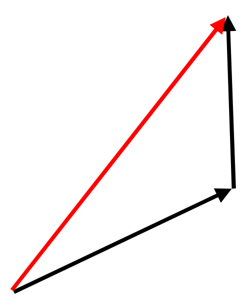
Math 150 Lecture Notes
Introduction to Vectors

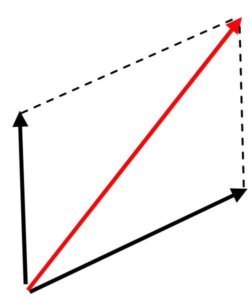
Quantities that are determined only by magnitude, i.e., length, mass, temperature, area, are called **scalars**.

A **vector** is a line segment (with magnitude) and an assigned direction. An arrow is used to specify the direction. Vector $\overrightarrow{AB}$ has **initial point** A and **terminal point** B. The **magnitude** or **length** of the vector is the length of the segment AB and is denoted by $\left|\overrightarrow{AB}\right|$.

Two vectors are **equal** if they have equal magnitude and the same direction.

Vector $\overrightarrow{AC}$ is the **sum** of vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ when it is the displacement $\mathbf{u} = \overrightarrow{AB}$ followed by the displacement $\mathbf{v} = \overrightarrow{BC}$.





Multiplication of a Vector by a Scalar
If a is a real number and $\mathbf{v}$ is a vector, then $a\mathbf{v}$ is a vector of magnitude $|a|\,|\mathbf{v}|$ and has the same direction as $\mathbf{v}$ if $a > 0$ or the opposite direction as $\mathbf{v}$ if $a < 0$.

The **difference** of two vectors $\mathbf{u}$ and $\mathbf{v}$ is defined by $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.

In the coordinate plane, a vector $\mathbf{v}$ can be represented as an ordered pair of real numbers, $\mathbf{v} = \langle a, b\rangle$, where a is the **horizontal component** of $\mathbf{v}$ and b is the **vertical component** of $\mathbf{v}$.

Component Form of a Vector
If a vector $\mathbf{v}$ is represented in the plan with initial point $P\,(x_1, y_1)$ and terminal point $Q\,(x_2, y_2)$, then $\mathbf{v} = \langle x_2 - x_1, y_2 - y_1\rangle$.

Two vectors are **equal** iff their corresponding components are equal.

The **magnitude** or **length** of a vector $\mathbf{v} = \langle a, b\rangle$ is $|\mathbf{v}| = \sqrt{a^2 + b^2}$

Algebraic Operations on Vectors

If $\mathbf{u} = \langle a_1, b_1 \rangle$ and $\mathbf{v} = \langle a_2, b_2 \rangle$, then

$$\mathbf{u} + \mathbf{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$$
$$\mathbf{u} - \mathbf{v} = \langle a_1 - a_2, b_1 - b_2 \rangle$$
$$c\mathbf{u} = \langle ca_1, cb_1 \rangle, \quad c \in \Re$$

Properties of Vectors

<u>Vector Addition</u>

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

$\mathbf{u} + \mathbf{0} = \mathbf{u}$

$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

<u>Length of a Vector</u>

$|c\mathbf{u}| = |c|\ |\mathbf{u}|$

<u>Multiplication by a Scalar</u>

$c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

$(cd)\mathbf{u} = c(d\mathbf{u}) = d(c\mathbf{u})$

$1\mathbf{u} = \mathbf{u}$

$0\mathbf{u} = \mathbf{0}$

$c\mathbf{0} = \mathbf{0}$

Vectors in Terms of I and j

The vector $\mathbf{v} = \langle a, b \rangle$ can be expressed in terms of **i** and **j** by $\mathbf{v} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j}$.

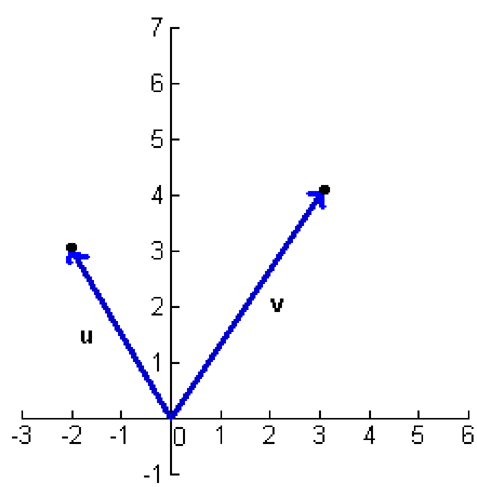
Horizontal and Vertical Components of a Vector

Let **v** be a vector with magnitude $|\mathbf{v}|$ and direction θ. Then $\mathbf{v} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j}$, where

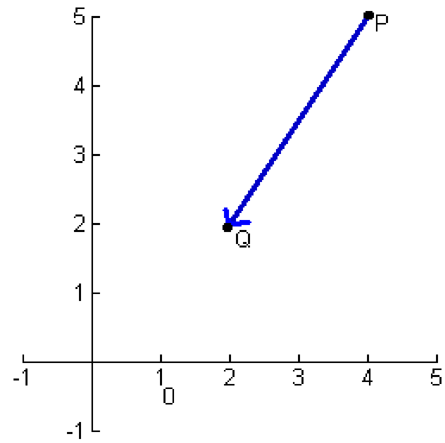
$a = |\mathbf{v}| \cos \theta$ and $b = |\mathbf{v}| \sin \theta$

We can express **v** as $\mathbf{v} = |\mathbf{v}| \cos \theta\mathbf{i} + |\mathbf{v}| \sin \theta\mathbf{j}$.

<u>Example 1:</u> Sketch $\mathbf{u} + 2\mathbf{v}$ using vectors **u** and **v** in the figure.



Example 2: Express the vector with initial point *P* and terminal point *Q* in component form.



Example 3: Find **u** – 2**v** and -3**u** + 4**v** for vectors **u** = ⟨2, -5⟩ and **v** = ⟨-3, 1⟩.

Example 4: Find |**u**|, |**v**|, |2**v**|, | ½ **u**|, and |**u** + **v**| for vectors **u** = 3**i** + 2**j** and **v** = -2**i** + **j**.

Example 5: Find the horizontal and vertical components of the vector with given length and direction, and write the vector in terms of the vectors **i** and **j**.

|**u**| = 60, θ = 120°

Example 6: Find the magnitude and direction (in degrees) of the vector $\mathbf{u} = \langle -5, 12\rangle$.

Example 7: A river flows due south at 4 miles per hour. An alligator heads due east swimming at 3 miles per hour relative to the water. Find the true velocity of the alligator as a vector.

http://hypertextbook.com/physics/foundations/vector-addition/worksheet-addition.pdf
http://hypertextbook.com/physics/foundations/vector-addition/worksheet-subtraction.pdf
http://hypertextbook.com/physics/foundations/vector-addition/worksheet-words.pdf