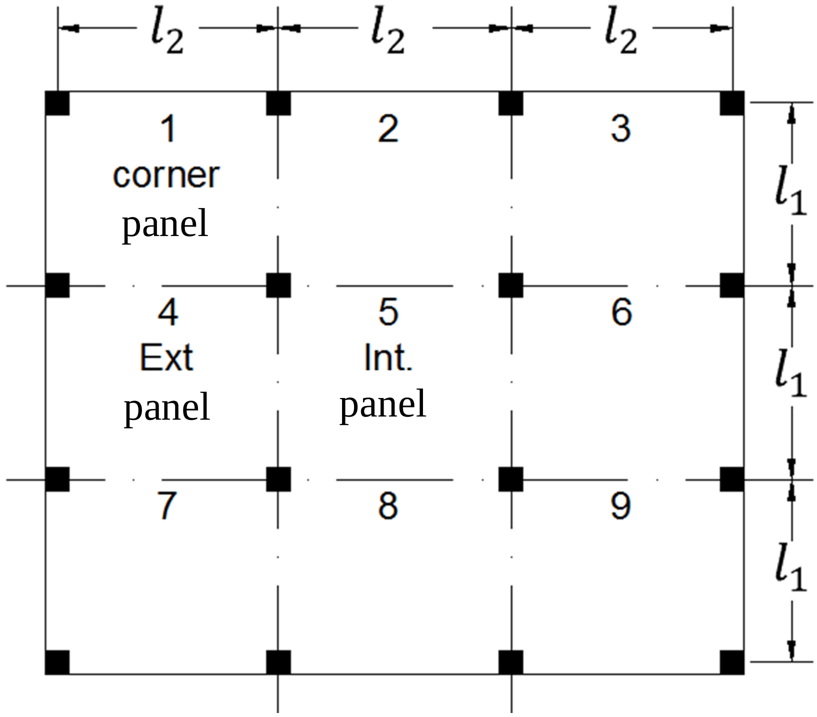



In all cases

$l_1$: is the span in the direction of the moment analysis (c. to c.).

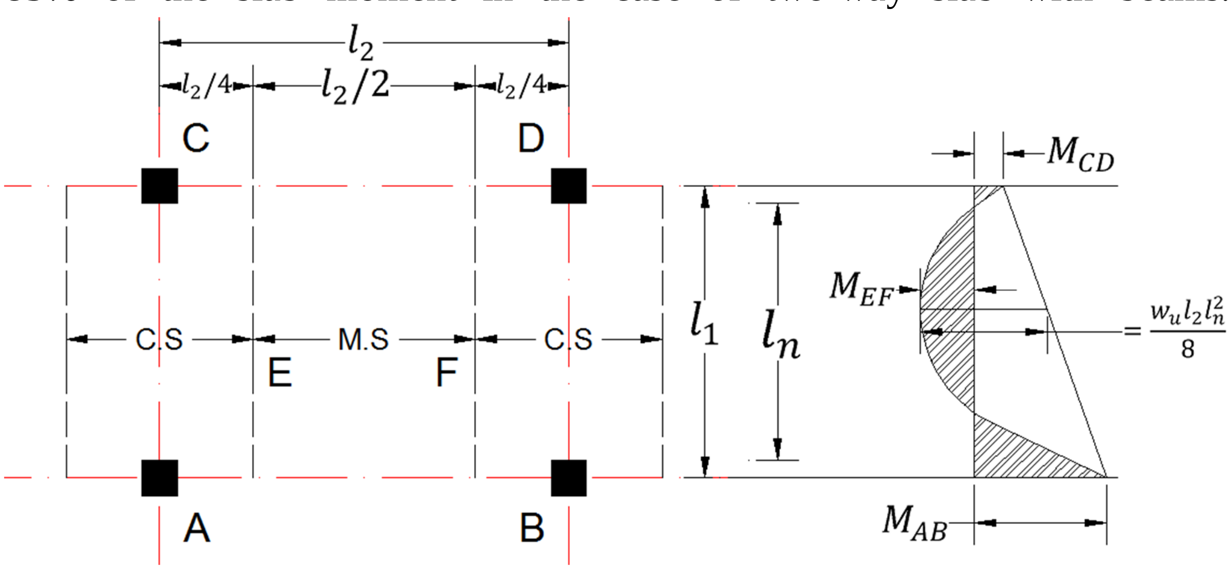
$l_2$: is the span in the lateral direction (transvers to$l_1$c.to.c).

$l_n$: clear span in $l_1$direction.



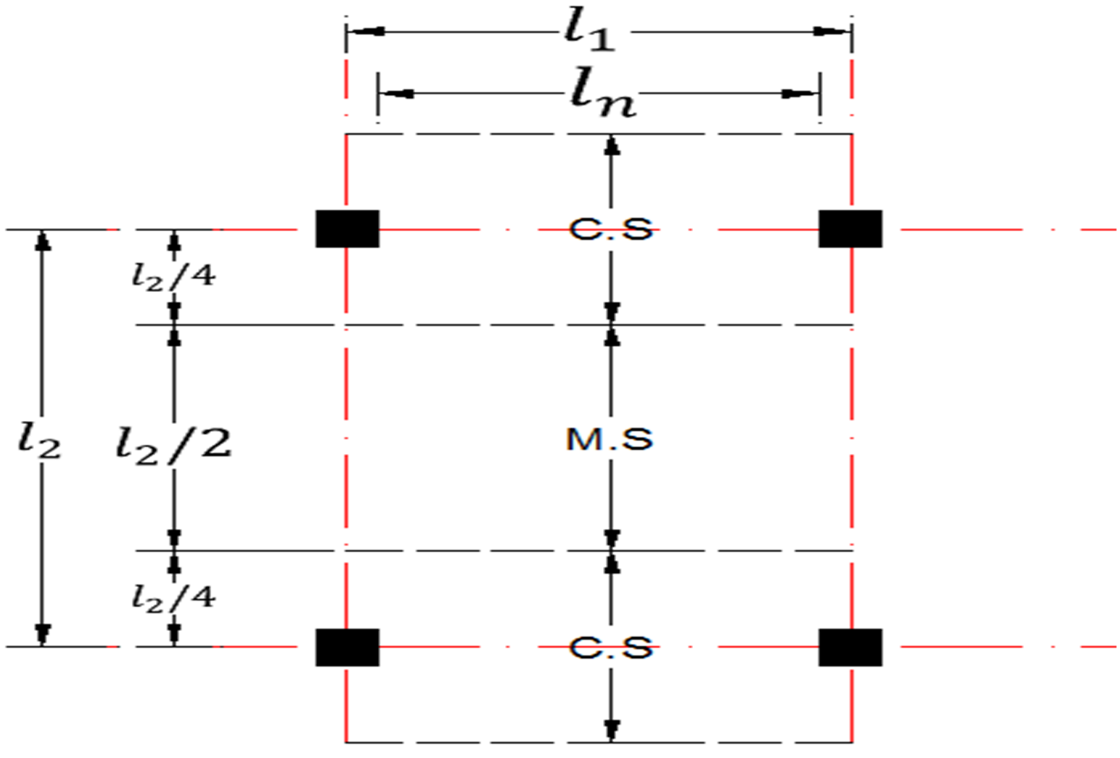

| | | |
|---|---|---|
| $l_1$ *direction* ↕ | Panels (1,2,3,7,8,9) | Exterior slab |
| | Panels (4,5,6) | Interior slab |
| $l_2$ *direction* ↔ | Panels (1,4,7,3,6,9) | Exterior slab |
| | Panels (2,5,8) | Interior slab |

In case of **flat slab** the column strip is more critical than middle strip because it work as beam carrying the middle strip load to the column therefore column strip need more reinforcement. Plus that the beam takes 85% of the slab moment in the case of two-way slab with beams.



