Since the casting conditions have not changed, the mold constant B is unchanged. The *V*/*A* ratio of the new casting is

$$t_r = \mathrm{B}\left(\frac{V_r}{A_r}\right)^2 = (22)\left(\frac{V_r}{A_r}\right)^2 = 11.05 \text{ min}$$

$$\left(\frac{V_r}{A_r}\right)^2 = 0.502 \text{ in.}^2 \quad \text{or} \quad \frac{V_r}{A_r} = 0.709 \text{ in.}$$

If *x* is the required thickness for our redesigned casting, then

$$\frac{V_r}{A_r} = \frac{(\pi/4)d^2x}{2(\pi/4)d^2 + \pi dx} = \frac{(\pi/4)(18)^2(x)}{2(\pi/4)(18)^2 + \pi(18)(x)} = 0.709 \text{ in.}$$

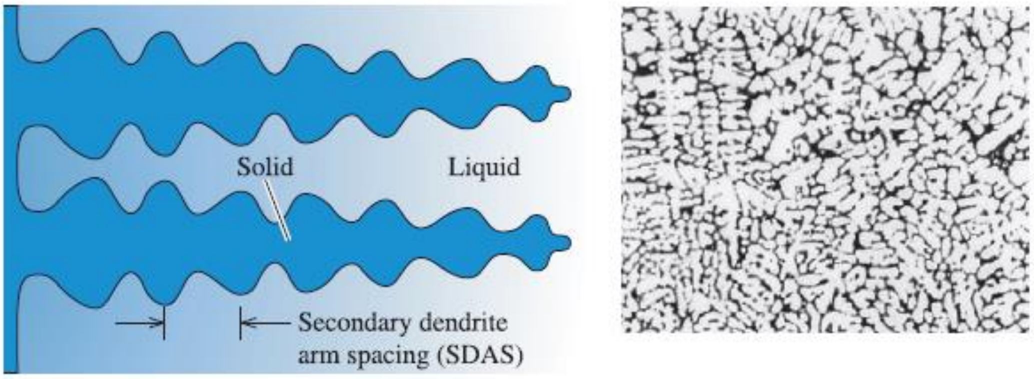
Therefore, *x* = 1.68 in.

This thickness provides the required solidification time, while reducing the overall weight of the casting by more than 15%.

Solidification begins at the surface, where heat is dissipated into the surrounding mold material. The rate of solidification of a casting can be described by how rapidly the thickness *d* of the solidified skin grows:

$$d = \mathrm{k}_{\text{solidification}}\sqrt{t} - \mathrm{c}_1$$

where *t* is the time after pouring, $\mathrm{k}_{\text{solidification}}$ is a constant for a given casting material and mold, and c1 is a constant related to the pouring temperature



(a) (b)

Figure 9-5 (a) The secondary dendrite arm spacing (SDAS). (b) Dendrites in an aluminum alloy (× 50). *(From ASM Handbook, Vol. 9, Metallography and Microstructure (1985), ASM International, Materials Park, OH 44073-0002.)*

Effect on Structure and Properties

The solidification time affects the size of the dendrites. Normally, dendrite size is characterized by measuring the distance between the secondary dendrite arms (Figure 9-5). The **secondary dendrite arm spacing** (SDAS) is reduced when the casting freezes more rapidly. The finer, more extensive dendritic network serves as a more efficient conductor of the latent heat to the undercooled liquid. The SDAS is related to the solidification time by

$$\text{SDAS} = k t_s^{m} \quad (9\text{-}6)$$

where m and k are constants depending on the composition of the metal. This relationship is shown in Figure 9-6 for several alloys. Small secondary dendrite arm spacing's are associated with higher strengths and improved ductility (Figure 9-7). Rapid solidification processing is used to produce exceptionally fine secondary dendrite arm spacing's; a common method is to produce very fine liquid droplets that freeze into solid particles. This process is known as spray atomization. The tiny droplets freeze at a rate of about 10^4°C/s, producing powder particles that range from5–100 μm. This cooling rate is not rapid enough to form a metallic glass, but does produce a fine dendritic structure. By carefully consolidating the solid droplets by powder metallurgy processes, improved properties in the material can be obtained. Since the particles are

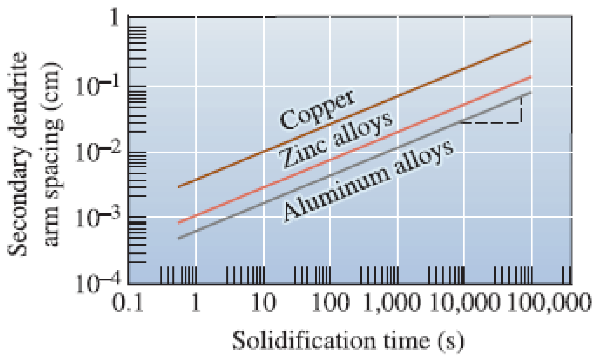


Figure 9-6 The effect of solidification time on the secondary dendrite arm spacings of copper, zinc, and aluminum.

