CHAPTER 20 Ducts

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20.1 Introduction

The conditioned air (cooled or heated) from the air conditioning equipment must be properly distributed to rooms or spaces to be conditioned in order to provide comfort conditions. When the conditioned air cannot be supplied directly from the air conditioning equipment to the spaces to be conditioned, then the ducts are installed. The duct systems convey the conditioned air from the air conditioning equipment to the proper air distribution points or air supply outlets in the room and carry the return air from the room back to the air conditioning equipment for reconditioning and recirculation.

It may be noted that the duct system for proper distribution of conditioned air cost nearly 20 to 30 per cent of the total cost of the equipments required and the power required by fans forms the substantial part of the running cost. Thus, it is necessary to design the air duct system in such a way that the capital cost of ducts and the cost of running the fans is lowest.

20.2 Classification of Ducts

The ducts may be classified as follows :

1. ***Supply air duct.*** The duct which supplies the conditioned air from the air conditioning equipment to the space to be conditioned is called supply air duct.

2. ***Return air duct.*** The duct which carries the recirculating air from the conditioned space back to the air conditioning equipment is called return air duct.

3. ***Fresh air duct.*** The duct which carries the outside air is called fresh air duct.

4. ***Low pressure duct.*** When the static pressure in the duct is less than 50 mm of water gauge, the duct is said to be a low pressure duct.

5. ***Medium pressure duct.*** When the static pressure in the duct is up to 150 mm of water gauge, the duct is said to be a medium pressure duct.

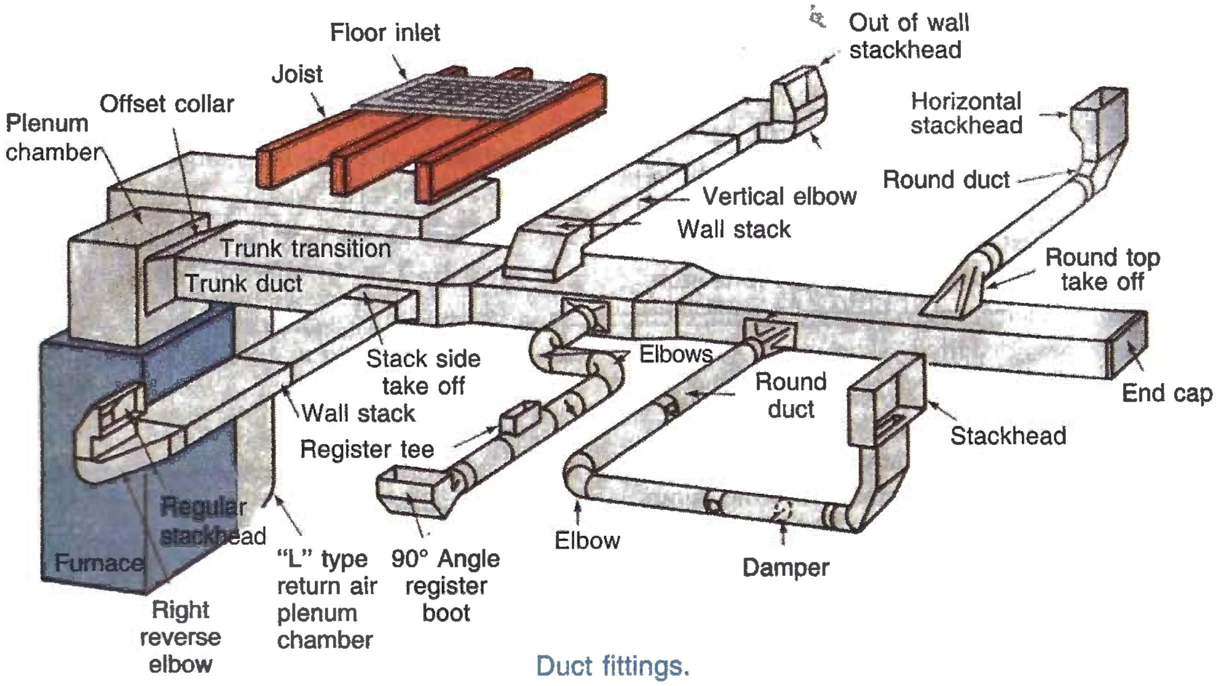
6. ***High pressure duct.*** When the static pressure in the duct is from 150 to 250 mm of water gauge, the duct is said to be a high pressure duct.

7. ***Low velocity duct.*** When the velocity of air in the duct is up to 600 m/min, the duct is said to be a low velocity duct.

8. ***High velocity duct.*** When the velocity of air in the duct is more than 600 m/min, the duct is said to be a high velocity duct.

20.3 Duct Material

The ducts are usually made from galvanised iron sheet metal, aluminium sheet metal or black steel. The most commonly used duct material in air conditioning systems is galvanised sheet metal, because the zinc coating of this metal prevents rusting and avoids the cost of painting. The sheet thickness of galvanised iron (*G.I.*) duct varies from 26 gauge (0.55 mm) to 16 gauge (1.6 mm). The aluminium is used because of its lighter weight and resistance to moisture. The black sheet metal is always painted unless they withstand high temperature.

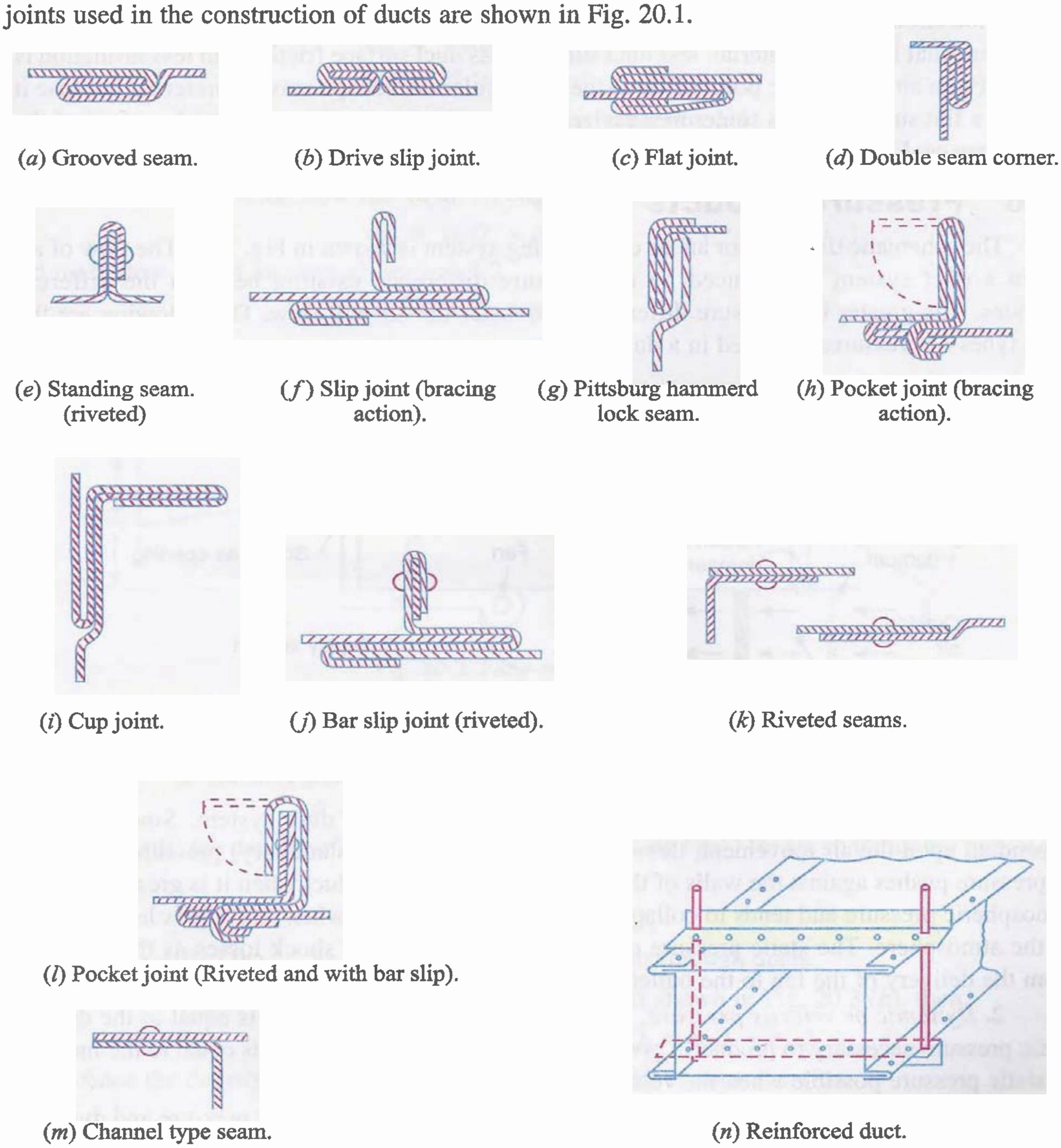


Duct fittings.

Now-a-days, the use of non-metal ducts has increased. The resin bonded glass fibre ducts are used because they are quite strong and easy to manufacture according to the desired shape and size. They are used in low velocity applications less than 600 metres/min and for static pressures below 5 mm of water gauge. The cement asbestos ducts may be used for underground air distribution and for exhausting corrosive materials. The wooden ducts may be used in places where moisture content in the air is not very large.