$\frac{1}{2}(M_{ab} + M_{cd}) + M_{ef} = \frac{1}{8}w_u l_2 l_n^2$

A similar requirement exists in the perpendicular direction.




## Deflection control of two-way slab:

### Design limits (ACI-code 2014):

**Minimum slab thickness:**

**8.3.1.1** For nonprestressed slabs without interior beams spanning between supports on all sides, having a maximum ratio of long-to-short span of 2, overall slab thickness h shall not be less than the limits in Table 8.3.1.1 and shall be at least the value in (a) or (b) unless the calculated deflection limits of 8.3.2 are satisfied:

(a) Slabs without drop panels as defined in 8.2.4..............................125 mm.

(b) Slabs with drop panels as defined in 8.2.4 ..................................100 mm.

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]**

| $f_y$, MPa[2] | Without drop panels[3] | | | With drop panels[3] | | |
|---|---|---|---|---|---|---|
| | Exterior panels | | Interior panels | Exterior panels | | Interior panels |
| | Without edge beams | With edge beams[4] | | Without edge beams | With edge beams[4] | |
| 280 | $\ell_n/33$ | $\ell_n/36$ | $\ell_n/36$ | $\ell_n/36$ | $\ell_n/40$ | $\ell_n/40$ |
| 420 | $\ell_n/30$ | $\ell_n/33$ | $\ell_n/33$ | $\ell_n/33$ | $\ell_n/36$ | $\ell_n/36$ |
| 520 | $\ell_n/28$ | $\ell_n/31$ | $\ell_n/31$ | $\ell_n/31$ | $\ell_n/34$ | $\ell_n/34$ |

[1]$\ell_n$ is the clear span in the long direction, measured face-to-face of supports (mm).

[2]For $f_y$ between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3]Drop panels as given in 8.2.4.

[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if $\alpha_f$ is less than 0.8. The value of $\alpha_f$ for the edge beam shall be calculated in accordance with 8.10.2.7.

**8.3.1.2** For nonprestressed slabs with beams spanning between supports on all sides, overall slab thickness h shall satisfy the limits in Table 8.3.1.2 unless the calculated deflection limits of 8.3.2 are satisfied.

**Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides**

| $\alpha_{fm}$[1] | Minimum $h$, mm | | |
|---|---|---|---|
| $\alpha_{fm} \le 0.2$ | 8.3.1.1 applies | | (a) |
| $0.2 < \alpha_{fm} \le 2.0$ | Greater of: | $\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+5\beta\left(\alpha_{fm}-0.2\right)}$ | (b)[2],[3] |
| | | 125 | (c) |
| $\alpha_{fm} > 2.0$ | Greater of: | $\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta}$ | (d)[2],[3] |
| | | 90 | (e) |

[1]$\alpha_{fm}$ is the average value of $\alpha_f$ for all beams on edges of a panel and $\alpha_f$ shall be calculated in accordance with 8.10.2.7.

[2]$\ell_n$ is the clear span in the long direction, measured face-to-face of beams (mm).

[3]$\beta$ is the ratio of clear spans in long to short directions of slab.

**8.3.1.2.1** At discontinuous edges of slabs conforming to 8.3.1.2, an edge beam with $\boldsymbol{\alpha_f \ge 0.80}$ shall be provided, or the minimum thickness required by (b) or (d) of Table 8.3.1.2 shall be increased by at least 10 percent in the panel with a discontinuous edge.

## Direct design method of two way slabs (8.10.2 ACI-2014):

Limitations:

Moments in two-way slab can be found using direct design method subject to the following restrictions:

**8.10.2.1** There shall be at least three continuous spans in each direction.

**8.10.2.2** Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.

**8.10.2.3** Panels shall be rectangular, with a ratio of longer to shorter panel dimensions measured center-to-center of supports, not exceed 2.

**8.10.2.4** Columns offset shall not exceed 10 percent of the span in direction of offset from either axis between centerlines of successive columns.

**8.10.2.5** All loads shall be due to gravity only and uniformly distributed over an entire panel.

**8.10.2.6** Unfactored live load shall not exceed two times the unfactored dead load.

$$\frac{L.L}{D.D} \leq 2.0$$

**8.10.2.1** For a panel with beams between supports on all sides, Eq. (8.10.2.7a) shall be satisfied for beams in the two perpendicular directions

$$0.2 \leq \frac{\alpha_{f1} l_2^2}{\alpha_{f2} l_1^2} \leq 5.0 \qquad (8.10.2.7a)$$

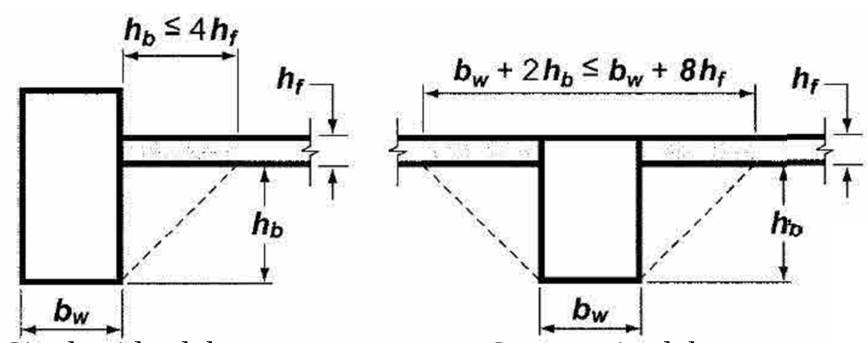
Where $\alpha_{f1}$and$\alpha_{f2}$are calculated by:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (8.10.2.7b)$$

