Basic Probability definitions and Rules

A probability is a measure of likelihood that a future event will occur ; it can assume a value between 0 and 1, inclusive.

Outcome is the result of a random trail.

An experiment is any process that generates well - defined out comes.

A sample space is the set of all possible outcomes for an experiment and it is usually denoted by the english capital letter S or the greack letter Ω omega.

An event is a collection of outcomes of an experiment, and it is usually denoted by Capital letters A, B, C, ...

Sample space

the Sample space for a random experiment is the set of all possible events that may occur. Exactly one and only one of the events will occur.

ex1

the Sample space for tossing a coin contains just two events, head and tail, which may be conveniently expressed in set notation as

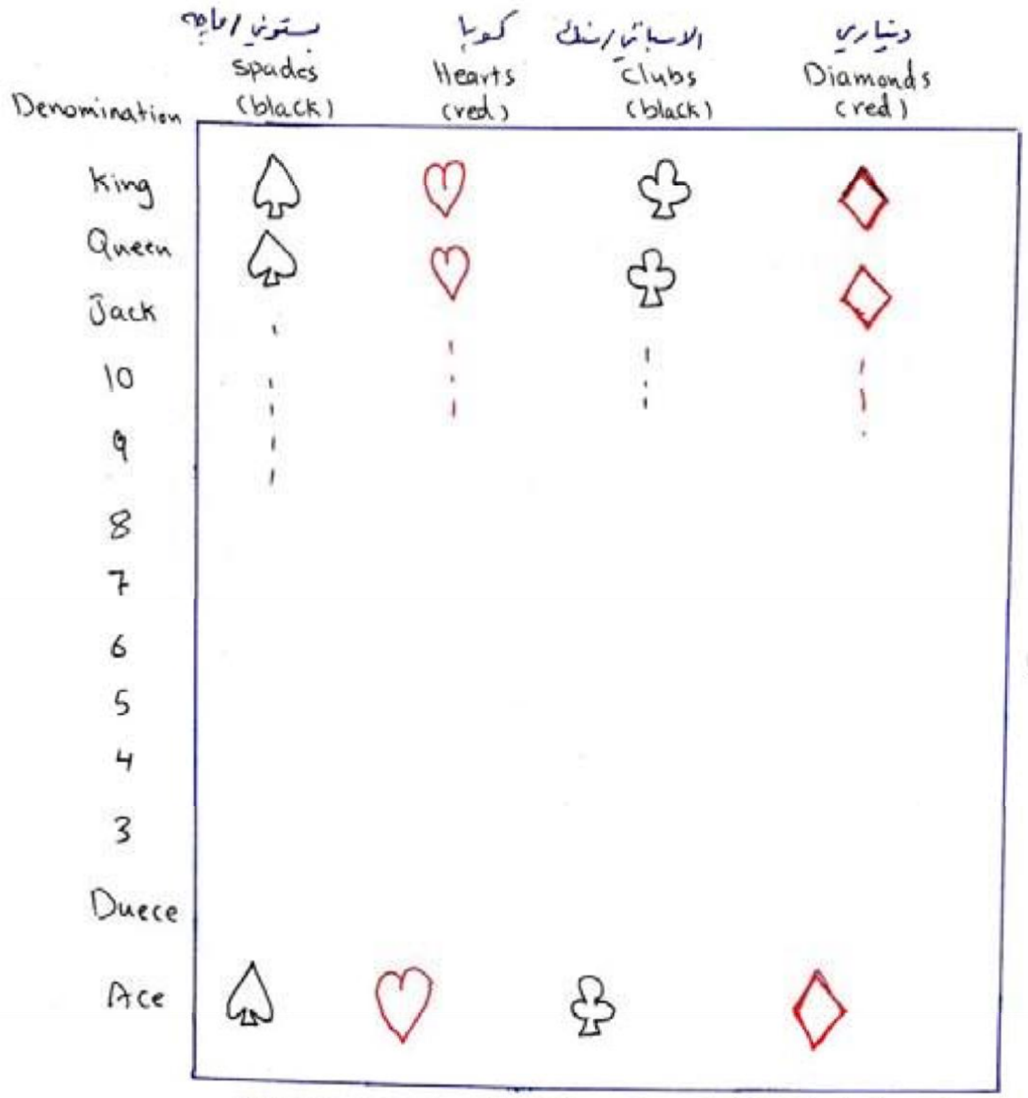
$$\Omega = \{\text{Head}, \text{Tail}\}$$

Head and Tail are the elements of a random experements sample space. One and only one of these events must occur.

ex2

Consider the random experiment - drawing a card from a fully shuffled deck of 52 ordinary playing cards. Here we have

Sample space = { ace of spades, deuce of spades, ..., queen of diamond , King of diamonds }



Sample space describing the card selected randomly from a fully shuffleds deck of 52 ordinary playing cards

it would be possible to construct many different sample spaces for the same random experiment.

ex3 if we were interested only in the suit of the selected card, we might have used the set:

Ω = { spade, heart, club, diamond }

ex4

Consider the outcomes when three different coins - a penny, a nickel and a dime - are tossed so that each will show either a head or a tail. Notice that the events are ordered triple such as (H, T, H) which represents the outcomes "head for the penny, tail for the nickle, head for the dime". Even though three coins are tossed, this may be viewed as a single random experiment and the outcome (H, T, H) is a single elementary event. below is the sample space for the tossing of three coins.

Penny Nickle Dime

Ω [redacted] = { (H, H, H), (H, H, T), (H, T, H), (H, T, T),
(T, H, H), (T, H, T), (T, T, H), (T, T, T) }

ex5

if we are describing the sample space for three tosses of the same coin, the elementary events would be represented in precisely the same manner - as eight ordered triples. In that case the random experiment would have three stages, so that (H, T, H) would represent the event "head for the first toss, tail for the second toss, head for the third toss".

event

the outcomes of a random experiment are called events. A preliminary step in finding an event's probability is to identify all of the possible outcomes of the random experiment with which the event is associated.

The elements of a random experiment's sample space are the simplest outcomes or elementary events.

we may, however be interested in more complex outcomes. for example, consider the sample space for the outcome of the toss of a six - sided symmetrical die:

Ω [redacted] = {1, 2, 3, 4, 5, 6}

the outcome "an even-valued face" is a composit event, occurring whenever any one of the elementary events 2, 4 or 6 results. we denote this event by

Even-valued face = {2, 4, 6}

---

below, some of the most important terms which are used in discussing probabilities and their applications, are defined:

Two events are independent if the occurrence of one in no way influences the occurrence of the other.

The Complement of any event is the collection of outcomes that are not contained in the event.