20.4 Duct Construction

The sheet metal ducts expand and contract as they heat and cool. The fabric joints are often used to absorb this movement. In order to prevent most of the fan and furnace noise from travelling along the duct metal, the fabric joints should also be used where ducts fasten to a furnace or an air conditioner. But, in fact, most duct joints are made of sheet metal. The various types of sheet metal joints used in the construction of ducts are shown in Fig. 20.1.



(*a*) Grooved seam. (*b*) Drive slip joint. (*c*) Flat joint. (*d*) Double seam corner.

(*e*) Standing seam. (riveted) (*f*) Slip joint (bracing action). (*g*) Pittsburg hammerd lock seam. (*h*) Pocket joint (bracing action).

(*i*) Cup joint. (*j*) Bar slip joint (riveted). (*k*) Riveted seams.

(*l*) Pocket joint (Riveted and with bar slip).

(*m*) Channel type seam. (*n*) Reinforced duct.

Fig. 20.1. Sheet metal duct joints.

The joint should be airtight and strong. Many joints are riveted for added strength and tightness. Many of these joints are also sealed with special duct tape to make them leakproof. The sealants are put in the duct seam for the same purpose.

When ducts travel through unconditioned space, they are often insulated to reduce noise as well as to decrease the rate of heat loss or gain through their walls. The insulation is fastened to the duct with adhesives. In some cases, metal clips are used to hold the insulation in place. For larger ducts and ducts under high pressures, the duct reinforcement as shown in Fig.20.1 is recommended. The duct reinforcement prevents bulging or collapsing of the duct and keeps the seams from separating. They also help to reduce noise associated with the duct vibration.

20.5 Duct Shape

The ducts may be made in circular, rectangular or square shapes. From an economical point of view, the circular ducts are preferred because the circular shape can carry more air in less space. This means that less duct material, less duct surface, less duct surface friction and less insulation is needed. From an appearance point of view, the rectangular duct shape may be preferred because it presents a flat surface that is sometimes easier to work with in relation to the finish surface of the room or space. From the practical point of view, the square duct may be preferred.

20.6 Pressure in Ducts

The schematic diagram for an air conditioning system is shown in Fig. 20.2. The flow of air within a duct system is produced by the pressure difference existing between the different locations. The greater the pressure difference, the faster the air will flow. The following are the three types of pressures involved in a duct system.

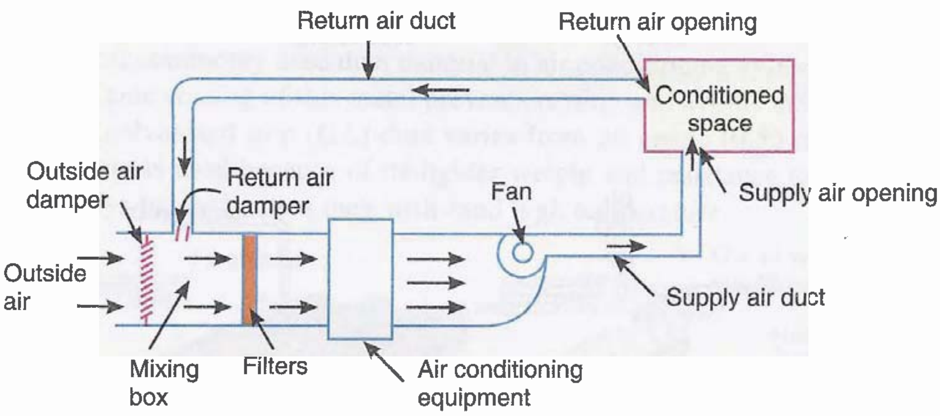


Fig. 20.2. Schematic diagram for an air conditioning system.

1. *Static pressure.* The static pressure always exists in a duct system. Since it is not dependent upon the air movement, therefore, it is called static (or stationary) pressure. This type of pressure pushes against the walls of the duct. It tends to burst a duct when it is greater than the atmospheric pressure and tends to collapse the confining envelope when its force is less than that of the atmosphere. The static pressure overcomes the friction and shock losses as the air flows from the delivery of the fan to the outlet of the duct.

2. *Dynamic or velocity pressure.* The dynamic or velocity pressure is equal to the drop in static pressure necessary to produce a given velocity of flow. Conversely, it is equal to the increase of static pressure possible when the velocity is reduced to zero.

3. *Total pressure.* The total pressure is the algebraic sum of the static pressure and dynamic or velocity pressure. Mathematically, total pressure of air,

$$p_T = p_s + p_v$$

where p_s = Static pressure of air, and

p_v = Dynamic or velocity pressure of air.

The total pressure overcomes the pressure losses caused by the various obstacles on the way from the fan to the conditioned space.

Note : The static and total pressure may either be positive or negative. The dynamic or velocity pressure is always positive.

20.7 Continuity Equation for Ducts

Consider the flow of air through a duct between the two sections 1-1 and 2-2, as shown in Fig. 20.3 (*a*).

Let Q_1 = Quantity of air passing through section 1-1,

m_1 = Mass flow rate of air through section 1-1,

A_1 = Cross-sectional area of duct at section 1-1,

V_1 = Velocity of air at section 1-1,

ρ_1 = Density of air at section 1-1,

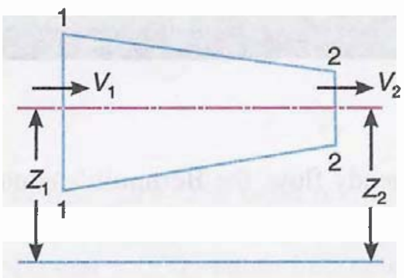
Q_2, m_2, A_2, V_2 and ρ_2 = Corresponding values at section 2-2.

We know that mass flow rate of air through section 1-1,

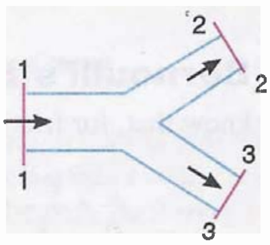
$$m_1 = \rho_1 Q_1 = \rho_1 A_1 V_1 \quad \ldots (\because Q_1 = A_1 V_1) \quad \ldots (i)$$

and mass flow rate of air through section 2-2,

$$m_2 = \rho_2 Q_2 = \rho_2 A_2 V_2 \quad \ldots (\because Q_2 = A_2 V_2) \quad \ldots (ii)$$



(*a*)



(*b*)

Fig. 20.3. Flow of air through a duct.

Since the mass flow rate of air through sections 1-1 and 2-2 is same, therefore equating equations (*i*) and (*ii*), we get

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Normally, the density of air is assumed constant (1.2 kg / m^3) for air conditioning purposes, therefore

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad Q_1 = Q_2$$

This is called the *continuity equation* for flowing air through ducts.

When the duct 1 branches into two ducts 2 and 3 as shown in Fig. 20.3 (*b*), then

$$m_1 = m_2 + m_3 \quad \text{or} \quad \rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

Since the density of air is assumed constant in a duct system, therefore

$$Q_1 = Q_2 + Q_3$$

Coaxial ducts.

20.8 Bernoulli's Equation for Ducts

We know that, for frictionless, incompressible and steady flow, the Bernoulli's equation is

$$\frac{p}{\rho} + \frac{V^2}{2} + g\,Z = \text{constant}$$

or $$p + \frac{\rho V^2}{2} + \rho\, g\, Z = \text{constant}$$

Applying this equation to the two cross-sections 1-1 and 2-2 of a duct,

$$p_{s1} + \frac{\rho_1 (V_1)^2}{2} + \rho_1\, g\, Z_1 = p_{s2} + \frac{\rho_2 (V_2)^2}{2} + \rho_2\, g\, Z_2$$

Since $\rho_1 = \rho_2$ and $Z_1 = Z_2$, therefore the above expression may be written as

$$p_{s1} + \frac{\rho_1 (V_1)^2}{2} = p_{s2} + \frac{\rho_2 (V_2)^2}{2}$$

or $$p_{s1} + p_{v1} = p_{s2} + p_{v2}$$

where p_{s1} and p_{s2} = Static gauge pressure, and

p_{v1} and p_{v2} = Velocity pressure.

From the above expression, we see that when the flow is frictionless and there is no pressure drop between the two sections, then the total pressure at the two sections will be equal. In other words, the total pressure,

$$p_T = p_{s1} + p_{v1} = p_{s2} + p_{v2} \qquad \ldots (i)$$

In actual practice, there is always a pressure drop in a duct due to friction and other causes such as sudden changes in the cross-section and direction. If p_L is the total pressure drop or loss between the two sections 1-1 and 2-2, then equation (*i*) is written as

$$p_{s1} + p_{v1} = p_{s2} + p_{v2} + p_L \quad \text{... (ii)}$$

In case a fan or blower is introduced between the two sections of a duct, then equation (*ii*) may be written as

$$p_{s1} + p_{v1} + p_{TF} = p_{s2} + p_{v2} + p_L \quad \text{... (iii)}$$

where p_{TF} = Rise in pressure due to fan work and is known as fan total pressure..

Notes : **1.** The pressures in the duct are usually expressed in mm of water.