$\alpha_{f1}$=$\alpha_f$ in direction $l_1$

$\alpha_{f2}$=$\alpha_f$ in direction $l_2$

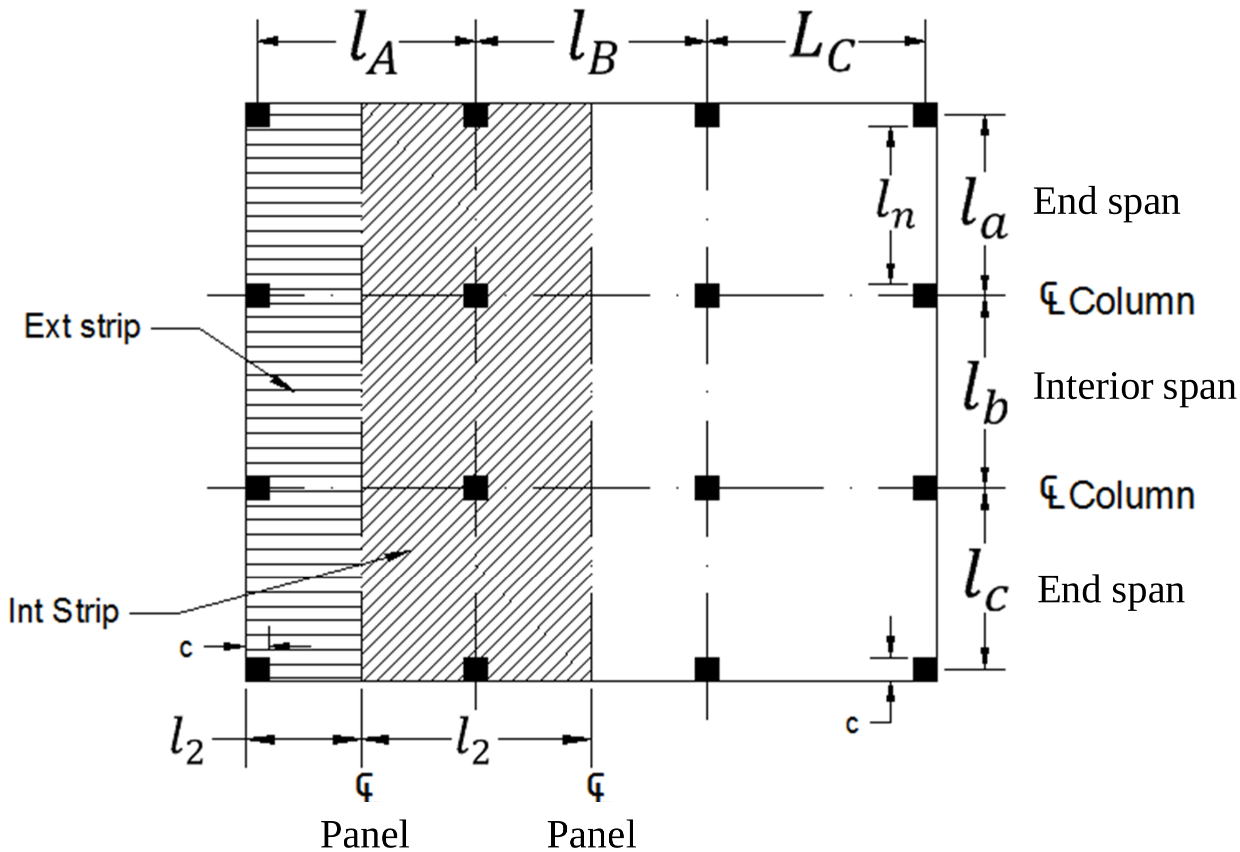
In the case of monolithic construction (two-way slab with beams)




Single side slab Symmetric slab

*Fig. R8.4.1.8—Examples of the portion of slab to be included with the beam under 8.4.1.8.*

$\alpha_f = \frac{E_{cb} I_b}{E_{cs} Is}$ , $I_s = \frac{l_2 h_f^3}{12}$




For internal strip $l_2 = \frac{l_A + l_B}{2}$

For external strip $l_2 = \frac{l_A}{2} + \frac{c}{2}$

Total static moment for end span

$M_o = \frac{w_u l_2 l_n^2}{8}$

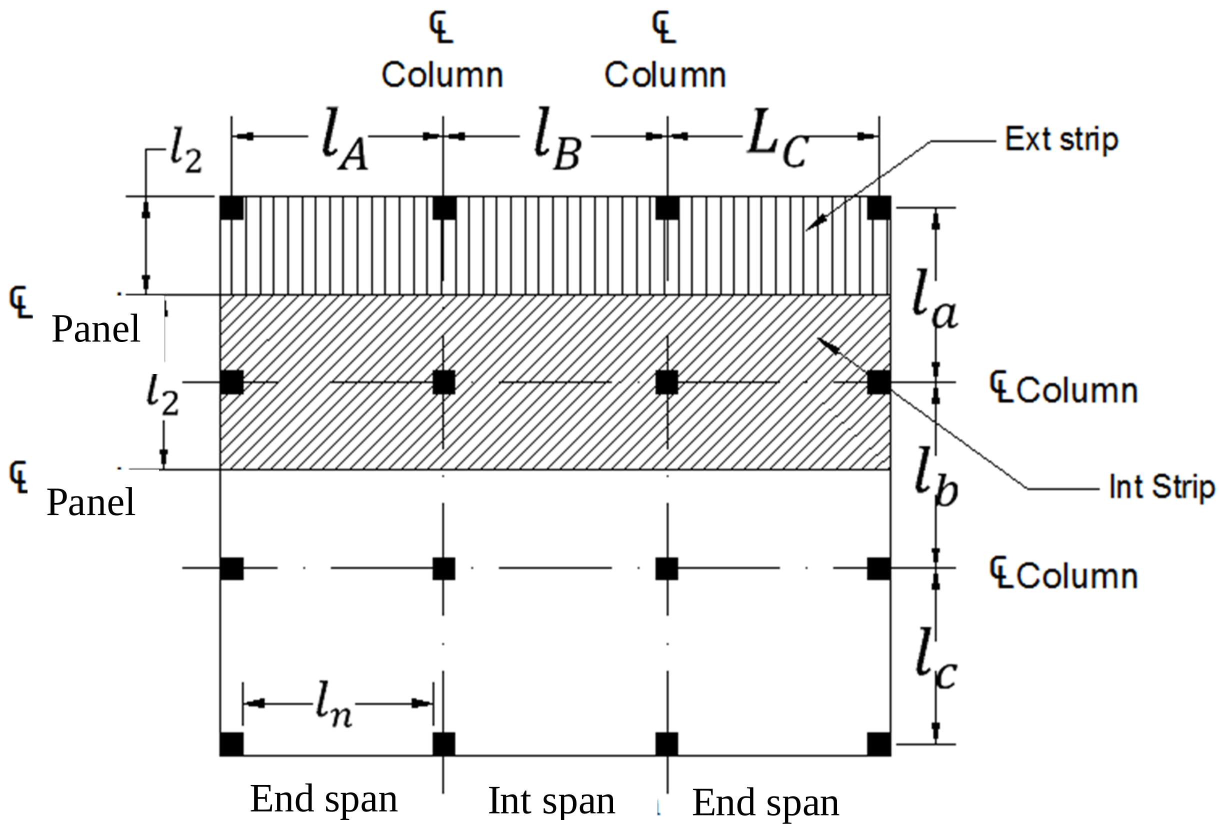
$l_1 = l_a$

$l_n \rightarrow l_1$

$l_n \rightarrow$ clear span face to face of columns, capitals, brackets or walls.

$l_n \geq 0.65\, l_1$

For other direction:




For internal strip $l_2 = \frac{l_a+l_b}{2}$

For external strip $l_2 = \frac{l_a}{2} + \frac{c}{2}$

Total static moment for end span, $M_o = \frac{w_u l_2 l_n^2}{8}$

$l_1 = l_A$ $\qquad l_n \rightarrow l_1$

$l_n$: Clear span, circular or regular polygon shaped support shall be treated as square support with the same area.