Events are matually exclusive if the occurrence of one
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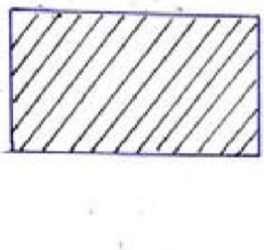
precludes the occurence of the other. in other words, if events A and B are matually exclusive, then A and B cannot occure simultaneously.

the sample space



the universal set

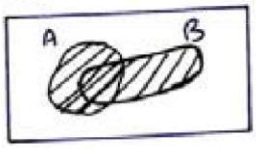
the event A; A occurs

the subset A

the complementary event A' of A; A does not occur

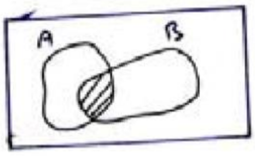
the complement A' of A

the union $A \cup B$
A or B or both
occur



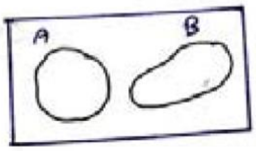
the union $A \cup B$

the intersection
$A \cap B$; both A
and B occur



the intersection $A \cap B$

A and B mutually
exclusive, A and
B cannot occur
simultaneously



A and B disjoint

Type of sample spaces

Example 1: throwing Dice

(a) Throwing a single Dice

let us throw a die. we introduce six outcomes, which we denote by $1, 2, \ldots, 6$. the Sample space Ω consist of these outcomes and can be written $\Omega = \{1, 2, \ldots, 6\}$. example of events are A = "number of points is odd" and B = "number of points is at most 2". Using the symbols for the outcomes, we can write these events

$$A = \{1, 3, 5\} \quad ; \quad B = \{1, 2\}$$

Note that we assumed that the die has six sided.

(b) Throwing two dice

let us throw two dice at the same time. It is then convenient to introduce 36 outcomes $u_1, u_2, \ldots, u_{36}$. these outcomes can also be written $(1,1), (1,2), \ldots, (6,6)$, where the first digit represents one die and the second digit the other. (we suppose that the dice have different colours so that they are distinguishable). the sample space consist of these 36 outcomes. As an example of an event we can take A = "the sum of the two dice is at most 3"; this event can be written

$$A = \{(1,1), (1,2), (2,1)\}$$

Hence the event A consist of three outcomes.

the Sample space of the two random experement above are said to contains قابل للعد numerably finite number of outcomes.

Example 2. Radioactive Decay انحلال اشعاعي للذرة

A sample of a radioactive substance مادة is available. A Geiger-Müller Counter registers the number of particles ذرة decaying تحلل during a certain time interval. This number can be $0, 1, 2, \ldots$, which are all possible outcomes. Thus the Sample space contains a قابل للعد numerably infinite number of outcomes.

Example 3. Strength of Material

From a batch of reinforcing (تعزيز، تقوية) bars (قضيب) a certain number are taken, and their tensil (شد) strengths are determined. The result (in Kp/mm^2) is say,

62.4 , 69.6 , 65.0 , 63.1 , 76.8 , ...

The possible values (= outcomes) are, at least theoretically, all numbers between 0 and ∞. We therefore let the sample space consist of all non negative numbers. It is impossible to write all these outcomes as one sequence. The event A might be, "tensil strength of a chosen bar is between 60.0 and 65.0".

In practice, the tensile strength is stated with a certain number of decimal places, perhaps with one decimal place as we did above. Then Ω consists only of the numbers $0.0, 0.1, 0.2, \ldots$, and the event A consist of just a finite number of outcomes $60.0, 60.1, \ldots, 65.0$. From a mathmatical point of veiw, it is generaly easir to disregard the discreteness (منفصل، غير مستمر) and to retain the continuous character of the sample space.

Definition:

If the number of outcomes is finite or denumerably infinite, Ω is said to be a **discrete sample space**. More particularly, if the number is finite, Ω is said to be a finite sample space.

If the number of outcomes is neither finite nor denumerably infinite, Ω is said to be a continuous sample space.

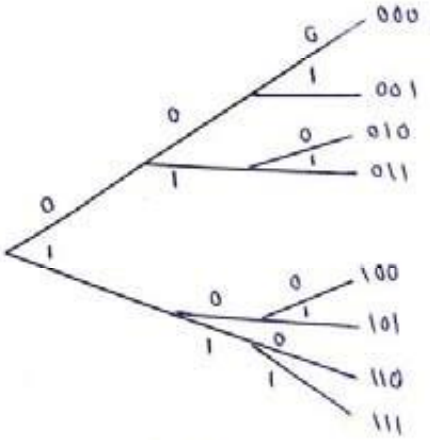
In Example 1 and 2 the sample space is discrete, in Example 1 it is actually finite, in Example 3 it is continuous.

Example 4

A coin is tossed repeatedly. let the tail be denoted by 0 and a head by 1.

(a) A given number of tosses

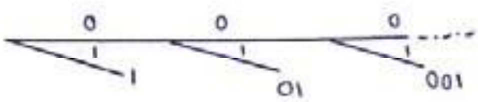
The result of n tosses can be represented by a binary n-digit number. (For example, if n=3, the possible outcomes are 000, 001, 010, 011, 100, 101, 110, 111) . we let each such number be an outcome. The number of possible outcomes is numerably finite. As an illustration we use a "tree diagram" for the case n = 3



three tosses with a coin

(b) Toss until a head is obtained

The coin is tossed until the first head is obtained. Using the representation by binary numbers, the possible outcomes can be written 1, 01, 001, 0001, ... , their number is numerably infinit. The tree diagram can be drawn as below:-



Tosses with a coin until a head is obtained

Probability

Suppose that an experiment has associated with a (finite) sample space Ω. A probability is a numerically valued function that assigns to every event A in Ω a real number $P(A)$ so that the following axioms hold.

Axiom 1: $0 \leq P(A) \leq 1$

Axiom 2: $P(\Omega) = 1$

Axiom (3): if A and B are mutually execlusive events in Ω, then

$$P(A \cup B) = P(A) + P(B)$$

Definition

if there are n equally likely possibilities, of which one must occure and s are regard as "success", then the probability of a "success" is

$\frac{s}{n}$ $n = |\Omega|$, $s = |A|$ where A = event

ex1 what is the probability of getting the event (head) from the toss of fair coin?

Sol since there is s=1 head among n=2 (head, tail), we find the probability of getting head is:-

$\Omega = \{H, T\}$ $|\Omega| = 2$

$p(\text{head}) = \frac{s}{n} = \frac{1}{2}$ $s = \{H\}$ $|s| = 1$

ex2 what is the probability of drawing the ace of spades from a shuffled deck of 52 ordinary playing cards?

Sol since there is s=1 ace of spade among n=52 cards, we find the probability of drawing an ace of spade is

$$p(\text{ace of spade}) = \frac{s}{n} = \frac{1}{52}$$

ex3 what is the probability of drawing an ace from a well-shuffled deck of 52 playing Cards?

SoL since there are $s=4$ ace among the $n=52$ Card we find that the probability of drawing an ace is

$$P(\text{ace}) = \frac{s}{n} = \frac{4}{52} = \frac{1}{13}$$

in the same manner, we find the probabilities of the below events which resulted when playing Cards:

$$P(\text{Club}) = \frac{13}{52} = \frac{1}{4}$$

$$P(\text{face Card}) = \frac{12}{52} = \frac{3}{13}$$ face cards هي فقط K,Q,J من كل مجموعة

$$P(5) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{even number denomination}) = \frac{20}{52} = \frac{5}{13}$$

[illegible]

ex4 what is the probability of getting an even number from tossing a six sided die.