2. The air flowing through the duct is taken as standard air unless it is stated. The standard air is the air which corresponds to 20°C, an atmospheric pressure of 1.013 bar (or 101.3 kN/m²) and a relative humidity of 62 per cent.

3. For the standard air, the mass density (ρ_a) is equal to 1.2 kg / m³ (or 11.72 N/m³).

4. If the velocity of air (V) flowing through the duct is in m/s, then velocity pressure in the duct,

$$p_v = \frac{\rho_a V^2}{2} = \frac{1.2 V^2}{2} = 0.6\ V^2\ \text{N/m}^2$$

$$= \frac{0.6 V^2}{9.81} = \frac{V^2}{16.35} = \left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

$$\text{... } (\because\ 1\ \text{N/m}^2 = \frac{1}{9.81}\ \text{mm of water})$$

When the velocity of air is in m / min, then velocity pressure in the duct,

$$p_v = \left(\frac{V}{60 \times 4.04}\right)^2 = \left(\frac{V}{242.4}\right)^2 \text{ mm of water}$$

Example 20.1. *The main air supply duct of an air conditioning system is 800 mm × 600 mm in cross-section, and carries 300 m³/min of standard air. It branches into two ducts of cross-section 600 mm × 500 mm and 600 mm × 400 mm. If the mean velocity in the larger branch is 480 m / min, find : 1. mean velocity in the main duct and the smaller branch, and 2. mean velocity pressure in each duct.*

Solution. Given : a_1 = 800 mm = 0.8 m ; b_1 = 600 mm = 0.6 m ; Q_1 = 300 m³/min = 5 m³/s ; a_2 = 600 mm = 0.6 m ; b_2 = 500 mm = 0.5 m ; a_3 = 600 mm = 0.6 m ; b_3 = 400 mm = 0.4 m ; V_2 = 480 m / min = 8 m / s

The cross-section of the duct is shown in Fig. 20.4.

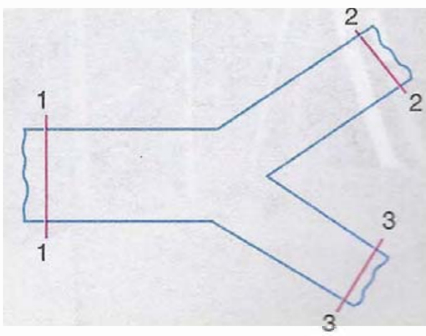


Fig. 20.4

Cross-sectional area of the main duct,

$$A_1 = a_1 b_1 = 0.8 \times 0.6 = 0.48\ \text{m}^2$$

Cross-sectional area of the larger branch,

$$A_2 = a_2 b_2 = 0.6 \times 0.5 = 0.3\ \text{m}^2$$

and cross-sectional area of the smaller branch,

$$A_3 = a_3 b_3 = 0.6 \times 0.4 = 0.24\ \text{m}^2$$

1. *Mean velocity in the main duct and the smaller branch*

We know that the mean velocity in the main duct,

$$V_1 = \frac{Q_1}{A_1} = \frac{5}{0.4} = 10.4\ \text{m/s}\ \textbf{Ans.}$$

Quantity of air passing through the larger branch,

$$Q_2 = A_2 V_2 = 0.3 \times 8 = 2.4 \text{ m}^3/\text{s}$$

∴ Quantity of air passing through the smaller branch,

$$Q_3 = Q_1 - Q_2 = 5 - 2.4 = 2.6 \text{ m}^3/\text{s}$$

We know that mean velocity in the smaller branch,

$$V_3 = \frac{Q_3}{A_3} = \frac{2.6}{0.24} = 10.8 \text{ m/s} \quad \textbf{Ans.}$$

2. *Mean velocity pressure in each duct*

We know that velocity pressure in the main duct,

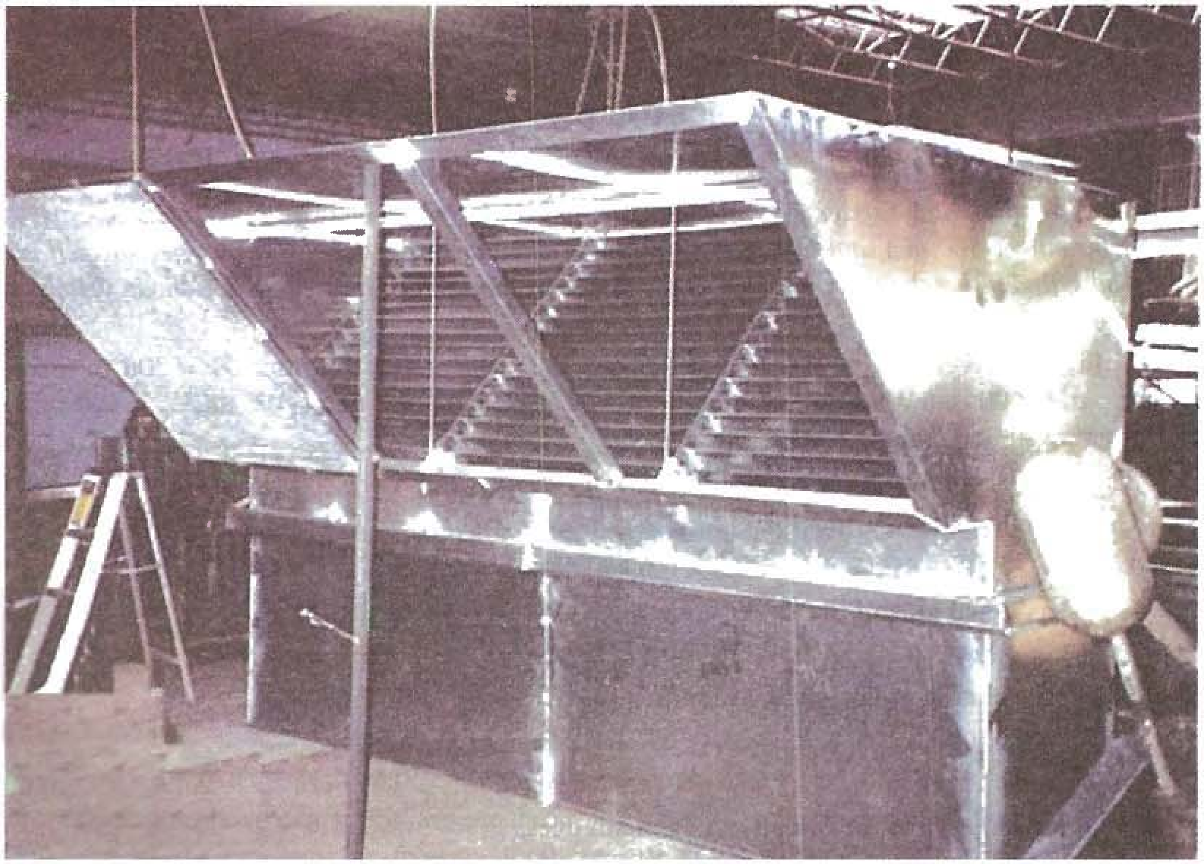
$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{10.4}{4.04}\right)^2 = 6.62 \text{ mm of water} \quad \textbf{Ans.}$$

Mean velocity pressure in the larger branch,

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{8}{4.04}\right)^2 = 3.92 \text{ mm of water} \quad \textbf{Ans.}$$

and mean velocity pressure in the smaller branch,

$$p_{v3} = \left(\frac{V_3}{4.04}\right)^2 = \left(\frac{10.8}{4.04}\right)^2 = 7.14 \text{ mm of water} \quad \textbf{Ans.}$$



Air intake ducts.

20.9 Pressure Losses in Ducts

A little consideration will show that the pressure is lost due to friction between the moving particles of the fluid (*i.e.* air) and the interior surfaces of a duct. When the pressure loss occurs in a straight duct, it is usually termed as *friction loss*. The pressure is also lost dynamically at the changes of direction such as in bends, elbows etc. and at the changes of cross-section of the duct. This type of pressure loss is usually termed as *dynamic loss*.

We shall now discuss these pressure losses, in detail, in the following pages.