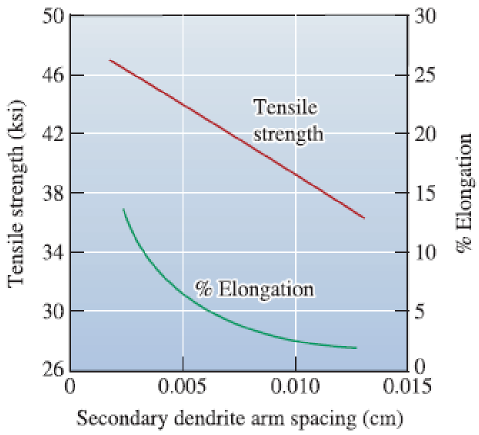


Figure 9-7 The effect of the secondary dendrite arm spacing on the mechanical properties of an aluminum casting alloy.

derived from a melt, many complex alloy compositions can be produced in the form of chemically homogenous powders. The following three examples discuss how Chvorinov's rule, the relationship between SDAS and the time of solidification, and the SDAS and mechanical properties can be used to design casting processes.

Example 9-3 *Secondary Dendrite Arm Spacing for Aluminum Alloys*

Determine the constants in the equation that describe the relationship between secondary dendrite arm spacing and solidification time for aluminum alloys (Figure 9-6).

SOLUTION

We could obtain the value of SDAS at two times from the graph and calculate k and m using simultaneous equations; however, if the scales on the ordinate and abscissa are equal for powers of ten (as in Figure 9-6), we can obtain the slope m from the log-log plot by directly measuring the slope of the graph. In Figure 9-6, we can mark five equal units on the vertical scale and 12 equal units on the horizontal scale. The slope is

$$\mathrm{m} = \frac{5}{12} = 0.42$$

A 4-
in.-

The constant k is the value of SDAS when $t_s = 1$ s, since

$$\log \mathrm{SDAS} = \log \mathrm{k} + \mathrm{m} \log t_s$$

If $t_s = 1$ s, $\mathrm{m} \log t_s = 0$, and SDAS = k, from Figure 9-6:

$$\mathrm{k} = 7 \times 10^{-4} \frac{\mathrm{cm}}{\mathrm{s}}$$

Example 9-4 *Time of Solidification*

diameter aluminum bar solidifies to a depth of 0.5 in. beneath the surface in 5 minutes. After 20 minutes, the bar has solidified to a depth of 1.5 in. How much time is required for the bar to solidify completely?

SOLUTION

From our measurements, we can determine the constants $k_{\text{solidification}}$ and c_1 in Equation 9-5:

$$0.5\text{ in.} = k_{\text{solidification}}\sqrt{(5\text{ min})} - c_1 \quad \text{or} \quad c_1 = k\sqrt{5} - 0.5$$

$$1.5\text{ in.} = k_{\text{solidification}}\sqrt{(20\text{ min})} - c_1 = k\sqrt{20} - (k\sqrt{5} - 0.5)$$

$$1.5 = k_{\text{solidification}}(\sqrt{20} - \sqrt{5}) + 0.5$$

$$k_{\text{solidification}} = \frac{1.5 - 0.5}{4.472 - 2.236} = 0.447\frac{\text{in.}}{\sqrt{\text{min}}}$$

$$c_1 = (0.447)\sqrt{5} - 0.5 = 0.5\text{ in.}$$

Solidification is complete when *d* = 2 in. (half the diameter, since freezing is occurring from all surfaces):

$$2 = 0.447\sqrt{t} - 0.5$$

$$\sqrt{t} = \frac{2 + 0.5}{0.447} = 5.59$$

$$t = 31.25 \text{ min}$$

In actual practice, we would find that the total solidification time is somewhat longer than 31.25 min. As solidification continues, the mold becomes hotter and is less effective in removing heat from the casting.

Example 9-5 *Design of an Aluminum Alloy Casting*

Design the thickness of an aluminum alloy casting with a length of 12 in., a width of 8 in., and a tensile strength of 40,000 psi. The mold constant in Chvorinov's rule for aluminum alloys cast in a sand mold is 45 min/in^2. Assume that data shown in Figures 9-6 and 9-7 can be used.