SoL the event "even-value face" resulting from a die toss occurs whenever any one of the 3 elementary events 2, 4, or 6 is the outcomes, and since there are exactly 6 possibile elementary events, then

$$P(\text{even-value face}) = \frac{s}{n} = \frac{3}{6} = \frac{1}{2}$$

ex5 what is the probability of rolling 3 or 4 with a six sided die?

SoL since $n=6$ and $s=2$, we find that the probability of rolling 3 or 4 is

$$P(\text{rolling 3 or 4}) = \frac{s}{n} = \frac{2}{6} = \frac{1}{3}$$

ex5 if H stand for Head and T for tail, the four possible outcomes for two flips of coin are: HH, HT, TH, TT. if it can be assumed that these four possibilities are equally likely, what are the probability of getting zero, one, or two heads?

Sol since $n=4$ and $s=1$ for zero head, $s=2$ for one head, and $s=1$ for two heads. Then

the probability of getting zero head $= \frac{1}{4}$
the probability of getting one head $= \frac{2}{4}$
the probability of getting two heads $= \frac{1}{4}$.

Note
two important observation follow from the probability diffinition:-

First: an event that is certain to occur will have the same value in both the numerator and denominator, thus such events will occure in all of the experiments, and:
P(certain event) = 1

the event "the next president of the united state will be at least 45 years old" is a certain event and has a probability of 1.

Second: At the other extrem, an imposible events probability will always have a 0 in the numerator; thus, such an event will occure in none of the experiments, and;

P(imposible event) = 0

those events that are not possible results of a random experiment are imposible events.

the event "getting the number 8 from tossing a six-sidded symmetrical die" is an imposible event and has the probability of zero.

ex
- if records show that 504 of 813 automatic dishwashers sold by a Large retailer required repairs within the warranty year, what is the probability that an automatic dishwasher sold by the retailer will not require repairs within the warranty year?

Sol since $813 - 504 = 309$ of the dishwashers did not require repairs, we estimate the probability as $\frac{309}{813} = 0.38$

مثل اعلاه →

Theorem 1 : Complement Theorem

$$P(\bar{A}) = 1 - P(A)$$

Proof : A and $\bar{A}$ are mutually exclusive and $A \cup \bar{A} = \Omega$.

If Axioms 3 and 2 are used, we obtain

$$P(A) + P(\bar{A}) = P(A \cup \bar{A}) = P(\Omega) = 1$$

which gives the theorm.

ex using Theorm 1 we can solve the above example as follow:

Let event A = Automatic dish washer required repairs

$P(A) = \frac{504}{813} = 0.62$ the probability of dish washers the required repair.

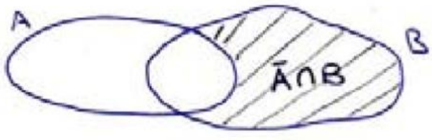
$P(\bar{A}) = 1 - P(A) = 1 - 0.62$

$= 0.38$ the probability of dish washer that not required repair

Theorem 2 : Addition theorem for Two Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof : In order to determine the probability of the event $A \cup B$ one has to add the probabilities of all "points" withen the outer contour. This can be done by first taking all points in A and adding all points in B (using Axiom 3), but then the common region $A \cap B$ has appeared twice, and the corresponding probality $P(A \cap B)$ must be subtracted.



A and B are not mutually exclusive

A strict proof runs as follows. we have

$$A \cup B = A \cup \bar{A}B$$
$$B = AB \cup \bar{A}B$$

(For brevity, we have written AB instead of $A \cap B$).

Look at the right-hand members of the relations. The events A and $\bar{A}B$ are mutually exclusive, as are also the events AB and $\bar{A}B$. the addition formula in Axiom 3 gives

$$P(A \cup B) = P(A) + P(\bar{A}B)$$ يتحول الاتحاد الى + عند
$$P(B) = P(AB) + P(\bar{A}B)$$ ادخال الاحتمال

A subtraction leads to the theorem.

For mutually exclusive events the third term of the formula (of theorm 2) is zero, and the expression reduces to the addition formulla, as it should.

Theorem 3: Boole's Inequality

$$P(A \cup B) \leq P(A) + P(B)$$

Proof: This Theorm follows from theorm 2, for $P(A \cap B)$ is always greater than or equal zero.

Definitions

1. the Union of two events A, B denoted by A or B which is the set of all the elements of both sets.

2. the intersection of two sets A, B denoted by A and B is the set containing only those elements common to both.

3. A collection of several events from the same random experiment is mutually exclusive if and only if the intersection of the corresponding events sets is empty; that is, if the event sets have no elementary event in common. Stated differently, when several events are mutually exclusive, at the most one event may occur.

Example1 Consider the composit event "ace or heart" describing the properties of a card drawn from a completely shuffled deck of 52 ordinary playing cards. ~~From the addition law~~ what is the probability of this event.

Sol: From the addition law:

$$P(\text{ace or heart}) = P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart i.e, ace of hearts})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

Example2 what is the probability of getting King or Queen.

Sol since the events "King", "Queen" are mutually exclusive, thus.

$$P(\text{King or Queen}) = P(\text{King}) + P(\text{Queen})$$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52}$$

Example3 suppose we wish to find the probability of "at least one head in three tosses of a fair coin".

Sol: "At least one" mean "some" and the opposite of "some" is none. the complementary event is therefore "no heads".

we have seen that only one of the eight equally likely outcomes (T,T,T) produces no heads, thus:

$$P(\text{at Least 1 head}) = 1 - P(\text{no head})$$
$$= 1 - P(T,T,T)$$
$$= 1 - \frac{1}{8} = \frac{7}{8}$$

Example

a bank records indicate that of a total of 1,000 customers, 800 have checking accounts, 600 have saving accounts, and 500 have both. What is the probability that a customer selected at random will have either a checking or savings account?

Sol let the events have a checking account (denoted by A) and have a savings account (denoted by B), which are not mutually exclusive. therefor the Computation is: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= 0.8 + 0.6 - 0.5$$
$$= 0.9$$

Example

the Diet Cola Company has an automatic machine that fills bottles with ounces of the Firm's beverage. Most of the bottles are filled properly; however, sometimes bottles are overfilled, and sometimes they are underfilled. A random sample of 1,000 bottles tested showed:

Event	Ounces	Number of bottles	Probability
A	< 16	20	0.02
B	$= 16$	940	0.94
C	> 16	40	0.04
		1000	1.0

what is the probability that a particular bottle will be either underfilled or overfilled?

Sole since that the above events are matually exclusive then

$$P(A \text{ or } C) = P(A) + P(C)$$

$$= 0.02 + 0.04$$

$= 0.06$ is the probability that aparticular bottle will be either underfilled or over filled.

theorem 4: Multiplication rule

to use the multiplication rule, first we must determin whether the events are independent or dependent. Only the multiplication rule for independent events will be discussed here, the multiplication rule for dependent events will be discussed later.

For events that are independent

$$P(A \text{ and } B) = P(A) \times P(B)$$

ex what is the probability of getting three heads from [redacted] tossing the same coin three times.