20.10 Pressure Loss due to Friction in Ducts

The pressure loss due to friction in ducts may be obtained by using the D'Arcy's formula or the Fanning's equation, *i.e.*,

$$p_f = \frac{f L \rho_a V^2}{2m} \qquad \ldots (i)$$

where p_f = Pressure loss due to friction in N/m^2,

f = Friction factor depending upon the surface of the duct (dimensionless),

L = Length of the duct in metres,

V = Mean velocity of the air flowing through the duct in m/s, and

m = Hydraulic mean depth in metres

$$= \frac{\text{Cross-sectional area of the duct } (A)}{\text{Wetted perimeter of the duct } (P)}$$

In air conditioning, the pressure loss due to friction in ducts is generally expressed in mm of water. From equation (*i*),

$$p_f = \frac{f L}{m}\left(\frac{\rho_a V^2}{2}\right) = \frac{f L}{m} \times p_v \text{ mm of water} \qquad \ldots (ii)$$

We have seen in Art. 20.8 that the velocity pressure in the duct for standard air is given by

$$p_v = \frac{\rho_a V^2}{2} = \left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

Thus equation (*ii*) may be written as

$$p_f = \frac{f L}{m}\left(\frac{V}{4.04}\right)^2 \text{ mm of water} \qquad \ldots (iii)$$

Notes : 1. When air is at a temperature t° C, then pressure loss due to friction in the duct is given by

$$p_f = \frac{f L}{m}\left(\frac{V}{4.04}\right)^2\left(\frac{273+20}{273+t}\right)$$

$$= \frac{f L}{m}\left(\frac{V}{4.04}\right)^2\left(\frac{293}{273+t}\right) \text{ mm of water}$$

2. For a circular duct of diameter D, the hydraulic mean depth,

$$m = \frac{A}{P} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4}$$

3. For a rectangular duct of sides a and b, the hydraulic mean depth,

$$m = \frac{A}{P} = \frac{ab}{2(a+b)}$$

Example 20.2. *A duct of 15 m length passes air at the rate of 90 m³/min. Assuming the friction factor as 0.005, calculate the pressure drop in the duct in mm of water when (a) the duct is circular of diameter 0.3 m ; and (b) the duct is of 0.3 m square cross-section.*

Solution. Given : L = 15 m ; Q = 90 m³/min ; f = 0.005

(a) Pressure drop in a circular duct

Let D = Diameter of the circular duct = 0.3 m ... (Given)

Cross-sectional area of the duct,

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(0.3)^2 = 0.07 \text{ m}^2$$

∴ Velocity of air passing through the duct,

$$V = \frac{Q}{A} = \frac{90}{0.07} = 1285.7 \text{ m/min} = 21.4 \text{ m/s}$$

Wetted perimeter of the duct,

$$P = \pi D = \pi \times 0.3 = 0.94 \text{ m}$$

∴ Hydraulic mean depth of the duct,

$$m = \frac{A}{P} = \frac{0.07}{0.94} = 0.074 \text{ m}$$

We know that pressure drop in a duct,

$$p_f = \frac{f\,L}{m}\left(\frac{V}{4.04}\right)^2 = \frac{0.005 \times 15}{0.074}\left(\frac{21.4}{4.04}\right)^2 \text{ mm of water}$$

$$= 28.47 \text{ mm of water}$$ **Ans.**

(b) Pressure drop in a square duct

Let $a = b$ = Sides of the square duct = 0.3 m ... (Given)

Cross-sectional area of the duct,

$$A = a \times b = 0.3 \times 0.3 = 0.09 \text{ m}^2$$

∴ Velocity of air passing through the duct,

$$V = \frac{Q}{A} = \frac{90}{0.09} = 1000 \text{ m/min} = 16.67 \text{ m/s}$$

Wetted perimeter of the duct,

$$P = 2(a + b) = 2(0.3 + 0.3) = 1.2 \text{ m}$$

∴ Hydraulic mean depth of the duct,

$$m = \frac{A}{P} = \frac{0.09}{1.2} = 0.075$$

We know that pressure drop in a duct,

$$p_f = \frac{f\,L}{m}\left(\frac{V}{4.04}\right)^2 = \frac{0.005 \times 15}{0.075}\left(\frac{16.67}{4.04}\right)^2 \text{ mm of water}$$

$$= 17.02 \text{ mm of water}$$ **Ans.**

20.11 Friction Factor for Ducts

The value of friction factor, for smooth ducts, may be obtained by using the following expressions :

1. For laminar flow, the friction factor,

$$f = \frac{64}{R_N}$$

2. For turbulent flow, the friction factor,

$$f = \frac{0.3164}{(R_N)^{0.25}}$$

where R_N is the Reynold number. It is defined as the ratio of the inertia forces to the viscous forces. It may be noted that the Reynold number is a dimensionless quantity and gives us the information about the type of flow (*i.e.* *laminar or turbulent). Mathematically, Reynold number,

$$R_N = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho_a V^2}{\mu V / D} = \frac{\rho_a DV}{\mu} = \frac{DV}{\mu / \rho_a} = \frac{DV}{K}$$

where
V = Velocity of air in m/s ,
D = Diameter of the duct in metres,
ρ_a = Mass density of air in kg / m³,
μ = Absolute or dynamic viscosity in N-s/m²,
K = Kinematic viscosity in m²/s = μ / ρ_a.

The following table shows the values of absolute viscosity and mass density of air at different temperatures and at atmospheric pressure of 1.013 bar.

Table 20.1. Values of absolute viscosity and density of air at different temperatures.

Temperature in °C	*Absolute or dynamic viscosity (μ) in N-s / m²*	*Density in kg / m³*
0	17.24×10^{-6}	1.29
10	17.71×10^{-6}	1.25
20	18.18×10^{-6}	1.20
30	18.65×10^{-6}	1.16
40	19.12×10^{-6}	1.13
50	19.60×10^{-6}	1.09

In case of rough pipes or ducts, the friction factor (f) depends upon the roughness factor e / D, where e is the absolute roughness of the surface and D is the diameter of the duct. Table 20.2 shows the values of e for different types of commercial pipes or duct work.

The friction factor (f) for rough pipes or ducts may be obtained from the following equation :

$$f = \frac{1}{\left[1.74 - 2\log\left(\frac{2e}{D}\right)\right]^2}$$

The values of friction factor (f) for different Reynold numbers (R_N) and different roughness factors (e/D) may be read directly from the Moody chart as shown in Fig. 20.5.

* When the Reynold number (R_N) is less than 2000, the flow is said to be *laminar flow.* But if the Reynold number is between 2000 and 4000, the flow is neither laminar nor turbulent flow. When the Reynold number exceeds 4000, the flow is said to be *turbulent flow.*

It may be noted that the values given in Fig. 20.5 are independent of the fluid and apply equally well to water, air or other gases when they are evaluated in terms of Reynold number.

Note : When the friction factor 'f' is obtained from the Moody chart as shown in Fig. 20.5, then the pressure loss due to friction in a circular duct of diameter 'D' is given by

$$p_f = \frac{f L}{D} \times p_V = \frac{f L}{D}\left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

Table 20.2. Recommended surface roughness (e) for different material pipes or ducts.

Types of pipe or duct	*Absolute surface roughness (e) in mm*
Smooth-drawn tubes (glass, brass and lead)	0.0015
Commercial steel or wrought iron pipe	0.045
Galvanised iron or steel air ducts	0.15
Cast iron	0.255
Riveted steel (light weight, small rivets)	0.9
Riveted steel (heavy weight, large rivets)	9.0
Smooth concrete	0.3
Average concrete	1.2
Very rough concrete	3.0
Brick conduits	9.0
Wood-stave conduits	0.9

Example 20.3. *A galvanised steel duct of 0.4 m diameter and 20 m long carries air at 20°C and 1.013 bar. If the flow rate of air through the duct is 60 m³/min, determine the pressure loss due to friction in (a) N/m² ; and (b) in mm of water.*

Solution. Given : $D = 0.4$ m ; $L = 20$ m ; $t = 20$°C ; $p = 1.013$ bar ; $Q = 60$ m³/min $= 1$ m³/s

We know that velocity of air in the duct,

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{1 \times 4}{\pi (0.4)^2} = 7.96 \text{ m/s}$$

From Table 20.1, we find that for air at 20°C and atmospheric pressure of 1.013 bar, mass density of air,

$$\rho_a = 1.2 \text{ kg/m}^3$$

and absolute or dynamic viscosity

$$\mu = 18.18 \times 10^{-6} \text{ N-s/m}^2$$

∴ Reynold number,

$$R_N = \frac{\rho_a D V}{\mu} = \frac{1.2 \times 0.4 \times 7.96}{18.18 \times 10^{-6}} = 210\,165$$

Now from Table 20.2, surface roughness for a galvanised steel duct,

$$e = 0.15 \text{ mm} = 0.000\,15 \text{ m}$$

∴ Roughness factor,

$$\frac{e}{D} = \frac{0.000\,15}{0.4} = 0.000\,375$$

From the Moody chart as shown in Fig. 20.5, we find that corresponding to $R_N = 210\,165$ and $e/D = 0.000\,375$, the friction factor,