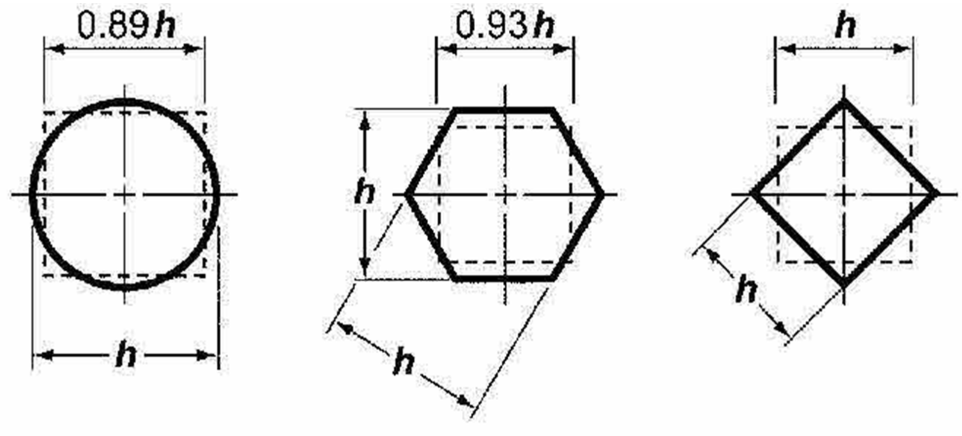



*Fig. R8.10.1.3—Examples of equivalent square section for supporting members.*

**<u>Negative and positive factored moment:</u>**

**<u>For interior span (8.10.4.1):</u>**

Negative Mu = $0.65M_o$
Positive Mu = $0.35\ M_o$

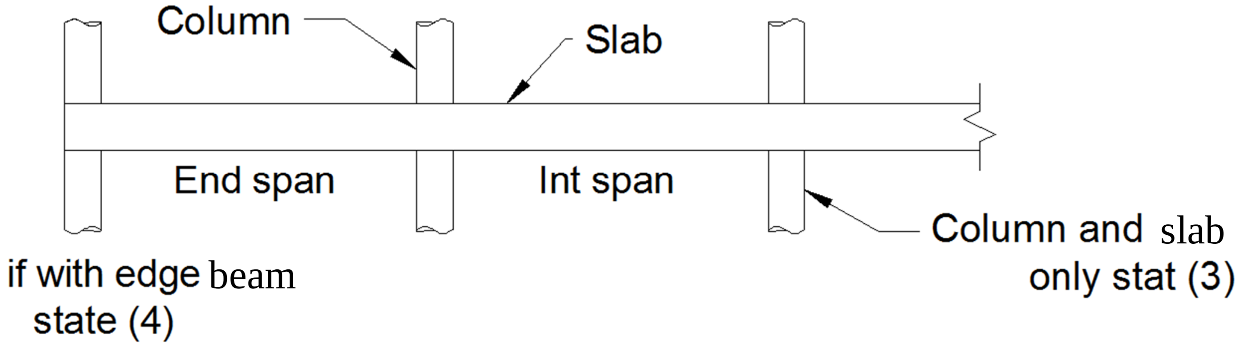
**<u>For end span:</u>**

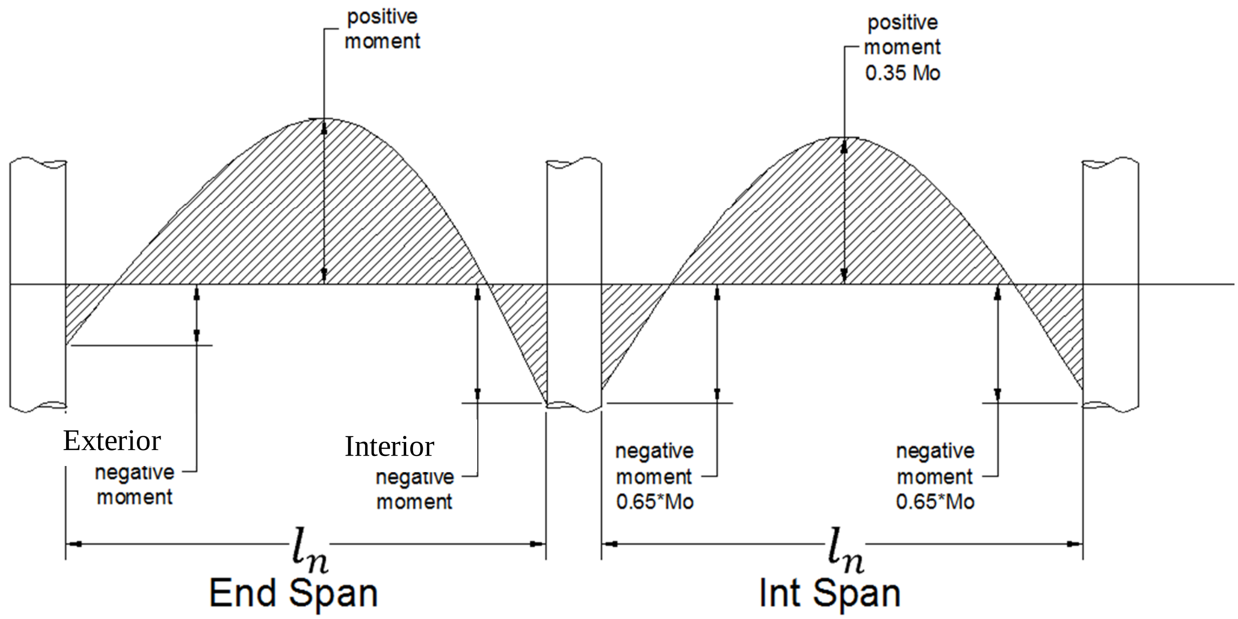
Use Table 8.10.4.2 (ACI-2014)

**8.10.4.2** In an end span, $M_o$ shall be distributed in accordance with Table 8.10.4.2.

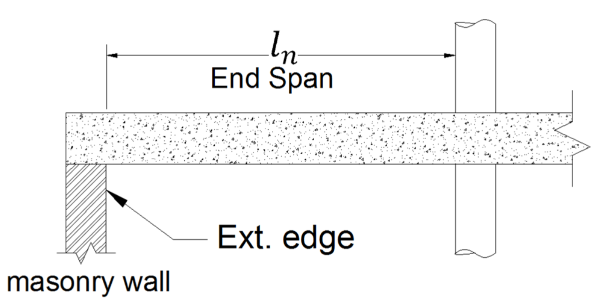
**Table 8.10.4.2—Distribution coefficients for end spans**

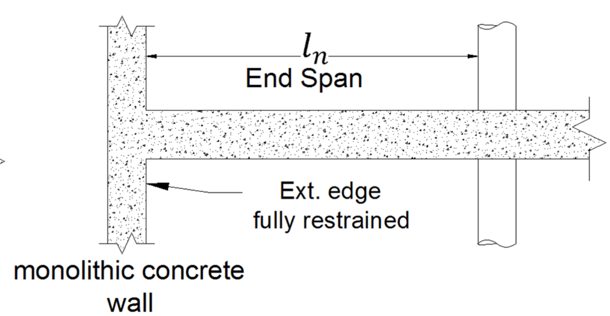
| | Exterior edge unrestrained | Slab with beams between all supports | Slab without beams between interior supports | | Exterior edge fully restrained |
|---|---|---|---|---|---|
| | | | Without edge beam | With edge beam | |
| Interior negative | 0.75 | 0.70 | 0.70 | 0.70 | 0.65 |
| Positive | 0.63 | 0.57 | 0.52 | 0.50 | 0.35 |
| Exterior negative | 0 | 0.16 | 0.26 | 0.30 | 0.65 |