Sol since the three flips are independence and each flip of head has the probability 0.50, so their produce

$(0.5)(0.5)(0.5) = 0.125$ which is the probability of flipping three heads in succession.

ex a computer company is considering introducing two new softwares to the local market. it has been discovered that the first software has a change of 50 % to be successful and about 75 % chance that the second will succeed. what is the probability that both sw will be successful?

sol we assume that the two softwares are independent of each others, then the multiplication rule calls for multiplying the two probabilities of success to find the desired probability :-

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$= 0.50 * 0.75 = 0.375$$

hence, there is about 37.5 % chance that both products will be successful.

Example (Defective Units)

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multiplication Rule

In a batch of manufactured units, 2% of the units have the wrong weight, 5% have the wrong colour, and 1% have both the wrong weight and the wrong colour. A unit is taken at random from the batch. What is the probability that the unit is defective in at least one of the two respects?

Sol let A = the unit have the wrong weight.
let B = the event have the wrong colour.

Then $P(A) = 0.02$, $P(B) = 0.05$, $P(A \cap B) = 0.01$.

using Theorem 2 ("Addition Theorem for two events"), we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.02 + 0.05 - 0.01$$
$$= 0.06$$

Example (Throwing Two Dice)

Find the probability that a throw with two well-made dice result in "one" on both dice.

Sol

Let A and B denote the events "first [redacted] die shows one" and "the second [redacted] die show one" respectively. we have

$P(A) = p(B) = \frac{1}{6}$.

since A and B are independent, we get

$P(A \cap B) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Second sol. [redacted]

there are 36 possibilities in all, 1 of which is favourable; that is, we have $P(A \cap B) = \frac{1}{36}$.

argument

Note the difference between the second [redacted] and the first solution. In the first solution, we introduce probabilities for each die separately, and hence there are 6 outcomes for each; in the second solution, we consider outcomes for both dice simultaneously, which leads to 36 outcomes.

Conditional Probability

The important concept of conditional probability will first be presented in a special example. We have eight pencils :-

1	2	3	4	5	6	7	8
R	R	R	G	G	G	G	G

where R denotes a red pencil and G a green pencil. One of the pencils is chosen at random with equal probability. If A is the event that "a red pencil is obtained", the classical definition of probability shows that :- $P(A) = \frac{3}{8}$

Let us now also classify the pencils with respect to hardness (H = hard pencil, S = soft pencil) :

1	2	3	4	5	6	7	8
R	R	R	G	G	G	G	G
H	H	S	H	H	H	S	S

If B is the event that "a hard pencil is obtained", we find that

$P(B) = \frac{5}{8}$

it is also found that

$P(A \cap B) = \frac{2}{8}$ (for only no. 1 and no. 2 are both red and hard)

We now ask : If a pencil has been chosen and found to be red, what is the probability that it is hard ? this probability is denoted by $P(B \backslash A)$ and is called " the conditional probability of B given A).

Since we know the pencil is red, there are now only three possible cases (no 1, 2, 3), two of which are hard. It is therefore reasonable to take $P(B \backslash A) = \frac{2}{3}$, which can also be written

$$P(B \backslash A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{2}{8}}{\frac{3}{8}} = \frac{2}{3}$$

Definition : The expression

$$P(B \backslash A) = \frac{P(A \cap B)}{P(A)}$$

is called the Conditional probability of B given A

Naturally, the diffinition makes sense only when P(A) is positive.

The formula in the definition can also be expressed in the form

$$P(A \cap B) = P(A)\, P(B \backslash A)$$

or in words: The probability that two events both occurs is equal to the probability that one of them occurs, multiplied by the Conditional probability that the other occurs, given that the first one has occured.

Both versions are important. It is, of course, possible to interchange A and B, and so we have the extended formula

$$P(A \cap B) = P(A)\, P(B \backslash A) = P(B)\, P(A \backslash B)$$

Example (Drawings Without Replacement)

In a batch of 50 units there are 5 defectives. A unit is selected at random, and thereafter بعد ذلك one more from the remaining ones. Find the probability that both are defective.

Sol let A be the event that "the first unit is defective" and B the event that "the second unit is defective". It can seen that

$$P(A) = \frac{5}{50}$$

If A occurs, there remain 49 units, 4 of which are defective. Hence we conclude that

$$P(B \backslash A) = \frac{4}{49}$$

and the formula from definition produce the probability which we seek

$$P(A \cap B) = P(A) \cdot P(B \backslash A) = \frac{5}{50} \cdot \frac{4}{49} = \frac{2}{245}$$

Example

Suppose that five motors are available for use, and one is to be selected. Motor 2 is defective, motors 1 and 2 come from Supplier I, and motors 3, 4, 5 come from supplier II.

A randomly selected motor has probability $\frac{1}{5}$ of being defective.

However, if a motor is selected and then observed to have come from supplier I, the probability that it is defective will change. Of the two motors from Supplier I, one is defective. Thus the probability of observing a defective motor given that it is from supplier I should be $\frac{1}{2}$.

1	2	3	4	5
ok	defective	ok	ok	ok
I	I	II	II	II

let us use the Conditional probability diffinition.

assume A be the event that "the selected motor is defective"

B be the event that "the selected motor come from supplier I".

$P(A) = \frac{1}{5}$

$P(B) = \frac{2}{5}$

$P(A \cap B) = \frac{1}{5}$

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$$

Example

In an experement of selecting a card from a fully shuffled deck. Suppose another person draws a card from the deck without letting you see it, but in a brief glimpse you see enough to know it must be a face card. what is the probability that the selected card is a king?

Sol Since there are only 12 face card in the deck. and the deck contains 4 kings.

Our answer is not $\frac{4}{52}$, since our surreptitiously gained information indicates that some of the 52 cards are impossible to be selected. The sample space has been restricted to 12 face cards, so that the only remaining uncertainty is which of the 12 cards has been removed. In a sence, there is a new "sample space" having as elementary events the 12 face cards.

Using basic concept of probability, we can determin that the probability of a king is $\frac{4}{12}=\frac{1}{3}$. thus we can state that the conditional probability of a king given face card is

$$P(\text{King} \mid \text{face Card}) = \frac{4}{12} = \frac{1}{3}$$

we may use the conditional probability formulla as below:

$$P(\text{King} \mid \text{face}) = \frac{P(\text{King and face})}{P(\text{face})} = \frac{\frac{4}{52}}{\frac{12}{52}} = \frac{4}{12} = \frac{1}{3}$$

Example If H stands for heads and T for tails, the four equally likely outcomes for two flips of a balanced coin are

HH, HT, TH, and TT

If A denote the event of getting head on the first flip of the coin.
B denot the event of getting head on the Second flip of the coin.

find (a) $P(A)$ [redacted], $P(B)$

(b) $P(A \cap B)$

(c) $P(B \backslash A)$

Sol Using the formula $\frac{s}{n}$ for equally likely outcomes we get

(a) $P(A) = \frac{2}{4} = 0.5$ and $P(B) = \frac{2}{4} = 0.5$

(b) $P(A \cap B) = \frac{1}{4} = 0.25$

(c) $P(B \backslash A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.25}{0.50} = 0.5$

example a six sided die is throughn, the outcomes are

$\Omega = \{1, 2, 3, 4, 5, 6\}$

let
A : we obtain an odd no.
B : we obtain an even no.