$$f = 0.016$$

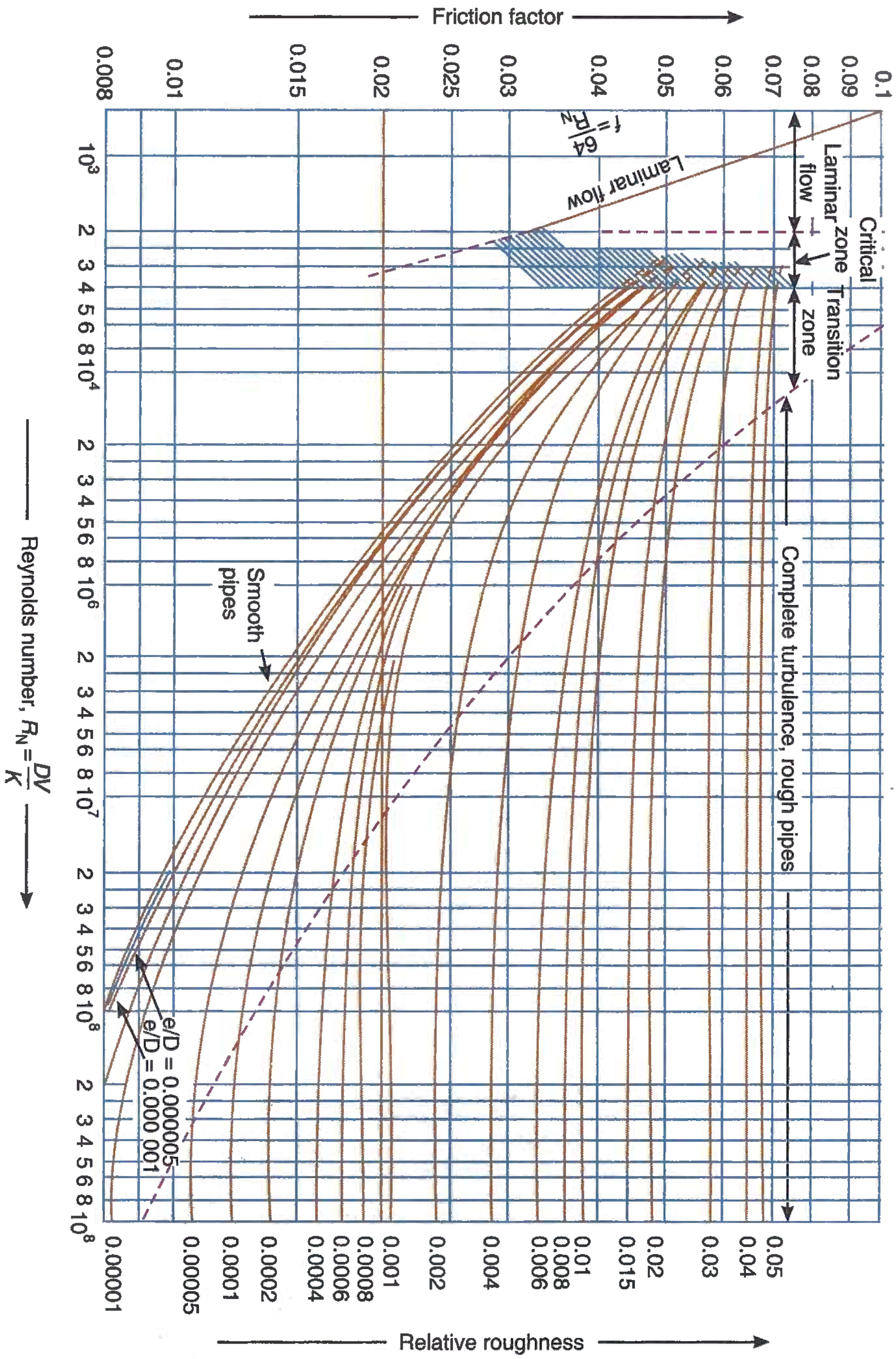


Fig. 20.5. Moody chart showing friction factors for fluid flow in circular pipes or ducts.

(a) Pressure loss due to friction in N/m²

We know that pressure loss due to friction,

$$p_f = \frac{f\,L}{D} \times p_v = \frac{f\,L}{D}\left(\frac{V^2}{2} \times \rho_a\right)$$

$$= \frac{0.016 \times 20}{0.4}\left(\frac{(7.96)^2}{2} \times 1.2\right) = 30.4 \text{ N/m}^2 \textbf{ Ans.}$$

(b) Pressure loss due to friction in mm of water

We know that pressure loss due to friction,

$$p_f = \frac{f\,L}{D} \times p_v = \frac{f\,L}{D}\left(\frac{V}{4.04}\right)^2$$

$$= \frac{0.016 \times 20}{0.4}\left(\frac{7.96}{4.04}\right)^2 = 3.1 \text{ mm of water } \textbf{Ans.}$$

20.12 Equivalent Diameter of a Circular Duct for a Rectangular Duct

In order to find the equivalent diameter of a circular duct for a rectangular duct for the same pressure loss per unit length, we shall consider the following two cases :

1. When the quantity of air passing through the rectangular and circular ducts is same

Let Q = Quantity of air passing through the rectangular and circular ducts,

a = Longer side of the rectangular duct,

b = Shorter side of the rectangular duct,

D = Equivalent diameter of the circular duct,

A_R = Cross-sectional area of the rectangular duct = $a.b$,

P_R = Wetted perimeter of the rectangular duct = $2\,(a + b)$,

A_C = Cross-sectional area of the equivalent circular duct = $\frac{\pi}{4} \times D^2$,

P_C = Wetted perimeter of the equivalent circular duct = πD, and

ρ_a = Mass density of air.

Spiral ducts.

Velocity of air passing through the circular duct,

$$V_C = \frac{Q}{A_C}$$

and velocity of air passing through the rectangular duct,

$$V_R = \frac{Q}{A_R}$$

We know that pressure loss due to friction,

$$p_f = \frac{f L \rho_a V^2}{2m} = \frac{f L \rho_a}{2m}\left(\frac{Q}{A}\right)^2 \qquad \ldots \left(\because V = \frac{Q}{A}\right)$$

and hydraulic mean depth,

$$m = \frac{\text{Cross-sectional area of duct } (A)}{\text{Wetted perimeter of duct } (P)}$$

∴ Pressure loss due to friction for the circular duct,

$$p_{fC} = \frac{f L \rho_a}{2}\left(\frac{P_C}{A_C}\right)\left(\frac{Q}{A_C}\right)^2 = \frac{f L \rho_a Q^2}{2}\left(\frac{P_C}{(A_C)^3}\right) \qquad \ldots (i)$$

and pressure loss due to friction for the rectangular duct,

$$p_{fR} = \frac{f L \rho_a}{2}\left(\frac{P_R}{A_R}\right)\left(\frac{Q}{A_R}\right)^2 = \frac{f L \rho_a Q^2}{2}\left(\frac{P_R}{(A_R)^3}\right) \qquad \ldots (ii)$$

Since the pressure loss, friction factor, length, density and quantity of air for the circular and rectangular ducts is same, therefore from equations (*i*) and (*ii*),

$$\frac{P_C}{(A_C)^3} = \frac{P_R}{(A_R)^3} \quad \text{or} \quad \frac{\pi D}{\left(\frac{\pi}{4} \times D^2\right)^3} = \frac{2(a+b)}{(ab)^3}$$

$$\therefore \quad \frac{32}{\pi^2 D^5} = \frac{a+b}{a^3 b^3} \quad \text{or} \quad D^5 = \frac{32\, a^3 b^3}{\pi^2 (a+b)}$$

or

$$D = \left[\frac{32\, a^3 b^3}{\pi^2 (a+b)}\right]^{1/5} = 1.265\left(\frac{a^3 b^3}{a+b}\right)^{1/5} \qquad \ldots (iii)$$

2. *When the velocity of air passing through the rectangular and circular ducts is same*

Let V = Velocity of air passing through the rectangular and circular ducts.

We know that the pressure loss due to friction for a circular duct,

$$p_{fC} = \frac{f L \rho_a V^2}{2}\left(\frac{P_C}{A_C}\right) \qquad \ldots (iv)$$

and pressure loss due to friction for a rectangular duct,

$$p_{fR} = \frac{f L \rho_a V^2}{2}\left(\frac{P_R}{A_R}\right) \qquad \ldots (v)$$

Since the pressure loss, velocity of air, friction factor, density and length for the circular and rectangular ducts is same, therefore from equations (*iv*) and (*v*),

$$\frac{P_C}{A_C} = \frac{P_R}{A_R} \quad \text{or} \quad \frac{\pi D}{\frac{\pi}{4} \times D^2} = \frac{2(a+b)}{ab}$$

$$\therefore \qquad D = \frac{2ab}{a+b} = \frac{2a}{a/b+1} \qquad \ldots (vi)$$

where a / b is known as *aspect ratio.*

Note : The aspect ratio, for rectangular ducts, should not be greater than 8 in any case.

Example 20.4. *A rectangular duct section of 500 mm × 350 mm size carries 75 m³/min of air having density of 1.15 kg / m³. Determine the equivalent diameter of a circular duct if (a) the quantity of air carried in both the cases is same, and (b) the velocity of air in both the cases is same.*

If f = 0.01 for sheet metal, find the pressure loss per 100 m length of duct.