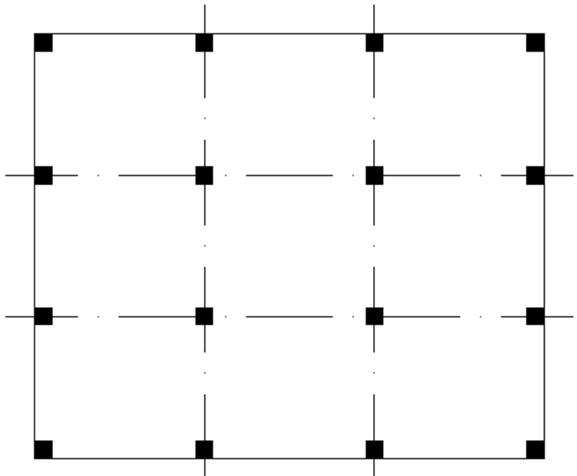


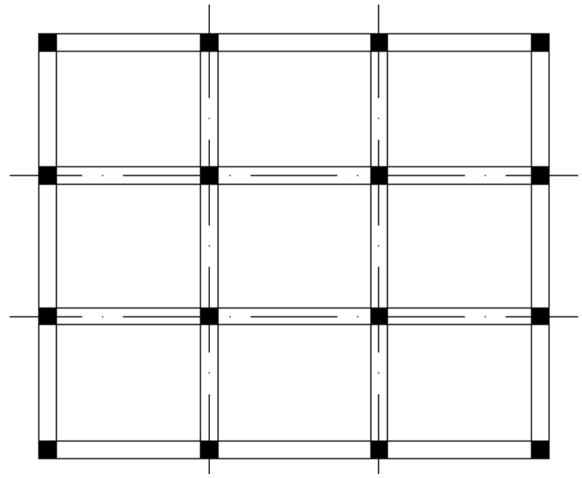

**8.10.4.5** Negative moment $M_u$ shall be the greater of the two negative $M_u$ calculated for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.



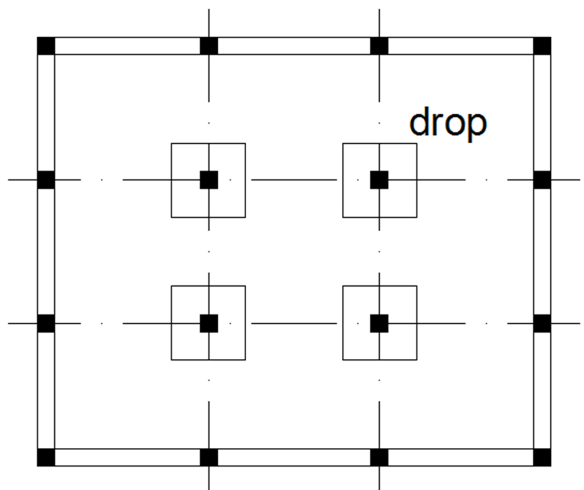





Flat plate without edge beam



Slab with beams between supports




Flat slab with edge beam

**Factored moments in column strips**

**8.10.5.1** The column strips shall resist the portion of interior negative $M_u$ in accordance with Table 8.10.5.1.

**Table 8.10.5.1—Portion of interior negative $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\ell_2/\ell_1$ | | |
|---|---|---|---|
| | 0.5 | 1.0 | 2.0 |
| 0 | 0.75 | 0.75 | 0.75 |
| ≥1.0 | 0.90 | 0.75 | 0.45 |

Note: Linear interpolations shall be made between values shown.

**8.10.5.2** The column strips shall resist the portion of exterior negative $M_u$ in accordance with Table 8.10.5.2.

**Table 8.10.5.2—Portion of exterior negative $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\beta_t$ | $\ell_2/\ell_1$ 0.5 | 1.0 | 2.0 |
|---|---|---|---|---|
| 0 | 0 | 1.0 | 1.0 | 1.0 |
| | ≥2.5 | 0.75 | 0.75 | 0.75 |
| ≥1.0 | 0 | 1.0 | 1.0 | 1.0 |
| | ≥2.5 | 0.90 | 0.75 | 0.45 |

Note: Linear interpolations shall be made between values shown. $\beta_t$ is calculated using Eq. (8.10.5.2a), where $C$ is calculated using Eq. (8.10.5.2b).

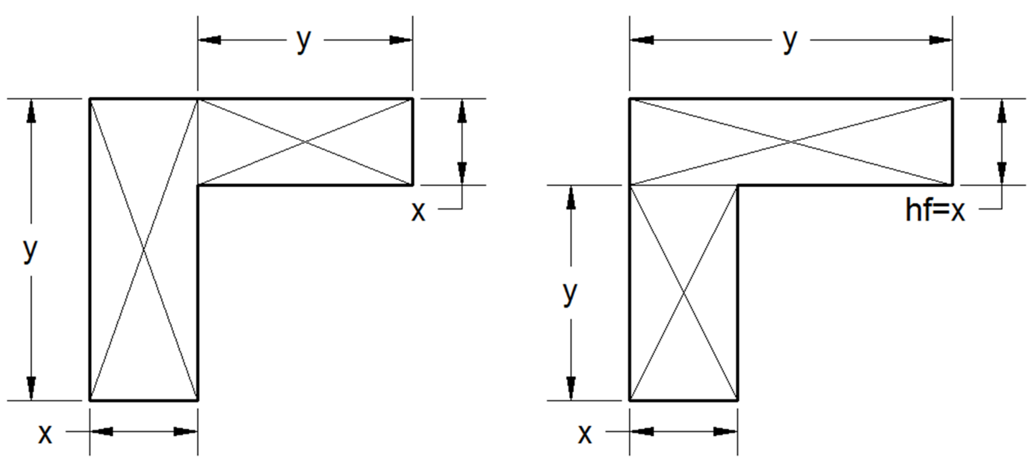
The relative restrained provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter $\beta_t$, defined as:

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad (8.10.5.2a)$$

The constant C for T- or L- section is calculated by dividing the section into separate rectangular parts, each having smaller dimension (x) and larger dimension (y), and summing the values of C for each part.

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{(x^3y)}{3} \qquad (8.10.5.2b)$$

The subdivision can be done in such away as to maximize C.




**8.10.5.5** The column strips shall resist the portion of positive $M_u$ in accordance with Table 8.10.5.5.

**Table 8.10.5.5—Portion of positive $M_u$ in column strip**

| $\alpha_{f1}\ell_2/\ell_1$ | $\ell_2/\ell_1$ | | |
|---|---|---|---|
| | **0.5** | **1.0** | **2.0** |
| 0 | 0.60 | 0.60 | 0.60 |
| ≥1.0 | 0.90 | 0.75 | 0.45 |

Note: Linear interpolations shall be made between values shown.

**Factored moments in beams**

**8.10.5.7.1** Beams between supports shall resist the portion of column strip $M_u$ in accordance with Table 8.10.5.7.1.

**Table 8.10.5.7.1—Portion of column strip $M_u$ in beams**

| $\alpha_{f1}\ell_2/\ell_1$ | Distribution coefficient |
|---|---|
| 0 | 0 |
| ≥1.0 | 0.85 |

Note: Linear interpolation shall be made between values shown.

**8.10.5.7.2** In addition to moments calculated according to 8.10.5.7.1, beams shall resist moments caused by factored loads applied directly to the beams, including the weight of the beam stem above and below the slab.