find (a) $P(A)$, $P(B)$

(b) $P(A \cap B)$

(c) $P(B \backslash A)$

Sol (a) $P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{3}{6} = \frac{1}{2}$

(b) $P(A \cap B) = \emptyset$ because $A \cap B = \emptyset$

(c) $P(B \backslash A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\emptyset}{\frac{1}{2}} = \emptyset$ لانه هنا الـ Conditional
[illegible]

The General Multiplication Rule

If two events A and B are such that $p(B\backslash A) = p(B)$, that is, if the probability that B occurs is the same whether or not it is known that A has accured, then it is reasonable to say that A and B are independent. Thus, from the conditional probability formula, we can obtain the formula:-

$$p(B) = \frac{p(A \cap B)}{p(A)}$$

multiplication of both sides by $p(A)$ results in the following formula:

$$\boxed{p(A \cap B) = p(A) \cdot p(B)}$$ special multiplication rule (independent events)

and A, B are said to be independent events.

hence;

if A, B, C are independent events then

$$p(A \cap B \cap C) = p(A) \cdot p(B) \cdot p(C)$$

we can also use the conditional probability formula to obtain the multiplication rule for the dependent events.

$$p(A\backslash B) = \frac{p(A \cap B)}{p(B)}$$

multiplication both sides with $p(B)$ results in the following formula which is called the general multiplication rule, which enables us to calculate the probability that two events will both occur

$$\boxed{p(A \cap B) = p(B) \cdot p(A\backslash B)}$$ the general multiplication Rule

As it does not matter which event is referred to as A and which is reffered to as B, the above formula can also be written as

$$\boxed{p(A \cap B) = p(A) \cdot p(B\backslash A)}$$

Exmple

what is the probability of getting two aces in a row when two Cards are drawn from an ordinary deck of 52 playing Cards, if

(a) the first Card is replaced befor the second Card is drawn?

(b) the first Card is not replaced before the second card is drawn?

Sol

let A : the first drawn Card is ace.

B : the second drawn Card is ace.

(a) since there are four aces among the 52 Cards, we get

$$P(A) = \frac{4}{52} \quad , \quad P(B) = \frac{4}{52}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

(b) Since there are only three aces among the 51 Cards which remain after one ace has been removed from the deck, we get

$$P(A) = \frac{4}{52}$$

$P(B) = \frac{3}{51}$ which is equivalent to $P(B \setminus A)$

$$P(A \text{ and } B) = P(A) \cdot P(B \setminus A)$$

$$= \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$$

Independence of Events [Lawrance P123]

Definition

Two events A and B are said to be Independent if the Chance of one is unaffected by the occurrence of the other; that is, if

$$P(A \mid B) = P(A)$$

whenever the above equality holds, so must the following

$$P(B \mid A) = P(B)$$

ex a card is drawn from a fully shuffled deck of 52 ordinary playing Cards. are the events "jack" and "face Card" ~~are~~ dependent events or not.

sol

$$P(\text{jack} \mid \text{face}) = \frac{P(\text{jack and face})}{P(\text{face})}$$

$$= \frac{4/52}{12/52} = \frac{1}{3}$$

$$P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$$

since $P(\text{jack}) \neq P(\text{jack} \mid \text{face})$

$\therefore$ the events "jack" and "face Card" are dependent.

argument The events "jack" and "face" are dependent is evident from the fact that knowing a card is a face card increases the probability that the other event will occur from what the probability would have been without the knowledge of the first event.

example

Suppose that a foreman must select one worker for a special job, from a pool of four available workers, number 1, 2, 3, and 4. He select the workers by mixing the four names and randomly selecting one. Let A denote the event that worker 1 or 2 is selected, B the event that worker 1 or 3 is selected, and C the event that worker 1 is selected. Are A and B independent? Are A and C independent?

Sol Since the name is selected at random, a reasonable assumption for probabilistic model is to assign a probability of $\frac{1}{4}$ to each individual worker. then

$$P(A) = \frac{1}{2} \quad , \quad P(B) = \frac{1}{2} \quad , \text{ and } \quad P(C) = \frac{1}{4}$$

Since the intersection $A \cap B$ contains only worker no. 1, then

$P(A\cap B) = \frac{1}{4}$. " محسوبه "

Now

$$P(A\cap B) = \frac{1}{4} = P(A) \cdot P(B) \quad \text{or} \quad P(A\backslash B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

So A and B are independent. $\therefore P(A) = P(A\backslash B)$

Since the intersection $A \cap C$ contains only worker no. 1, then

$$P(A\cap C) = \frac{1}{4} \qquad \text{or} \quad P(A\backslash C) = \frac{P(A\cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

but

$$P(A\cap C) = \frac{1}{4} \neq P(A) \cdot P(C)$$

So A and C are not independent. $\therefore P(A) \neq P(A\backslash C)$

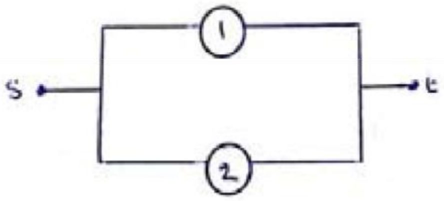
A and C are said to be dependent because the fact that C occurs changes the probability that A occurs.

A : worker 1 or 2 selected ملاحظة الفرض سيزال عند التركيز على ان الحدث

B : worker 1 or 3 selected

Example [Richard P44]

A section of an electrical circuit has two relays in parallel, as shown below. The relays operate independently, and when a switch is thrown, each will close properly with probability of 0.8. Find the probability that current will flow from s to t.



Let O denote an open relay and C a closed relay. The four sample events for this experiment are given by:-

	Relay 1	Relay 2
$E_1 = \{($	O	$O)\}$
$E_2 = \{($	O	$C)\}$
$E_3 = \{($	C	$O)\}$
$E_4 = \{($	C	$C)\}$

Since the relays operate independently, we can find the probabilities for these sample events as follows.

$$P(E_1) = P(O) \cdot P(O) = (0.2)(0.2) = 0.04$$
$$P(E_2) = P(O) \cdot P(C) = (0.2)(0.8) = 0.16$$
$$P(E_3) = P(C) \cdot P(O) = (0.8)(0.2) = 0.16$$
$$P(E_4) = P(C) \cdot P(C) = (0.8)(0.8) = 0.64$$

If A denote the event that current will flow from s to t then

$$A = E_2 \cup E_3 \cup E_4$$

or $\bar{A} = E_1$

$\therefore \quad P(A) = 1 - P(\bar{A})$ "At least one of the relays must be close for current to flow"

$= 1 - P(E_1) = 1 - 0.04 = 0.96$ which is the same as $P(E_2) + P(E_3) + P(E_4)$

Common Errors in Applying the Laws of Probability
Lawrence P129

Some of the most prevalent errors committed in the determination of probability values are due to the improper use of the law of probability. A few of the common mistakes are listed here:

(1) Using the addition law to find the probability of the union of several events when they are not mutually exclusive, without correcting for the double counting of possible occurrences.

example

Suppose that casualty insurance underwriters have established that the probabilities of a city experiencing one of the following natural disasters in the next decade are

disasters	Probability
tornado	0.7
flood	0.5
earthquack	0.6

we cannot say that the probability of suffering tornado or flood in the next decade is equal to $0.7 + 0.5 = 1.1$.