Solution. Given : a = 500 mm = 0.5 m ; b = 350 mm = 0.35 m ; Q = 75 m³/min ; ρ_a = 1.15 kg / m³ ; f = 0.01 ; L = 100 m

(a) Equivalent diameter of a circular duct if the quantity of air carried in both the cases is same

When the quantity of air carried by rectangular and circular ducts is same, then equivalent diameter of a circular duct,

$$D = 1.265\left(\frac{a^3 b^3}{a+b}\right)^{1/5} = 1.265\left[\frac{(0.5)^3 (0.35)^3}{0.5+0.35}\right]^{1/5}$$

$$= 1.265\left(\frac{0.005\,36}{0.85}\right)^{0.2} = 1.265 \times 0.363 = 0.46 \text{ m} \quad \textbf{Ans.}$$

(b) Equivalent diameter of a circular duct if the velocity of air in both the cases is same

When the velocity of air passing through the rectangular and circular ducts is same, then equivalent diameter of a circular duct,

$$D = \frac{2ab}{a+b} = \frac{2 \times 0.5 \times 0.35}{0.5+0.35} = 0.41 \text{m} \quad \textbf{Ans.}$$

Pressure loss

We know that velocity of air passing through the duct,

$$V = \frac{Q}{A} = \frac{Q}{ab} = \frac{75}{0.5 \times 0.35} = 428.6 \text{ m/min} = 7.143 \text{ m/s}$$

and mean hydraulic depth of the duct,

$$m = \frac{A}{P} = \frac{ab}{2(a+b)} = \frac{0.5 \times 0.35}{2\,(0.5+0.35)} = 0.103 \text{ m}$$

$$\therefore \text{ Pressure loss,} \qquad p_f = \frac{f L \rho_a V^2}{2m} = \frac{0.01 \times 100 \times 1.15\,(7.143)^2}{2 \times 0.103} = 284.8 \text{ N/m}^2$$

$$= 284.8 / 9.81 = 29 \text{ mm of water} \quad \textbf{Ans.}$$

$$\ldots \left(\because\ 1 \text{ N/m}^2 = \frac{1}{9.81} \text{ mm of water}\right)$$

Example 20.5. *A duct 2m by 1m in size carrying conditioned air runs in a straight line for 50 m from the supply fan. It divides into two parts each of 80 m long and 2 m by 1 m in cross-section as shown in Fig. 20.6.*

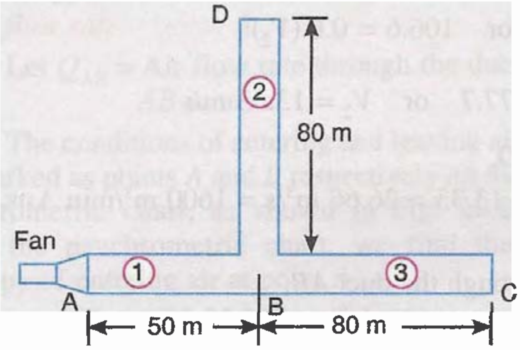


Fig. 20.6

Insulated air ducts.

If the quantity of air discharged at C is 1600 m³/min, calculate the quantity discharged at D and the static pressure at the fan outlet A. Calculate the duct friction loss in N/m² taking the value of friction factor as 0.005.

Solution. Given : $a = 2$ m ; $b = 1$ m ; $L_1 = 50$ m ; $L_2 = 80$ m ; $L_3 = 80$ m ; $Q_3 = 1600$ m³/min ; $f = 0.005$

Quantity of air discharged at D

Let Q_2 = Quantity of air discharged at D.

Cross-sectional area of ducts AB, DB and BC,

$$A_1 = A_2 = A_3 = a \times b = 2 \times 1 = 2 \text{ m}^2$$

Velocity of air in duct BC,

$$V_3 = \frac{Q_3}{A_3} = \frac{1600}{2} = 800 \text{ m/min} = 13.33 \text{ m/s}$$

We know that velocity pressure,

$$p_v = \frac{V^2}{2} \times \rho_a = \frac{V^2}{2} \times 1.2 = 0.6\, V^2 \text{ N/m}^2$$

... (Taking $\rho_a = 1.2$ kg/m³)

∴ Velocity pressure in duct BC,

$$p_{v3} = 0.6\,(V_3)^2 = 0.6\,(13.33)^2 = 106.6 \text{ N/m}^2$$

Since each duct is 2 m by 1 m, therefore hydraulic mean depth for ducts AB, BD, and BC is

$$m_1 = m_2 = m_3 = \frac{ab}{2(a+b)} = \frac{2 \times 1}{2(2+1)} = \frac{1}{3}$$

∴ Pressure loss due to friction in duct BC,

$$p_{f3} = \frac{f L_3}{m_3}(p_{v3}) = \frac{0.005 \times 80}{1/3} \times 106.6 = 128 \text{ N/m}^2$$

and total pressure at B,

$$p_{TB} = p_{f3} + p_{v3} = 128 + 106.6 = 234.6 \text{ N/m}^2 \quad \text{... (i)}$$

Let p_{v2} = Velocity pressure in the duct BD, and

V_2 = Velocity in the duct BD.

We know that the pressure loss due to friction in the duct BD,

$$p_{f2} = \frac{f L_2}{m_2}(p_{v2}) = \frac{0.005 \times 80}{1/3}(p_{v2}) = 1.2\,(p_{v2}) \text{ N/m}^2 \quad \text{... (ii)}$$

From equations (*i*) and (*ii*), we get

$$p_{v2} = \frac{234.6}{1.2} = 106.6 \text{ N/m}^2$$

We know that $p_{v2} = 0.6\,(V_2)^2$ or $106.6 = 0.6\,(V_2)^2$

$$\therefore \quad (V_2)^2 = \frac{106.6}{0.6} = 177.7 \quad \text{or} \quad V_2 = 13.33 \text{ m/s}$$

We know that quantity of air discharged at *D*,

$$Q_2 = A_2 V_2 = 2 \times 13.33 = 26.66 \text{ m}^3/\text{s} = 1600 \text{ m}^3/\text{min}$$ **Ans.**

Static pressure at the fan outlet

We know that the quantity of air flowing through the duct *AB*,

$$Q_1 = Q_2 + Q_3 = 1600 + 1600 = 3200 \text{ m}^3/\text{min}$$

∴ Velocity of air in the duct *AB*,

$$V_1 = \frac{Q_1}{A_1} = \frac{3200}{2} = 1600 \text{ m / min} = 26.67 \text{ m / s}$$

Velocity pressure in the duct *AB*,

$$p_{v1} = 0.6\,(V_1)^2 = 0.6\,(26.67)^2 = 427 \text{ N/m}^2$$

Pressure loss due to friction in the duct *AB*,

$$p_{f1} = \frac{f L_1}{m_1} \times p_{v1} = \frac{0.005 \times 50}{1/3} \times 427 = 320 \text{ N/m}^2$$

Total pressure at the fan outlet (*i.e.* at *A*),

$$p_{TA} = p_{TB} + p_{f1} = 234.6 + 320 = 554.6 \text{ N/m}^2$$

∴ Static pressure at the fan outlet (*i.e.* at A),

$$p_{SA} = p_{TA} - p_{v1} = 554.6 - 427 = 127.6 \text{ N/m}^2$$ **Ans.**

Example 20.6. *Fig. 20.7 shows a duct system containing a dehumidifying coil. The air enters at a dry bulb temperature of 26° C and relative humidity of 60% and leaves at a dry bulb temperature of 14°C and relative humidity of 90%. The chilled water enters the cooling coil at the rate of 15 kg / s and its temperature rise is 4.03°C. The velocity in the main duct is 10 m/s. The aspect ratio for each duct is 2 : 1. The main duct line is A-B-C with pressure drop per unit length being same and there is no damper in the system. The pressure drop due to 90° bend is equivalent to 3 times the diameter of the duct at the point considered.*

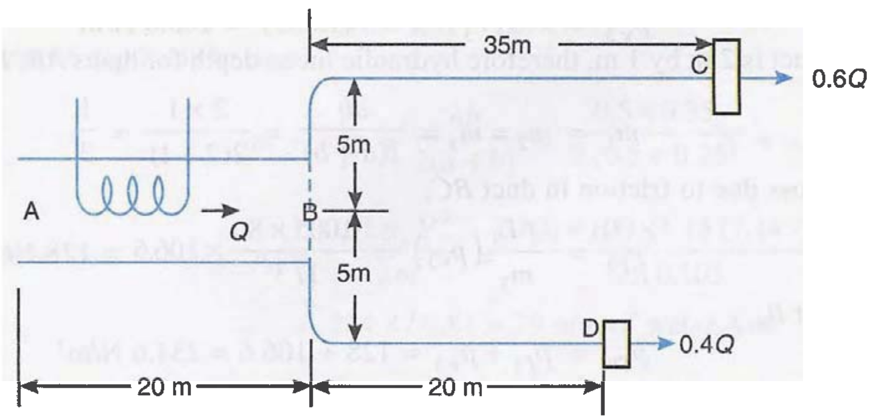


Fig. 20.7

Determine : 1. Air flow rate (m³/s) ; 2. Main duct dimensions ; 3. Size of duct BC ; 4. Total pressure ; and 5. Dimensions of duct BD.

Solution. Given : t_{dA} = 26°C ; ϕ_A = 60% ; t_{dB} = 14° C ; ϕ_B = 90% ; m_w = 15 kg / s ; Δt_w = 4.03°C ; V_{AB} = 10 m / s ; Aspect ratio = a / b = 2 : 1; $p_b = 3D_d$; L_{AB} = 20 m ; L_{BC} = 5 + 35 = 40 m ; L_{BD} = 5 + 20 = 25 m

1. Air flow rate

Let Q_{AB} = Air flow rate through the duct *AB* in m³/s.

