Tsai-Hill Failure Criterion

The Tsai-Hill failure criterion for an orthotropic lamina under plane stress conditions is:

on-axis

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1$$

The Tsai-Hill failure criterion for uniaxial **off-axis** strength,

$$\frac{cos^4\theta}{X^2} + \left[\frac{1}{S^2} - \frac{1}{X^2}\right] cos^2\theta\, sin^2\theta + \frac{sin^4\theta}{Y^2} = \frac{1}{\sigma_x^2}$$

Hoffman Failure Criterion

To account for different strengths in tension and compression, Hoffman added linear terms to Hill's equation (the basis for the Tsai-Hill criterion). For plane stress in the 1-2 plane, the failure criterion is

$$-\frac{\sigma_1^2}{X_c X_t}+\frac{\sigma_1\ \sigma_2}{X_c X_t}-\frac{\sigma_2^2}{Y_c Y_t}+\frac{X_c+X_t}{X_c X_t}\ \sigma_1\ +\ \frac{Y_c+Y_t}{Y_c Y_t}\ \sigma_2+\frac{\tau_{12}^2}{S_{12}^2}=1$$

Tsai-Wu Failure Criterion

$$\frac{\sigma_1^2}{X^2} + 2F_{12}\sigma_1\,\sigma_2 + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1$$

$$F_{12} = \frac{1}{2\sigma^2}\left[1 - \left[\frac{1}{X_t} + \frac{1}{X_c} + \frac{1}{Y_t} + \frac{1}{Y_c}\right]\sigma + \left[\frac{1}{X_t X_c} + \frac{1}{Y_t Y_c}\right]\sigma^2\right]$$

B-Failure criterion associated with failure modes

Two different conditions of failure were considered in correspondence with a maximum strain in fibre direction and for matrix strain such as

Fibre fracture: $|\varepsilon_1| \geq \grave{\varepsilon}_f$

Transverse matrix cracking: $|\varepsilon_1| \, or \, |\varepsilon_2| \geq \grave{\varepsilon}_m$

where $\grave{\varepsilon}_f$ and $\grave{\varepsilon}_m$ are the tensile strains to failure of the fibre and matrix, respectively.

MACRO MECHANICAL BEHAVIOR OF A LAMINATE