Clearly, two or more of these disasters may occur over a ten-year Period simultenuously, and some may occur more than once.

(2) Using the addition law when the multiplication law should be used, and conversely. Remember that or signifies addition and that and signifies multiplication.

example The probability of drawing a red face card is the same as that for the event "red and face"; Recall that:

$$P(red) = \frac{26}{52} \quad \text{and} \quad P(face) = \frac{12}{52}$$

if we add these values

$$\frac{26}{52} + \frac{12}{52} = \frac{38}{52}$$

we obtain a meaningless result.

Thus, since red and face are independent events:

$$P(\text{red and face}) = \frac{26}{52} \times \frac{12}{52} = \frac{6}{52}$$

(3) Using the multiplication law for independent component events when the events are dependent.

A common commission of this error occurs when replacement is mistakenly assumed in calculating the probability of obtaining a particular sample result. As we have seen, removal of an item from a group changes the group's composition and hence the probabilities that future selections will be of a certain type.

(4) Improperly identifying the complement of an event.

For example, the complement of non is some (which may be expressed as "one or more" or "at least one").

The above examples, which may happened, illustrates how ludicrous results may be obtained through the incorrect application of probability laws.

RANDOM VARIABLES المتغيرات

PROBABILITY DISTRIBUTIONS

RANDOM VARIABLES

Random variables are very important in probability theory. The concept is uncomplicated, but it takes some time and energy to become familiar with it.

A random trail often produces a number associated with the outcome of the trail. This number is not known beforehand; it is determined by the outcome of the trail, that is, by chance. It is therefore called a Random Variable (abbreviated rv).

When there is only one single number at a time is associated with the outcome of each trail, the term one-dimensional rv is used.

Typical examples of a one-dimensional rv are the number of heads in a sequence of tosses of a coin, the gain of a player in a roulette game in Mont Carlo, the number of children in a randomly chosen family, and the length of the life of a randomly chosen citizen.

We have been somewhat careless in these examples. Strictly speaking, the random variable has nothing to do with the random trails themselves, for these trials are phenomena in the empirical world, while the Variables belong to the model we build by means of probability theory.

Example refer to the example P_{54}, in which two relays are operating in parallel. let the numerical event of interest is the number of relays that closed properly. If we denote the number of relays that close by X, then X can take on three possible values 0, 1, 2. we can assign probabilities to these values as follows:

$$P(X=0) = P(E_1) = 0.04$$

$$P(X=1) = P(E_2 \cup E_3) = P(E_2) + P(E_3) = 0.32$$

$$P(X=2) = P(E_4) = 0.64$$

Thus we have mapped the sample space of four simple events into a set of three meaningful real numbers, and attached a probability to each - the quantity X is called a random variable.

Definition A random variable (rv) is a real-value function whose domain is a sample space.

Random variables will be denoted by upper-case letters such as X, Y, and Z. The actual numerical values that a random variable can assume will be denoted by lower-case letters, such as x, y, and z. We can then talk about "the probability that X takes on the value x" denoted by $P(X=x)$.

In the relay example, the random variable, X, has only three possible values and it is a relatively simple matter to assign probabilities to these values. Such a random variable is called discrete.

Definition A random variable X is said to be discrete if it can take on only a finite number, or a countable infinity, of possible values x.

In this case:

(1) $P(X=x) = p(x) \geq 0$

(2) $\sum_{x} P(X=x) = 1$, where the sum is over all possible values x.

The function $p(x)$ is called the probability function of X.

It is often convenient to list the probabilities for a discrete random variable on a table. With X defined as the number of closed relays in the problem discussed above, the table is as follows:

x	$p(x)$
0	0.04
1	0.32
2	0.64
total	1.00

This listing is one way of representing the probability distribution of X.

Note The relation between a random variable's values and their probabilities is summarized by the probability distribution.

A second type of random variable comes about in an experiment such as measuring the lifelength X of a transistor. In this case there is an infinite number of possible values that X can assume. We cannot assign a positive probability to each possible outcome of the experiment because, no matter how small we might make the individual probabilities, they will sum to a value greater than one when accumulated over the entire sample space. We can, however, assign positive probabilities to intervals of real numbers in a manner consistent with the axioms of probability.

A random variable X that is associated with a continuum of values, such as in the lifelength example, is called continuous. The function $f(x)$, which models the relative frequency behavior of X, is called the probability density function.

Definition

A random variable X is said to be continuous if it can take on the infinite number of possible values associated with intervals of real numbers, and there is a function $f(x)$, called probability density function, such that

(1) $f(x) \geq 0$, for all x.

(2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

(3) $P(a \leq X \leq b) = \int_a^b f(x)\, dx$

Note that for a continuous random variable X,

$$P(X=a) = \int_a^a f(x)\, dx = 0 \quad \text{for any specific value } a.$$

The fact that we must assign zero probability to any specific value should not disturb us, since there is an infinite number of possible values that X can assume. For example, out of all the possible values that the lifelength of a transistor can take on, what is the probability

that the transistor we are using will last exactly 497.392 hours?
Assigning probability zero to this event does not rule out 497.392 as a possible lifelength, but it does say that the chance of observing this particular lifelength is ~~[illegible]~~ extremely small.

Example Throwing a Die

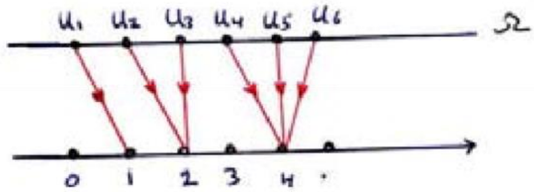
A well-made die is thrown once. A person receives one dollar for a "one", two dollars for a "two" or a Three, and four dollars for a "four", a "five" or a "six". The amount he gets in a rv X which can take the values 1, 2, and 4.

let u_i denote the outcome "die shows i". Formally, we have assigned numbers to the outcomes as follows:-

$u_1 \to 1$, $u_2 \to 2$, $u_3 \to 2$, $u_4 \to 4$, $u_5 \to 4$, $u_6 \to 4$

Thus, the rv X defines a function from the sample space to the real numbers. It has the domain $\Omega = \{u_1, u_2, \dots, u_6\}$ and the range $\{1, 2, 4\}$.

$P(X=1) = \frac{1}{6}$
$P(X=2) = \frac{2}{6}$
$P(X=4) = \frac{3}{6}$



Note

The rv is not always the measurement itself, it can be a function of this value. For instance, if the radius of a circle is measured and the quantity of interest is the area, we take $Y = \pi X^2$, where X is the measured radius. Hence, the function defined by rv Y is $u_x \to \pi x^2$.

Ex (life lengths of batteries)

Suppose that we have conducted an experiment designed to measure the lifelengths of a collection of batteries of a certain type.

let the random variable X represent the lifelength of those batteries, which has associated with it a probability density function of the form

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that the life length of a particular battery of this type is less than 2 or greater than 4 hours.

Sol we can clearly see that ~~that~~ most of the life lengths are real numbers; Thus, the random Variable X is a Continuous random Variable.

let A denote ~~that~~ the event that X is less than 2 hours.