The conditions of entering and leaving air are marked as points *A* and *B* respectively on the psychrometric chart, as shown in Fig. 20.8. From the psychrometric chart, we find that enthalpy of entering air at point *A*,

$$h_A = 58.4 \text{ kJ / kg of dry air}$$

Enthalpy of leaving air at point *B*,

$$h_B = 36.5 \text{ kJ / kg of dry air}$$

Specific volume of leaving air at point 2,

$$v_{sB} = 0.825 \text{ m}^3 \text{/ kg of dry air}$$

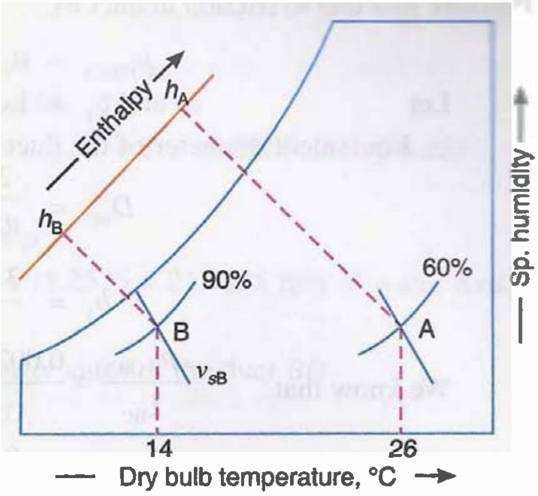


Fig. 20.8

Let *m* be the mass rate of air (in kg / s) flowing through the duct *AB*, then the decrease in enthalpy of air is equal to the heat carried away by chilled water.

or $$m(h_A - h_B) = m_w \times c_w \times \Delta t_w$$

$$m(58.4 - 36.5) = 15 \times 4.187 \times 4.03 = 253$$

... (Taking specific heat of water, c_w = 4.187 kJ / kg K)

and air flow rate, $$Q_{AB} = m / v_{sB} = 11.55 / 0.825 = 14 \text{ m}^3\text{/s} \quad \textbf{Ans.}$$

2. Main duct dimensions

Let a and b = Longer and shorter side of the main duct *AB*.

We know that cross-sectional area of the main duct *AB*,

$$A_{AB} = a \times b = 2b \times b = 2b^2 \quad ...(\because a/b = 2)$$

and quantity of air flowing through the main duct *AB*,

$$Q_{AB} = A_{AB} \times V_{AB} = 2b^2 \times 10 = 20\,b^2$$

$\therefore$ $$b^2 = Q_{AB} / 20 = 14 / 20 = 0.7 \quad \text{or} \quad b = 0.836 \text{ m} \quad \textbf{Ans.}$$

and $$a = 2b = 2 \times 0.836 = 1.672 \text{ m} \quad \textbf{Ans.}$$

3. Size of duct BC

We know that equivalent diameter of the duct *AB*,

$$D_{AB} = \frac{2ab}{a+b} = \frac{2 \times 1.672 \times 0.836}{1.672 + 0.836} = 1.115$$

and pressure loss due to friction in duct *AB* per metre length,

$$\frac{p_{f(AB)}}{L_{AB}} = \frac{0.002\ 268\,(Q_{AB})^{1.852}}{(D_{AB})^{4.973}} = \frac{0.002\ 268\,(14)^{1.852}}{(1.115)^{4.973}}$$

$$= \frac{0.3}{1.718} = 0.1746 \text{ mm of water / m length}$$

This is constant for all sections, *i.e.*

$$\frac{p_{f(AB)}}{L_{AB}} = \frac{p_{f(BC)}}{L_{BC}} = \frac{p_{f(BD)}}{L_{BD}} = 0.1746 \text{ mm of water / m length}$$

∴ Pressure loss due to friction in duct *AB*, ...(*i*)

$$p_{f(AB)} = 0.1746 \times L_{AB} = 0.1746 \times 20 = 3.492 \text{ mm of water}$$

Pressure loss due to friction in duct *BC*,

$$p_{f(BC)} = 0.1746 \times L_{BC} = 0.1746 \times 40 = 6.984 \text{ mm of water}$$

Let a_1 and b_1 = Longer and shorter side of the duct *BC*.

∴ Equivalent diameter of the duct *BC*,

$$D_{BC} = \frac{2a_1 b_1}{a_1 + b_1} = \frac{2 \times 2b_1 \times b_1}{2b_1 + b_1} = \frac{4b_1}{3} \quad \ldots (\because a_1 = 2b_1)$$

or

$$b_1 = \frac{3D_{BC}}{4} = 0.75\, D_{BC} \quad \ldots (ii)$$

We know that

$$\frac{p_{f(BC)}}{L_{BC}} = \frac{0.002\ 268\ (Q_{BC})^{1.852}}{(D_{BC})^{4.973}}$$

$$0.1746 = \frac{0.002\ 268\ (0.6 \times 14)^{1.852}}{(D_{BC})^{4.973}} = \frac{0.1168}{(D_{BC})^{4.973}}$$

...($\because Q_{BC} = 0.6\, Q_{AB}$)

or

$$D_{BC} = \left(\frac{0.1168}{0.1746}\right)^{1/4.973} = (0.669)^{0.2011} = 0.922 \text{ m}$$

We know that $b_1 = 0.75\, D_{BC} = 0.75 \times 0.922 = 0.6915$ m **Ans.**

...[From equation (*ii*)]

and $a_1 = 2b_1 = 2 \times 0.6915 = 1.383$ m **Ans.**

4. *Total pressure*

We know that velocity of air flowing through the duct *AB*,

$$V_{AB} = \frac{Q_{AB}}{A_{AB}} = \frac{Q_{AB}}{a \times b} = \frac{14}{1.672 \times 0.836} = 10 \text{ m/s}$$

and velocity of air flowing through the duct *BC*,

$$V_{BC} = \frac{Q_{BC}}{A_{BC}} = \frac{0.6\, Q_{AB}}{a_1 \times b_1} = \frac{0.6 \times 14}{1.383 \times 0.6915} = 8.783 \text{ m/s}$$

∴ Velocity pressure in duct *AB*,

$$p_{v(AB)} = \left(\frac{V_{AB}}{4.04}\right)^2 = \left(\frac{10}{4.04}\right)^2 = 6.127 \text{ mm of water}$$

and velocity pressure in duct *BC*,

$$p_{v(BC)} = \left(\frac{V_{BC}}{4.04}\right)^2 = \left(\frac{8.783}{4.04}\right)^2 = 4.726 \text{ mm of water}$$

The dynamic losses between *A* and *C* are as follows :

(*a*) Pressure loss at discharge opening at *C*

$= p_{v(BC)} = 4.726$ mm of water

(*b*) Elbow loss at *B* $= 0.25\, p_{v(BC)}$

$= 0.25 \times 4.726 = 1.1815$ mm of water

(*c*) Fitting loss at *B* $= 0.25\, [\, p_{v(AB)} - p_{v(BC)}\,]$

$= 0.25\, [6.127 - 4.726] = 0.35$ mm of water (*d*) Presure loss in dehumidifier coil

$= $ Usually 2 to 10 mm of water. Let us take it as 5 mm of water

$\therefore$ Total dynamic loss between *A* and *C*,

$$p_d = (a) + (b) + (c) + (d)$$
$$= 4.726 + 1.1815 + 0.35 + 5 = 11.2575 \text{ mm of water}$$

and total pressure (at fan exit)

$$= p_{f(AB)} + p_{f(AB)} + p_d$$
$$= 3.492 + 6.984 + 11.2575 = 21.7335 \text{ mm of water } \textbf{Ans.}$$

5. *Dimensions of duct BD*

Let a_2 and b_2 = Longer and shorter sides of the duct *BD*.

We know that equivalent diameter of the duct *BD*,

$$D_{BD} = \frac{2a_2b_2}{a_2+b_2} = \frac{2\times 2b_2\times b_2}{2b_2+b_2} = \frac{4b_2^2}{3b_2} = \frac{4b_2}{3} \qquad ...(\because a_2 = 2b_2)$$

or $b_2 = 0.75\, D_{BD}$

We know that

$$\frac{p_{f(BD)}}{L_{BD}} = \frac{0.002\,268\,(Q_{BD})^{1.852}}{(D_{BD})^{4.973}}$$

$$0.1746 = \frac{0.002\,268\,(0.4\times 14)^{1.852}}{(D_{BD})^{4.973}} = \frac{0.055}{(D_{BD})^{4.973}}$$

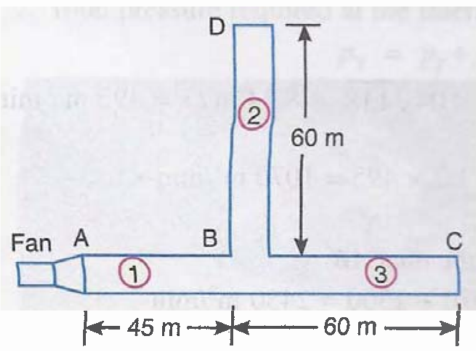
$$... (\because Q_{BD} = 0.4\, Q_{AB})$$

$$\therefore \quad D_{BD} = \left(\frac{0.055}{0.1746}\right)^{1/4.973} = (0.3157)^{0.2011} = 0.793 \text{ m}$$

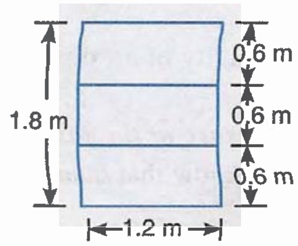
We know that $b_2 = 0.75\, D_{BD} = 0.75 \times 0.793 = 0.595$ m **Ans.**

and $a_2 = 2b_2 = 2 \times 0.595 = 1.19$ m **Ans.**

Example 20.7. *A ventilation duct 1.8 m by 1.2 m runs in a straight line for 45 m from the supply fan, then bifurcates into parts each 60 m long and each 1.8 m by 1.2 m, as shown in Fig. 20.9 (a). In order to reduce the air noise to a minimum, the branch BD is divided into three parts and lined with absorbent material of negligible thickness as shown in Fig. 20.9 (b). If the quantity of air discharged at C is 1360 m³/min, calculate the quantity of air discharged at D and the static pressure at the fan outlet. The friction factor 'f' is 0.0055 for sheet metal and 0.007 for the absorbent material.*