**Direct loads on beams**:- factored beam self-weight + factored wall weight

$$(w_u)_b = (h - hf)b_w * 24 * 1.2 + wall\ weight * 1.2$$

$$(M_o)_b = \frac{(w_u)_b l_n^2}{8}, \qquad l_n = clear\ span\ for\ beam$$

**Interior beam**

$$\text{Total negative moment} = 0.85\ \text{col. Strip moment} + 0.65\,(\text{M}_\text{o})_\text{b}$$

$$\text{Total pos. moment} = 0.85\ \text{col. Strip moment} + 0.35\,(\text{M}_\text{o})_\text{b}$$

**End beam (Use Table 8.10.4.2):**

$$\text{Internal negative moment} = 0.85\ \text{col. Strip moment} + \text{factor}\ (M_o)_b$$
$$\text{pos. moment} = 0.85\ \text{col. Strip moment} + \text{factor}\ (M_o)_b$$
$$\text{External neg. moment} = 0.85\ \text{col. Strip moment} + \text{factor}\ (M_o)_b$$

**Design of moment reinforcement for slab**

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2R_u m}{fy}}\right], \quad R_u = \frac{M_u}{\emptyset bd^2}$$

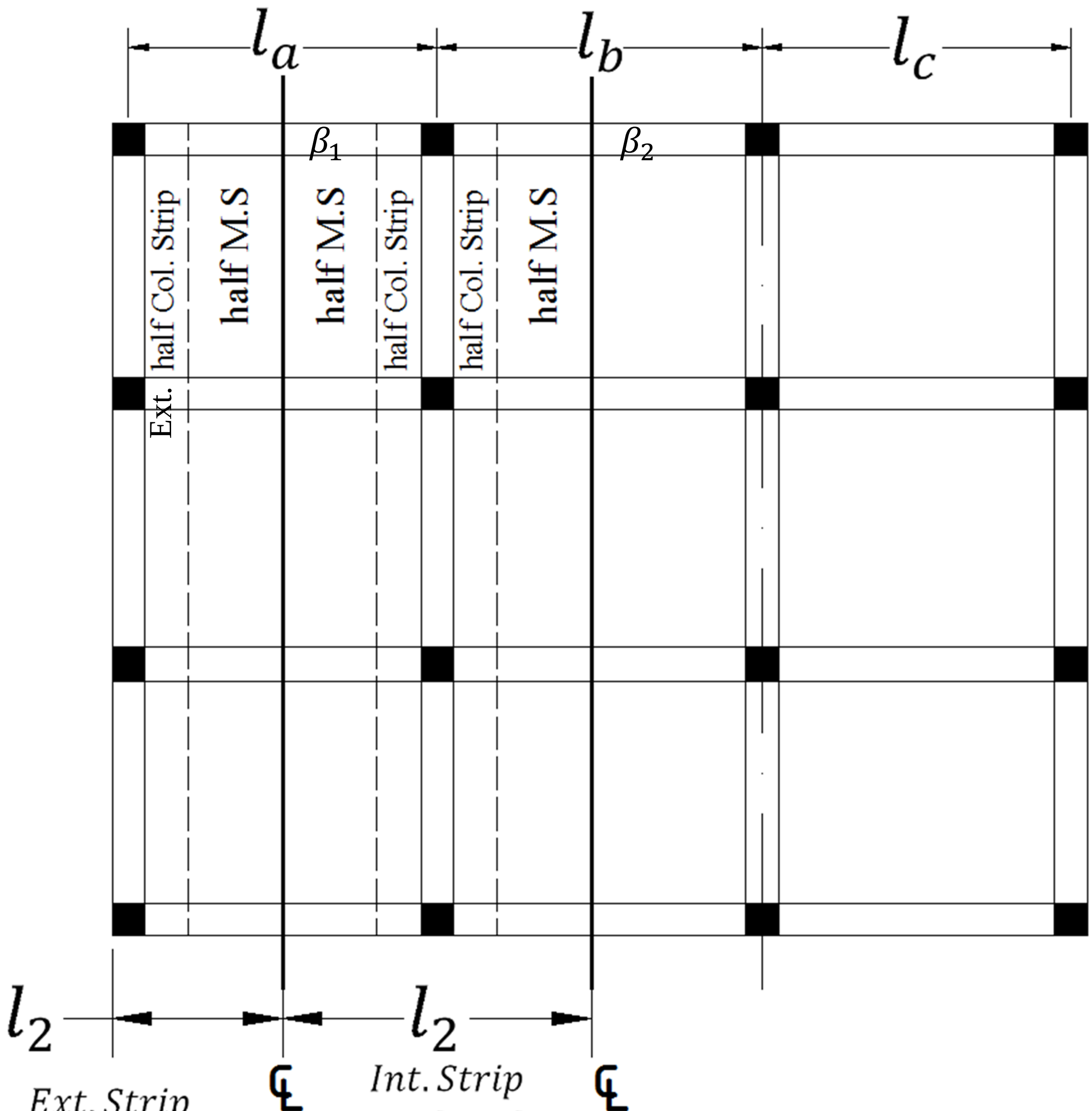
$$m = \frac{fy}{0.85 * fc'} \qquad As = \rho * bd$$

$$As_{min} = 0.002\ A_g \text{ for } f_y < 420\ N/mm^2$$

$$As_{min} = \frac{0.0018 * 420}{f_y} \text{ for } f_y \geq 420 \frac{N}{mm^2} \text{ or } As_{min} = 0.0014A_g$$

$$S_{main} = \frac{As(\text{provided by one bar})}{\text{total As (req.)}} * \text{width of strip}$$

$$S_{max} = 2t(2hf)$$




**Ext. Strip**

$$l_2 = \frac{l_a}{2} + \frac{b_w}{2}$$

**Int. Strip**

$$l_2 = \frac{l_a + l_b}{2}$$

$$\beta_t = \frac{\beta_1 + \beta_2}{2}$$

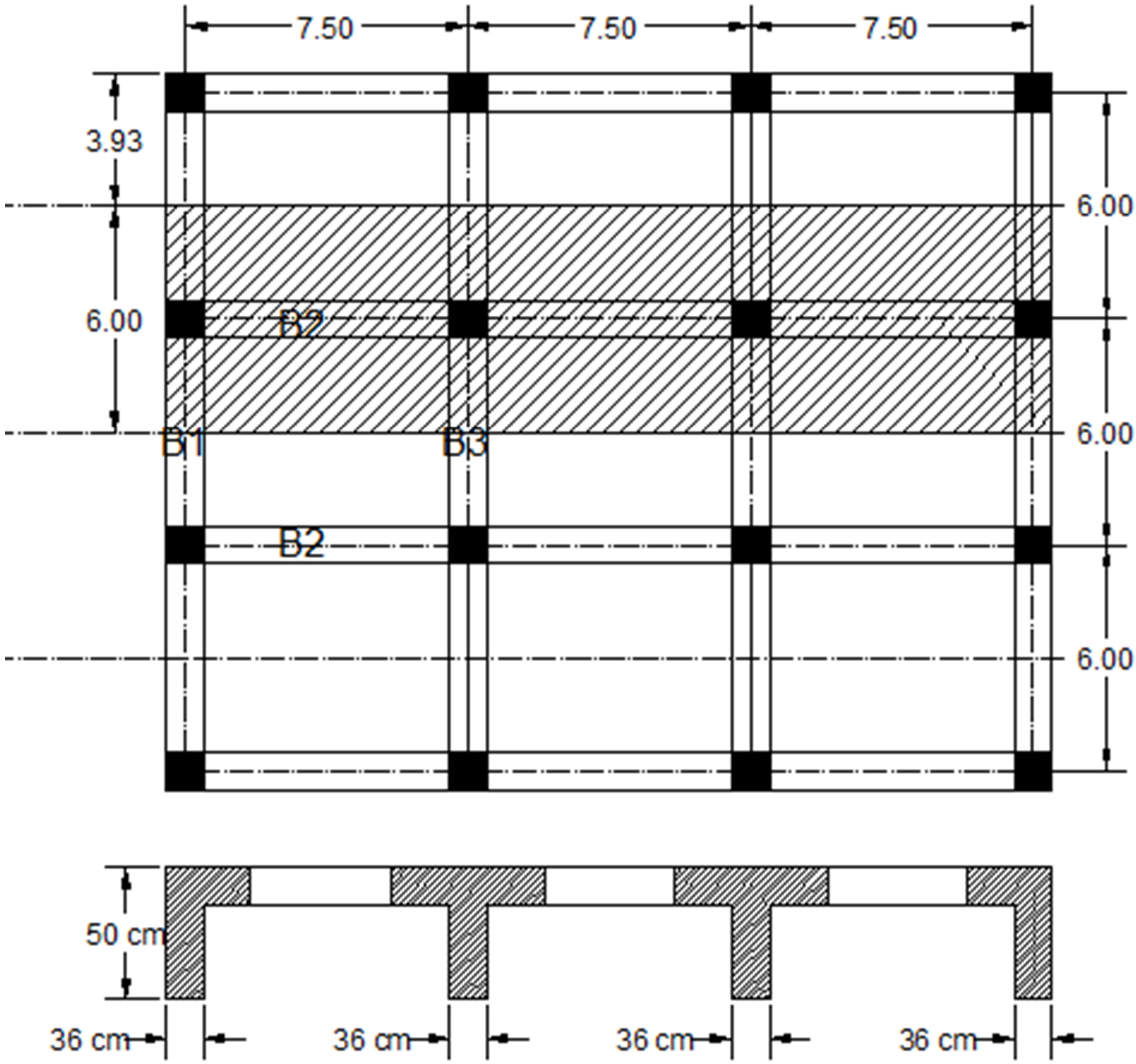
### 8.10.6 *Factored moments in middle strips*