B denote the event that X is greater than 4 hours.

then since A and B are mutually exclusive.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= \int_0^2 \frac{1}{2} e^{-x/2} dx + \int_4^\infty \frac{1}{2} e^{-\frac{x}{2}} dx \\ &= -e^{-x/2} \Big|_0^2 + \left(-e^{-x/2} \Big|_4^\infty\right) \\ &= (1 - e^{-1}) + (e^{-2}) \\ &= 1 - 0.368 + 0.135 \\ &= 0.767 \end{aligned}$$

Ex refer to the last example. Find the probability that a battery of this type ~~lasts~~ works more than 3 hours given that it has already been in use for more than 2 hours.

Sol we are interested in $P(X>3 \backslash X>2)$ and by the definition of conditional probability, we have

$$P(X>3 \backslash X>2) = \frac{P(X>3)}{P(X>2)}$$

since the intersection of the event $(X>3)$ and $(X>2)$ is the event $(X>3)$. Now

$$\frac{P(X>3)}{P(X>2)} = \frac{\int_3^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx}{\int_2^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx} = \frac{e^{-\frac{3}{2}}}{e^{-1}} = e^{-\frac{1}{2}} = 0.606$$

Example check whether the correspondence given by

$$P(x) = \frac{x+3}{15} \quad \text{for } x = 1, 2, 3$$

Can serve as the probability distribution of some random variable

Solution substituting $x = 1, 2, 3$ into $\frac{x+3}{15}$ we get

$$P(1) = \frac{4}{15}, \quad P(2) = \frac{5}{15}, \quad P(3) = \frac{6}{15}$$

Since none of these values is negative or greater than 1, and since their summation is

$$\frac{4}{15} + \frac{5}{15} + \frac{6}{15} = 1$$

the given function can serve as the probability distribution of some random variable.

Cumulative probability [Richard P61]

تنقل لعبد شرح

Probability distribution function
density

We sometimes study the behavior of random variables by Looking at their **cumulative** probabilities. that is, for any random variable X we may Look at $P(X \leq b)$ for any real number b. This is the Cumulative probability for X evaluted at b. Thus we can define a function $F(b)$ as $F(b) = P(X \leq b)$

Definition the ~~distribution~~ Cumulative function $F(b)$ for a random variable X is defined as

$$F(b) = P(X \leq b)$$

if X is discrete then:

$$F(b) = \sum_{x=-\infty}^{b} P(x)$$ where $P(x)$ is the probability distribution function

if X is continuous then:

$$F(b) = \int_{-\infty}^{b} f(x)\,dx$$ where $f(x)$ is the probability density function

Example The random variable X, denoting the number of relays closing properly, has the probability distribution function given by

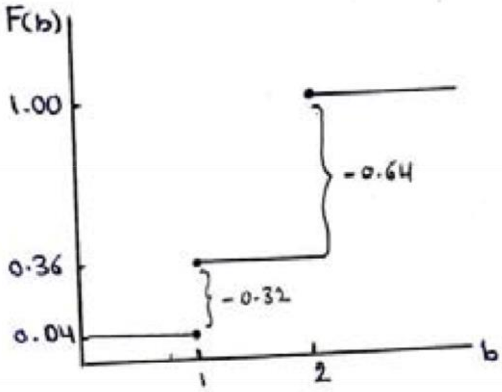
$$P(x) = \begin{cases} 0.04 & x = 0 \\ 0.32 & x = 1 \\ 0.64 & x = 2 \end{cases}$$

the Cumulative function for this random variable then has the form

$$F(b) = \begin{cases} 0 & b < 0 \\ 0.04 & 0 \leq b < 1 \\ 0.36 & 1 \leq b < 2 \\ 1.00 & 2 \leq b \end{cases}$$

Note

$P(X \leq 1.5) = P(X \leq 1.9) = P(X \leq 1) = 0.36$ because X is a discret rv.



A Cumulative Function for a discrete rv

Example

In the lifelength of batteries example, X has a probability density function given by:

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

then find the cummulative function for the above function.
Probability density

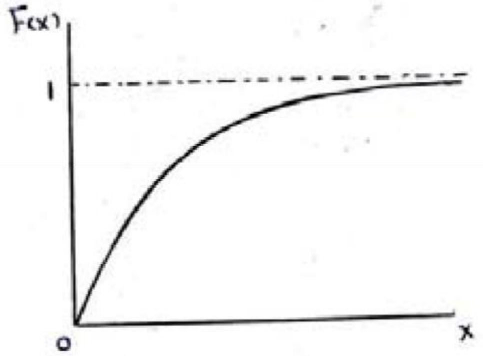
Sol

$$F(b) = P(X \leq b) = \int_{-\infty}^{b} f(x)\, dx$$

$$= \int_{0}^{b} \frac{1}{2} e^{-x/2}\, dx$$

$$= -e^{-x/2} \Big]_{0}^{b}$$

$$\therefore \quad F(b) = \begin{cases} 1 - e^{-b/2} & b > 0 \\ 0 & b \leq 0 \end{cases}$$



A Cumulative Function for a Continuous rv.

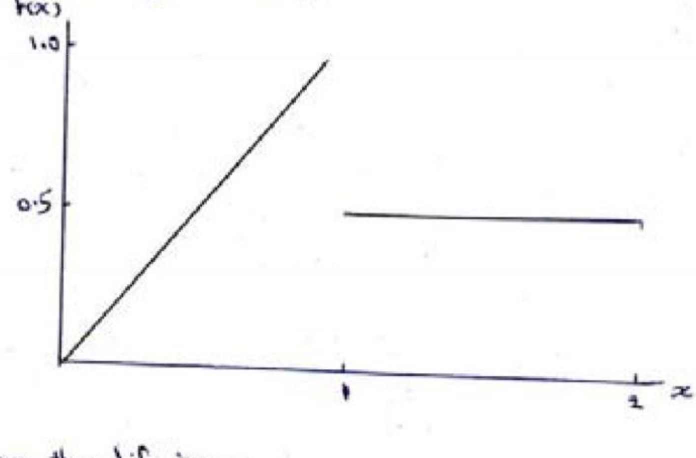
Example [schaffer P63]

A supplier of Kerosene has a 200-gallon tank filled at the begining of each week. His weekly demands show a relative frequency behavior that increases steadily up to 100 gallons, and then levels of between 100 and 200 gallons. letting X denote weekly demand in hundreds of gallons, suppose the relative frequencies for demand are modeled adequatly by

$$f(x) = \begin{cases} = 0 & x < 0 \\ = x & 0 \leq x \leq 1 \\ = 1/2 & 1 < x \leq 2 \\ = 0 & x > 2 \end{cases}$$

a. Find $F(b)$ for this random variable.

b. use $F(b)$ to find the probability that demand will exceed 150 gallons on a given week.



