Failure Theories (criteria) for Fiber-Reinforced Materials

The failure criteria proposed to predict lamina failure could be divided in two main groups:

A-Failure criterion not associated with failure modes

- **Maximum Stress Failure Criterion**
- **Maximum Strain Failure Criterion**
- **Tsai-Hill Failure Criterion**
- **Hoffman Failure Criterion**
- **Tsai-Wu Failure Criterion**

Maximum Stress Failure Criterion

In the maximum stress failure criterion, each and every one of the stresses in principal material coordinates must be less than the respective strengths; otherwise, fracture is said to have occurred.

On-axis loading

That is, for tensile stresses,

$\sigma_1 < X_t$ $\sigma_2 < Y_t$

and for compressive stresses,

$\sigma_1 > X_c$ $\sigma_2 > Y_t$

Also,

$|\tau_{12}| < S$

X = axial or longitudinal strength (1-direction)

Y = transverse strength (2-direction)

S = shear strength (1-2 coordinates)

Maximum Stress Failure Criterion

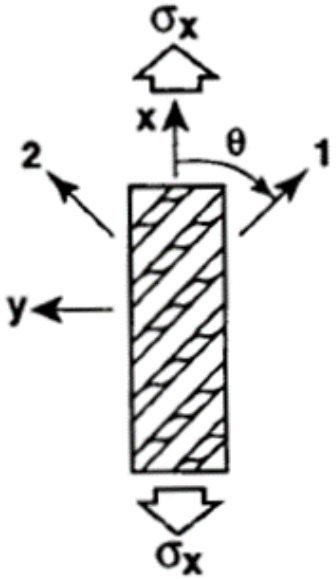
Off-axis loading

Therefore, the maximum uniaxial stress, σ_x, is the smallest of

$$\frac{X_c}{cos^2\theta} < \sigma_x < \frac{X_t}{cos^2\theta}$$

$$\frac{Y_c}{sin^2\theta} < \sigma_x < \frac{Y_t}{sin^2\theta}$$

$$|\sigma_x| < \left|\frac{S}{sin\theta\ cos\theta}\right|$$



$$\sigma_1 = \sigma_x \cos^2\theta$$

$$\sigma_2 = \sigma_x \sin^2\theta$$

$$\tau_{12} = -\sigma_x \sin\theta \cos\theta$$

Maximum Strain Failure Criterion

On-axis loading

Specifically, the material is said to have failed if one or more of the following inequalities is not satisfied:

$$\varepsilon_1 < X_{\varepsilon t} \qquad \varepsilon_2 < Y_{\varepsilon t} \qquad |\gamma_{12}| < S_{\varepsilon}$$

including for materials with different strength in tension and compression

$$\varepsilon_1 > X_{\varepsilon c} \qquad \varepsilon_2 > Y_{\varepsilon c}$$

$X_{\varepsilon t}$ $(X_{\varepsilon c})$ = maximum tensile (compressive) normal strain in the 1-direction

$Y_{\varepsilon t}$ $(Y_{\varepsilon c})$ = maximum tensile (compressive) normal strain in the 2-direction

S_{ε} = maximum shear strain in the 1-2 coordinates

Maximum Strain Failure Criterion

Off-axis loading

The maximum strain failure criterion for uniaxial off-axis loading can be written as

$$\frac{X_c}{cos^2\theta - \nu_{12}\, sin^2\theta} < \sigma_x < \frac{X_t}{cos^2\theta - \nu_{12}\, sin^2\theta}$$

$$\frac{Y_c}{sin^2\theta - \nu_{21}\, cos^2\theta} < \sigma_x < \frac{Y_t}{sin^2\theta - \nu_{21}\, cos^2\theta}$$

$$|\sigma_x| < \left|\frac{S}{sin\theta\, cos\theta}\right|$$