**8.10.6.1** That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

**8.10.6.2** Each middle strip shall resist the sum of the moments assigned to its two half middle strips.

**8.10.6.3** A middle strip adjacent and parallel to a wall-supported edge shall resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

Ex\\ A two-way reinforced concrete building floor system is composed of slab panels measuring 6*7.5 m in plan supported by column-line beams as shown in figure below. Using concrete with fc'=27.6 MPa and steel having fy=414 MPa, design typical exterior panel to carry a service live load of 6.9 kN/m$^2$ in addition to its own weight of the floor.



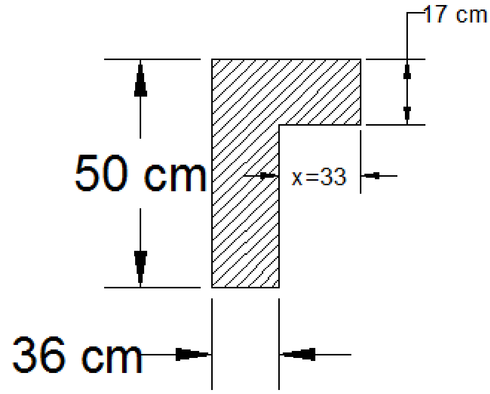
Sol:

Try h=17 cm

Either x=h-hf=500-170=330 mm=33 cm

Or x=4*hf=4*170=680 mm=68cm

Chose min. value (x=33 cm)



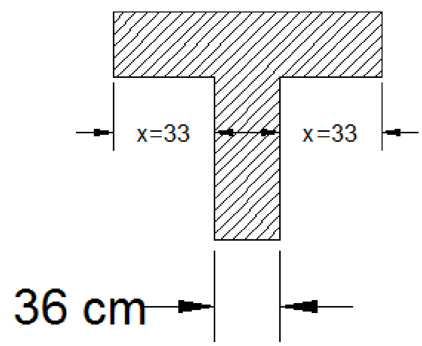

**Beam ($B_1$)**

$$I_{beam} = \frac{(36 * 50^3)}{12} * 1.5 = 562500\ cm^4$$

$$Slab\ strip\ width = \frac{7.5}{2} + \frac{0.36}{2} = 3.93\ m$$

$$I_{Slab} = \frac{393 * 100 * 17^3}{12} = 160901\ cm^4$$

$$\alpha_1 = \frac{E_{cb} * I_b}{E_{Cs} * I_S} = \frac{562500}{160901} = 3.5$$




**Beam ($B_2$)**

$$I_b = \frac{36 * 50^3}{12} * 2 = 750000\ cm^4$$

Slab strip width = 6m

$$I_S = \frac{600 * 17^3}{12} = 245650\ cm^4$$

$$\alpha_2 = \frac{750000}{245650} = 3.1$$

**Beam ($B_3$)**

$$I_b = 750000\ cm^4$$

slab strip width = 7.5 m

$$I_s = \frac{750 * 17^3}{12} = 307063\ cm^4$$

$$\alpha_3 = \frac{750000}{307063} = 2.44$$

$$\alpha_{fm} = \frac{3.5 + 3.1 + 3.1 + 2.44}{4} = 3.035$$

$$\beta = \frac{7.5 - 0.36}{6 - 0.36} = 1.27$$

$\alpha_{fm} > 2.0\ slab\ with\ interior\ beam\ use\ eq\ 9.13\ to\ fined\ h$

$$h = \frac{l_n\left(0.8 + \frac{fy}{1400}\right)}{36 + 9 * \beta} = \frac{7.14\left(0.8 + \frac{414}{1400}\right)}{36 + 9 * 1.27} * 10^3 = 165\ \text{mm} > 90\ \text{mm ok}$$

$\therefore h = 17\ cm\ is\ ok$

$\text{self} - \text{weight of slab} = \frac{17}{100} * 24 = 4.08\ \text{kN/m}^2$

$w_u = (1.2 * 4.08) + 1.6 * 6.9 = 16\ \text{kN/m}^2$

**Short span direction**

$$M_o = \frac{w_u l_2 l_n^2}{8}$$

$\boldsymbol{interior\ slab\ strip\ (7.5\ m\ width)}$

$$M_0 = \frac{16 * 7.5 * (5.64)^2}{8} = 477\ \text{kN.m}$$

sec 8.10.4.1

$+M = 0.35 * 477 = 167\ kN.m$

$-M = 0.65 * 477 = 310\ kN.$

$l_2/l_1 = \frac{7.5}{6} = 1.25$

$\frac{\alpha_1 l_2}{l_1} = \frac{2.44 * 7.5}{6} = 3.1$

sec 8.10.5.1 68% from $(-M) \rightarrow$ C.S

$0.68 * 310 = 211$ kN.m C.S

$0.85 * 211 = 179$ kN.m Beam

$0.15 * 211 = 32$ kN.m Slab C.S

$310 - 211 = 99$ kN.m M.S

sec 8.10.5.5 68% from $(+M) \rightarrow C.S$

$0.68 * 167 = 114$ kN.m C.S

$0.85 * 114 = 47$ kN.m beam

$0.15 * 114 = 17$ kN.m Slab C.S

$167 - 114 = 53$ kN.m M.S

**Exterior slab strip $(3.93\ \mathbf{m\ width})$**

$$M_o = \frac{16 * 3.93 * 5.643}{8} = 250\ kN.m$$

**sec. 8. 10. 4. 1**

$+M = 0.35 * 250 = 88\ kN.m$

$-M = 0.65 * 250 = 163\ kN.m$

$$l_2/l_1 = \frac{7.5}{6} = 1.25$$

$$\frac{\alpha_1 l_2}{l_1} = \frac{3.5 * 7.5}{6} = 3.1$$

**sec. 8. 10. 5. 1**

$68\%$ from $(-M) \rightarrow C.S$

$0.68 * 163 = 111\ kN.m \quad C.S$

$0.85 * 111 = 94\ kN.m$ beam

$0.15 * 111 = 17\ kN.m$ Slab $C.S$

$163 - 111 = 52\ kN.m \quad M.S$

$68\%$ from$(+M) \rightarrow C.S$

$0.68 * 88 = 60\ kN.m \quad C.S$

0.85*60= 51 kN.m beam

$0.15 * 60 = 9\ kN.m$ Slab $C.S$

$88 - 60 = 28\ kN.m \quad M.S$

| | Beam moment | Slab C.S moment | M.S moment |
|---|---|---|---|
| Interior slab strip 6m span | | | |
| Positive | 97 | 17 | 53 |
| Negative | 179 | 32 | 99 |
| Exterior slab strip 6m span | | | |
| Positive | 51 | 9 | 28 |
| Negative | 94 | 17 | 52 |