Solution a. From the difinition

$$F(b) = \int_{-\infty}^{b} f(x)\, dx$$

$$= 0 \qquad b < 0$$

$$= \int_0^b x\, dx = \left.\frac{x^2}{2}\right|_0^b = \frac{b^2}{2} \qquad 0 \leq b \leq 1$$

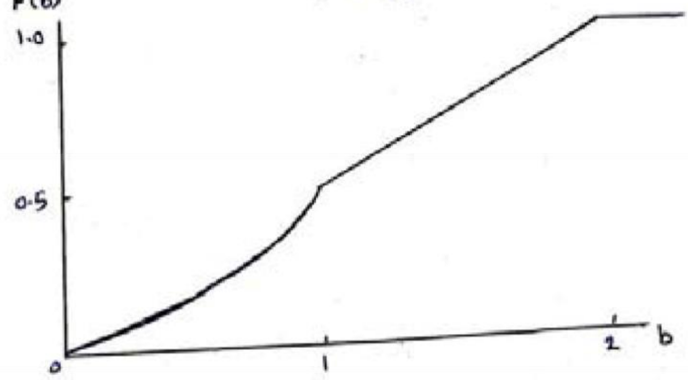
$$= \frac{1}{2} + \int_1^b \frac{1}{2}\, dx = \frac{1}{2} + \left.\frac{x}{2}\right|_1^b$$

$$= \frac{1}{2} + \frac{b}{2} - \frac{1}{2} = \frac{b}{2} \qquad 1 < b \leq 2$$

$$= 1 \qquad b > 2$$

b. The probability that demand will exceed 150 gallons is given by

$$P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - F(1.5)$$
$$= 1 - \frac{1.5}{2}$$
$$= 0.25$$



Note that $F(b)$ is continuous over the whole real line, even though $f(b)$ has two discontinuities.

The Binomial Distribution

There are many applied problems in which we are interested in the probability that an event will occur x times out of n. In Binomial distribution we are interested in the Probability of getting "x successes in n trails", or in other words, "x successes and $n-x$ failures in n attempts".

Essential characteristics of the Binomial distribution

1. There are n identical trails that lead to one of two outcomes: Success or failur.
2. The probability of each outcome remains constant from trail to trail. The probability of one of these outcomes, called success, is designated P; in other words, the probability of a success is the same for each trail.
3. The trails are all independent.

A good example of a Binomial experement is flipping a fair coin several times. There are only two possible outcomes for each trail, or flip (heads and tails); the probability of heads or tails remain constant from flip to flip (0.5 each), and the flips are independent of each other.

The probability of getting x successes in n independent trails is:

$$P(x) = C_x^n \, P^x (1-P)^{n-x} \quad \text{for } x = 0, 1, 2, \cdots, n$$

where C_x^n = Number of ways of getting exactly x success in n trails.

P = Probability of success on any one trail.

$1-P$ = Probability of failure on any one trail.

n = Number of trails

x = Number of successes in n trails.

معتاد، مألوف

It is customary to say here that the number of successes in n trails is a random variable having the Binomial probability distribution.

Ex a fair coin is to be flipped three times. what is the probability of obtaining exactly two heads?

Sol The set of outcomes for this experiment = $\left\{\begin{matrix} HHH, \underline{HTH}, \underline{HHT}, \underline{THH} \\ TTT, THT, TTH, HTT \end{matrix}\right\}$

let x represent the number of times we obtaining a head (number of successes)

it is clear that x follows the Binomial distribution.

the probability of getting x successes in n independent trail is:-

$$P(x) = C_x^n \, P^x (1-P)^{n-x}$$

where $n = 3$ the number of trail

$P = 0.5$ the probability of success on any trail

$$P(x=2) = C_2^3 \, (0.5)^2 \, (1-0.5)^{3-2}$$

$$= \frac{3!}{2!\,(3-2)!} (0.5)^2 (1-0.5)^1$$

لا تطلع $\leftarrow$

$$= 3 \times 0.25 \times 0.5 = 0.375 = \frac{3}{8}$$

Ex If the probability is 0.70 that any one registered voter will vote in a given elections; What is the probability that two of five registered voters will vote in the election?

Sol let x represent the number of registered voters that will actually vote (number of successes)

It is clear that x follows the Binomial distribution

where $x = 2$, $n = 5$, $P = 0.70$

$\boxed{e^0 = 1}$

$$P(x=2) = C_2^5 \ (0.7)^2 (1-0.7)^{5-2}$$

$$= \frac{5!}{2!\,(5-2)!} (0.7)^2 (0.3)^3$$

$$= 10 \times (0.7)^2 \times (0.3)^3 = 0.132$$

Ex Suppose that the probability is 0.60 that a car stolen in a certain city will be recovered. Find the probability that

(a) at most three of ten cars stolen in this city will be recovered.

(b) at least seven of ten cars stolen in this city will be recoverd.

Sol Let x represent the number of stolen cars that will be recovered.

$\therefore$ x follow the Binomial Distribution.

(a) $n = 10$, $p = 0.6$, $x = 0, 1, 2, 3$

$$P(x=0) = C_0^{10} (0.6)^0 (1-0.6)^{10} = (0.4)^{10} = 0.0001 \simeq 0$$

$$P(x=1) = C_1^{10} (0.6)^1 (1-0.6)^9 = 0.002$$

$$P(x=2) = C_2^{10} (0.6)^2 (1-0.6)^8 = 0.011$$

$$P(x=3) = C_3^{10} (0.6)^3 (1-0.6)^7 = 0.042$$

thus the probability that at most three of ten cars will be recoverd equal to: $0.000 + 0.002 + 0.011 + 0.042 = 0.055$

(b) $n = 10$, $p = 0.6$, $x = 7, 8, 9, 10$

$$P(x=7) = C_7^{10} (0.6)^7 (1-0.6)^3 = 0.215$$

$$P(x=8) = C_8^{10} (0.6)^8 (1-0.6)^2 = 0.121$$

$$P(x=9) = C_9^{10} (0.6)^9 (1-0.6)^1 = 0.040$$

$$P(x=10) = C_{10}^{10} (0.6)^{10} (1-0.6)^0 = (0.6)^{10} = 0.006$$

the probability that at least seven of ten cars will be recoverd equal to: $0.215 + 0.121 + 0.040 + 0.006 = 0.382$

Ex suppose that a Large lot of fuses contains 10 % defectives. If four fuses are randomly sampled from the lot, then

(a) find the probability that exactly one fuse is defective

(b) find the probability that at Least one fuse, in the sample of four is defective

Sol we assume that the four trails are independent and that the probability of observing a defective is the same (0.1) for each trail (This would be approximately true only if the lot is indeed Large). Thus the Binomial distribution provides a reasonable model for this experement, and we have, with x denoting the number of defectives:-

(a) $p(x=1) = C^4_1 \, (0.1)^1 (0.9)^3 = 0.2916$

(b) we have $x = 1, 2, 3, 4$

$$p(x \geq 1) = 1 - p(x=0)$$
$$= 1 - C^4_0 \, (0.1)^0 (0.9)^4$$
$$= 1 - (0.9)^4$$
$$= 0.3439$$

اعرفي

Ex

Determine the probability that a six-digit binary number contains a zero exactly twice.

Sol let x represent the number of times we obtain a zero

x follow the Binomial distribution

$n = 6 \qquad x = 2 \qquad P = \frac{1}{2}$

$P(x=2) = C^6_2 \, (\frac{1}{2})^2 (\frac{1}{2})^4 = \frac{15}{64}$