SOLUTION

In order to obtain a tensile strength of 42,000 psi, a secondary dendrite arm spacing of about 0.007 cm is required (see Figure 9-7). From Figure 9-6 we can determine that the solidification time required to obtain this spacing is about 300 s or 5 minutes.

From Chvorinov's rule

$$t_s = \mathrm{B}\left(\frac{V}{A}\right)^2$$

where B = 45 min/in.^{2} and x is the thickness of the casting. Since the length is 12 in. and the width is 8 in.,

$$V = (8)(12)(x) = 96x$$

$$A = (2)(8)(12) + (2)(x)(8) + (2)(x)(12) = 40x + 192$$

$$5 \text{ min} = (45 \text{ min/in.}^2)\left(\frac{96x}{40x + 192}\right)^2$$

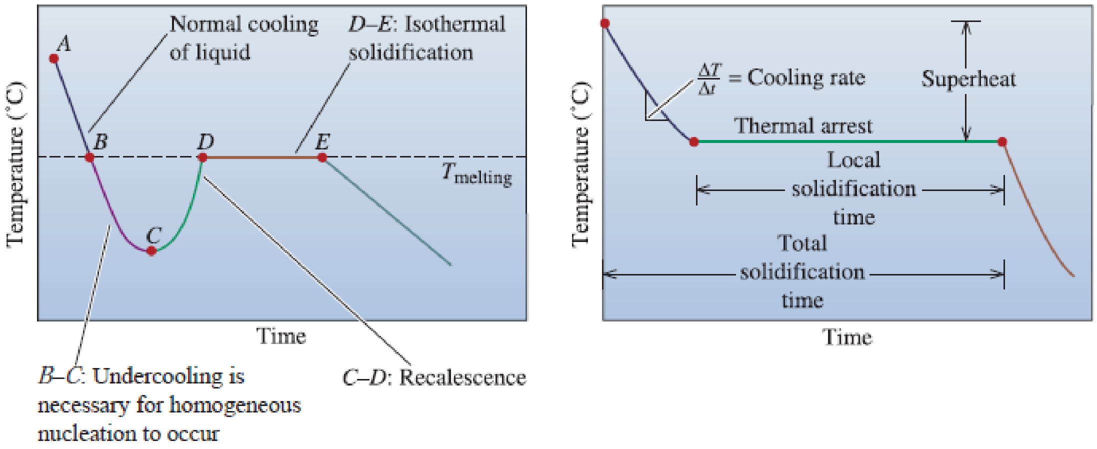
$$\frac{96x}{40x + 192} = \sqrt{(5/45)} = 0.333$$

$$96x = 13.33x + 64$$

$$x = 0.77 \text{ in.}$$

Cooling Curves

We can summarize our discussion at this point by examining cooling curves. A cooling curve shows how the temperature of a material (in this case, a pure metal) decreases with time [Figure 9-8 (a) and (b)]. The liquid is poured into a mold at the pouring temperature, point *A*. The difference between the pouring temperature and the freezing temperature is the superheat. The specific heat is extracted by the mold until the liquid reaches the freezing temperature (point *B*). If the liquid is not well-inoculated, it must be undercooled



(a) (b)

Figure 9-8 (a) Cooling curve for a pure metal that has not been well-inoculated. The liquid cools as specific heat is removed (between points *A* and *B*). Undercooling is thus necessary (between points *B* and *C*). As the nucleation begins (point *C*), latent heat of fusion is released causing an increase in the temperature of the liquid. This process is known as recalescence (point *C* to point *D*). The metal continues to solidify at a constant temperature (T_{melting}). At point *E*, solidification is complete. The solid casting continues to cool from this point. (b) Cooling curve for a well-inoculated, but otherwise pure, metal. No undercooling is needed. Recalescence is not observed. Solidification begins at the melting temperature.

(point *B* to *C*). The slope of the cooling curve before solidification begins is the cooling $\frac{\Delta T}{\Delta t}$e . As nucleation begins (point *C*), latent heat of fusion is given off, and the temperature rises. This increase in temperature of the undercooled liquid as a result of nucleation is known as **recalescence** (point *C* to *D*). Solidification proceeds isothermally at the melting temperature (point *D* to *E*) as the latent heat given off from continued solidification is balanced by the heat lost by cooling. This region between points *D* and *E*, where the temperature is constant, is known as the **thermal arrest**. A thermal arrest, or plateau, is produced because the evolution of the latent heat of fusion balances the heat being lost because of cooling. At point *E*, solidification is complete, and the solid casting cools from point *E* to room temperature. If the liquid is well-inoculated, the extent of