- **Long span direction ( slab strip 6 m)**

$$M_o = \frac{16 * 6 * 7.14^2}{8} = 612 \text{ kN.m}$$

sec 8.10.4.2

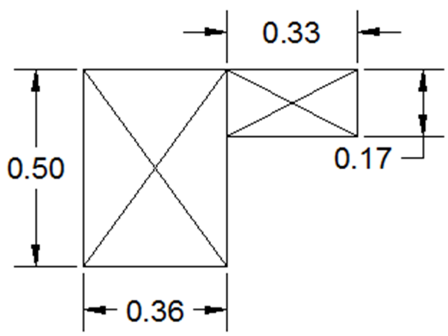
$$\text{Ext.} -M = 0.16 * 612 = 98 \text{ kN.m}$$

$$+M = 0.57 * 612 = 349 \text{ kN.m}$$

$$\text{Int} - M = 0.7 * 612 = 428 \text{ kN.m}$$

$$c = \sum \left(1 - 0.63 * \frac{x}{y}\right) * \frac{x^3 y}{3}$$

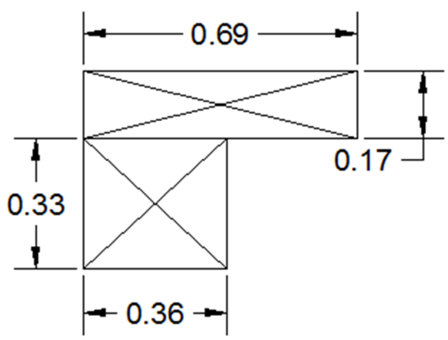
case 1




$$c = \left(1 - 0.63 * \frac{33}{36}\right) * \frac{33^3 * 36}{3} + \left(1 - 0.63 * \frac{17}{69}\right) * \frac{17^3 * 69}{3}$$

$$= 277 * 10^3\ cm^4$$

case 2




$$c = \left(1 - 0.63 * \frac{36}{50}\right) * \frac{36^3 * 50}{3} + \left(1 - 0.63 * \frac{17}{33}\right) * \frac{17^3 * 33}{3}$$

$$= 461 * 10^3 \text{ cm}^4$$

use $c = 461000\ cm^4$

$$\frac{l_2}{l_1} = \frac{6}{7.5} = 0.8 \qquad \frac{\alpha_1 l_2}{l_1} = 3.1 * 0.8 = 2.5$$

$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s}$$

Sec 8.10.5

$93\%\ from\ Ext. -M \rightarrow C.S$ $\quad$ $Table$ 8.10.5.2

$81\%\ from + M \rightarrow C.S$ $\quad$ $Table$ 8.10.5.5

$81\%\ from\ Tnt. -M \rightarrow C.S$ $\quad$ $Table$ 8.10.5.1

| | Beam moment | Slab C.S moment | M.S moment |
|---|---|---|---|
| Slab - beam strip (7.5m span) | | | |
| $Ext.-M$ (98) | 77 | 14 | 7 |
| $+M$ (349) | 240 | 42 | 66 |
| $Int.-M$ (428) | 295 | 52 | 81 |

$A_{min} = 0.002 * 1000 * 170 = 340\, mm^2$

$d_{long} = 170 - 20 - 1.5 * 12 = 132\, mm$

$d_{short} = 170 - 20 - \frac{12}{2} = 144\, mm$

$In\ long\ direction\ \rho_{min} = \frac{340}{132 * 1000} = 0.0025$

$In\ short\ direction\ \rho_{min} = \frac{340}{144 * 1000} = 0.0023$

$\rho_{max} = 0.0206$

$S_{max} = 2h = 2 * 170 = 340mm$

Required (d) for flexure:

$$Mu = \emptyset \rho fy\, b\, d^2 \left(1 - 0.59 * \rho * \frac{fy}{fc'}\right)$$

$$= 0.9 * 0.0206 * 414 * 1000 * d^2 \left(1 - 0.59 * 0.0206 * \frac{414}{276}\right)$$

In long direction

$$d = \sqrt{\frac{Mu}{6276}} = \sqrt{\frac{27 * 10^6}{6276}} = 65.6\, mm < 132\, mm\ o.k$$

In short direction

$$d = \sqrt{\frac{22 * 10^6}{6276}} = 59.2\, mm < 144\, mm\ o.k$$

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2 * Ru * m}{fy}}\right]$$

$$m = \frac{fy}{0.85fc'} = \frac{414}{0.85 * 272} = 17.65$$

$$Ru = \frac{\text{Mu}}{\text{Ø} * b * d^2} = \frac{16*10^6}{\text{Ø} * 1000 * 132^2} = 1.0203$$

$\rho = 0.0025$

$V_u @\ d\ from\ face\ of\ long\ beam$

$$V_u = 16\left(3 - \frac{360}{2*1000} - \frac{144}{1000}\right) = 42.8\ kN$$

$$\text{Ø}V_c = 0.17 * 0.75 * \sqrt{27.6} * 1000 * 144 * 10^{-3} = 96.45\ kN \rightarrow V_u < \text{Ø}V_c\ \ o.k$$

| | | location | $M_n$ $kN.m$ | b mm | d mm | $\frac{Mu}{b} * 1000$ | $\rho$ | $As\ mm^2$ | Number of bars |
|---|---|---|---|---|---|---|---|---|---|
| 7.5m span | Two half C.S | Ext. -M | 14 | 2640 | 132 | 5.3 | 0.0025 | 871 | 8 |
| | | +M | 42 | 2640 | 132 | 16 | 0.0025 | 871 | 8 |
| | | Int. -M | 52 | 2640 | 132 | 20 | 0.0031 | 1080 | 10 |
| | Two half M.S | Ext. -M | 7 | 3000 | 132 | 2.3 | 0.0025 | 990 | 9 |
| | | +M | 66 | 3000 | 132 | 22 | 0.0035 | 1386 | 13 |
| | | Int. -M | 81 | 3000 | 132 | 27 | 0.0043 | 1702 | 15 |
| 6 m span | Ext. half C.S | -M | 17 | 1320 | 144 | 12.8 | 0.0023 | 437 | 4 |
| | | +M | 9 | 1320 | 144 | 6.8 | 0.0023 | 437 | 4 |
| | M.S | -M | 99 | 4500 | 144 | 22 | 0.0029 | 1880 | 17 |
| | | +M | 53 | 4500 | 144 | 11.7 | 0.0023 | 1361 | 14 |
| | Int. half C.S | -M | 16 | 1320 | 144 | 12.1 | 0.0023 | 437 | 4 |
| | | +M | 8.5 | 1320 | 144 | 6.4 | 0.0023 | 437 | 4 |