(*a*)



(*b*)

Fig. 20.9

Solution. Given : a = 1.8 m ; b = 1.2 m ; L_1 = 45 m ; $L_2 = L_3$ = 60 m ; Q_3 = 1360 m³/min ; $f_1 = f_3$ = 0.0055 ; f_2 = 0.007

Quantity of air discharged at D

We know that cross-sectional area of the duct *AB* or *BC*,

$$A_1 = A_3 = a \times b = 1.8 \times 1.2 = 2.16 \text{ m}^2$$

∴ Velocity of air in the duct *BC*,

$$V_3 = \frac{Q_3}{A_3} = \frac{1360}{2.16} = 630 \text{ m/min} = 10.5 \text{ m/s}$$

and velocity pressure in the duct *BC*,

$$p_{v3} = \left(\frac{V_3}{4.04}\right)^2 = \left(\frac{10.5}{4.04}\right)^2 = 6.76 \text{ mm of water}$$

Hydraulic mean depth of the duct *AB* or *BC*,

$$m_1 = m_3 = \frac{ab}{2(a+b)} = \frac{1.8 \times 1.2}{2\,(1.8+1.2)} = 0.36 \text{ m}$$

Pressure loss due to friction in the duct *BC*,

$$p_{f3} = \frac{f_3 L_3}{m_3} \times p_{v3} = \frac{0.0055 \times 60}{0.36} \times 6.76 = 6.2 \text{ mm of water}$$

∴ Total pressure at *B*,

$$p_{TB} = p_{f3} + p_{v3} = 6.2 + 6.76 = 12.96 \text{ mm of water} \quad \ldots (i)$$

Let p_{v2} = Velocity pressure in the duct *BD*, and

V_2 = Velocity in the duct *BD*.

Since the duct *BD* is divided into three parts, each being 1.2 m by 0.6 m, therefore hydraulic mean depth of the duct *BD*,

$$m_2 = \frac{3 \times 1.2 \times 0.6}{3 \times 2\,(1.2+0.6)} = 0.2 \text{ m}$$

∴ Pressure loss due to friction in the duct *BD*,

$$p_{f2} = \frac{f_2 L_2}{m_2} \times p_{v2} = \frac{0.007 \times 60}{0.2} \times p_{v2} = 2.1\, p_{v2} \text{ mm of water}$$

and total pressure at *B*, $p_{TB} = p_{f2} + p_{v2} = 2.1\, p_{v2} + p_{v2} = 3.1\, p_{v2}$ mm of water ... (*ii*)

From equations (*i*) and (*ii*), we get

$$p_{v2} = \frac{12.96}{3.1} = 4.18 \text{ mm of water}$$

We know that $p_{v2} = \left(\dfrac{V_2}{4.04}\right)^2$

$$\therefore \quad V_2 = 4.04\sqrt{p_{v2}} = 4.04\sqrt{4.18} = 8.24 \text{ m/s} = 495 \text{ m/min}$$

Quantity of air discharged at *D*,

$$Q_2 = A_2 V_2 = 1.8 \times 1.2 \times 495 = 1070 \text{ m}^3\text{/min}$$ **Ans.**

Static pressure at the fan outlet

We know that quantity of air flowing through the duct *AB*,

$$Q_1 = Q_2 + Q_3 = 1070 + 1360 = 2430 \text{ m}^3\text{/min}$$

∴ Velocity of air in the duct *AB*,

$$V_1 = \frac{Q_1}{A_1} = \frac{2430}{2.16} = 1125 \text{ m/min} = 18.75 \text{ m/s}$$

Velocity pressure in the duct *AB*,

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{18.75}{4.04}\right)^2 = 21.5 \text{ mm of water}$$

Pressure loss due to friction in the duct *AB*,

$$p_{f1} = \frac{f_1 L_1}{m_1} \times p_{v1} = \frac{0.0055 \times 45}{0.36} \times 21.5 = 14.8 \text{ mm of water}$$

Total pressure at the fan outlet (*i.e.* at *A*),

$$p_{TA} = p_{TB} + p_{f1} = 12.96 + 14.8 = 27.76 \text{ mm of water}$$

∴ Static pressure at the fan outlet (*i.e.* at *A*),

$$p_{SA} = p_{TA} - p_{vA} = 27.76 - 21.15 = 6.26 \text{ mm of water } \textbf{Ans.}$$

... ($\because p_{vA} = p_{v1}$)

Example 20.8. *A rectangular duct 0.15 m by 0.12 m is 20 m long and carries standard air at the rate of 0.3 m³/s. Calculate the total pressure required at the inlet to the duct in order to maintain this flow and the air power. Assume that for the duct, the friction factor f = 0.005.*

Solution. Given : $a = 0.15$ m ; $b = 0.12$ m ; $L = 20$ m ; $Q = 0.3$ m³/s ; $f = 0.005$

Total pressure required at the inlet to the duct

Let p_T = Total pressure required at the inlet to the duct.

We know that cross-sectional area of the duct,

$$A = a \times b = 0.15 \times 0.12 = 0.018 \text{ m}^2$$

∴ Velocity of air in the duct,

$$V = \frac{Q}{A} = \frac{0.3}{0.018} = 16.67 \text{ m/s}$$

and velocity pressure,

$$p_v = \frac{\rho_a V^2}{2} = \frac{1.2 V^2}{2} = 0.6\, V^2 = 0.6\,(16.67)^2 = 166.7 \text{ N/m}^2$$

... ($\because \rho_a$ for standard air = 1.2 kg / m³)

Wetted perimeter of the duct,

$$P = 2(a + b) = 2(0.15 + 0.12) = 0.54 \text{ m}$$

Hydraulic mean depth of the duct,

$$m = \frac{A}{P} = \frac{0.018}{0.54} = 0.033 \text{ m}$$

We know that pressure loss due to friction in the duct,

$$p_f = \frac{f\,L}{m} \times p_v = \frac{0.005 \times 20}{0.033} \times 166.7 = 505.15 \text{ N/m}^2$$

∴ Total pressure required at the inlet to the duct,

$$p_T = p_f + p_v = 505.15 + 166.7 = 671.85 \text{ N/m}^2 \textbf{ Ans.}$$

Fastening duct using a portable electric drill and sheet metal screws.

Taped duct connection.

Air power

We know that the air power

$$= Q \times p_T = 0.3 \times 671.85 = 201.6 \text{ N-m/s} = 201.6 \text{ W} \quad \textbf{Ans.}$$

... ($\because$ 1 N–m/s = 1 W)

20.13 Friction Chart for Circular Ducts

We have already discussed in Art. 20.10, that the frictional pressure loss in ducts according to D' Arcy's formula or Fanning's equation is given by

$$p_f = \frac{f L \rho_a V^2}{2\, m}$$

and hydraulic mean depth for circular ducts,

$$m = \frac{A}{P} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4}$$

∴ Frictional pressure loss for circular ducts,

$$p_f = \frac{f L \rho_a V^2}{2}\left(\frac{4}{D}\right) = \frac{4 f L}{D}\left(\frac{\rho_a V^2}{2}\right)$$

$$= \frac{4 f L}{D} \times p_V \text{ (in N/m}^2\text{)} \qquad \ldots (i)$$

According to Fritzsche, the frictional pressure loss (p_f) in circular ducts may be obtained from the following relation :

$$p_f = \frac{0.014\,22 L\, (V)^{1.852}}{(D)^{1.269}} \text{ (in N/m}^2\text{)} \qquad \ldots (ii)$$

where V is in m/s and L and D are in metres.

We know that mean velocity of air flowing through the duct,

$$V = \frac{\text{Volume flow rate}}{\text{Cross-sectional area}} = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

Substituting this value of V in equation (*ii*), we have

$$p_f = \frac{0.222\,43 L\, (Q)^{1.852}}{(D)^{4.973}} \text{ N/m}^2 \qquad \ldots (iii)$$

$$= \frac{0.002\,268 L\, (Q)^{1.852}}{(D)^{4.973}} \text{ mm of water} \qquad \ldots (iv)$$

$$= \frac{0.012\,199 L\, (V)^{2.4865}}{(Q)^{0.6343}} \text{ N/m}^2 \qquad \ldots \text{(in terms of } V \text{ and } Q) \qquad \ldots (v)$$

The different equations as obtained above from the Fritzsche formula give quite accurate results.

The frictional pressure loss for circular ducts (in mm of water) for various velocities (in m/s) and duct diameters (in mm) may be obtained directly from the friction chart as shown in Fig. 20.10. In these charts, the vertical ordinates represent volume flow rate of air (Q) in m^3/s and the horizontal ordinates represent frictional pressure loss in mm of water per unit length of the circular duct (*i.e.* p_f / L). These charts are valid for 20°C and 1.013 bar and clean galvanised iron ducts with joints and seams having good commercial practice.

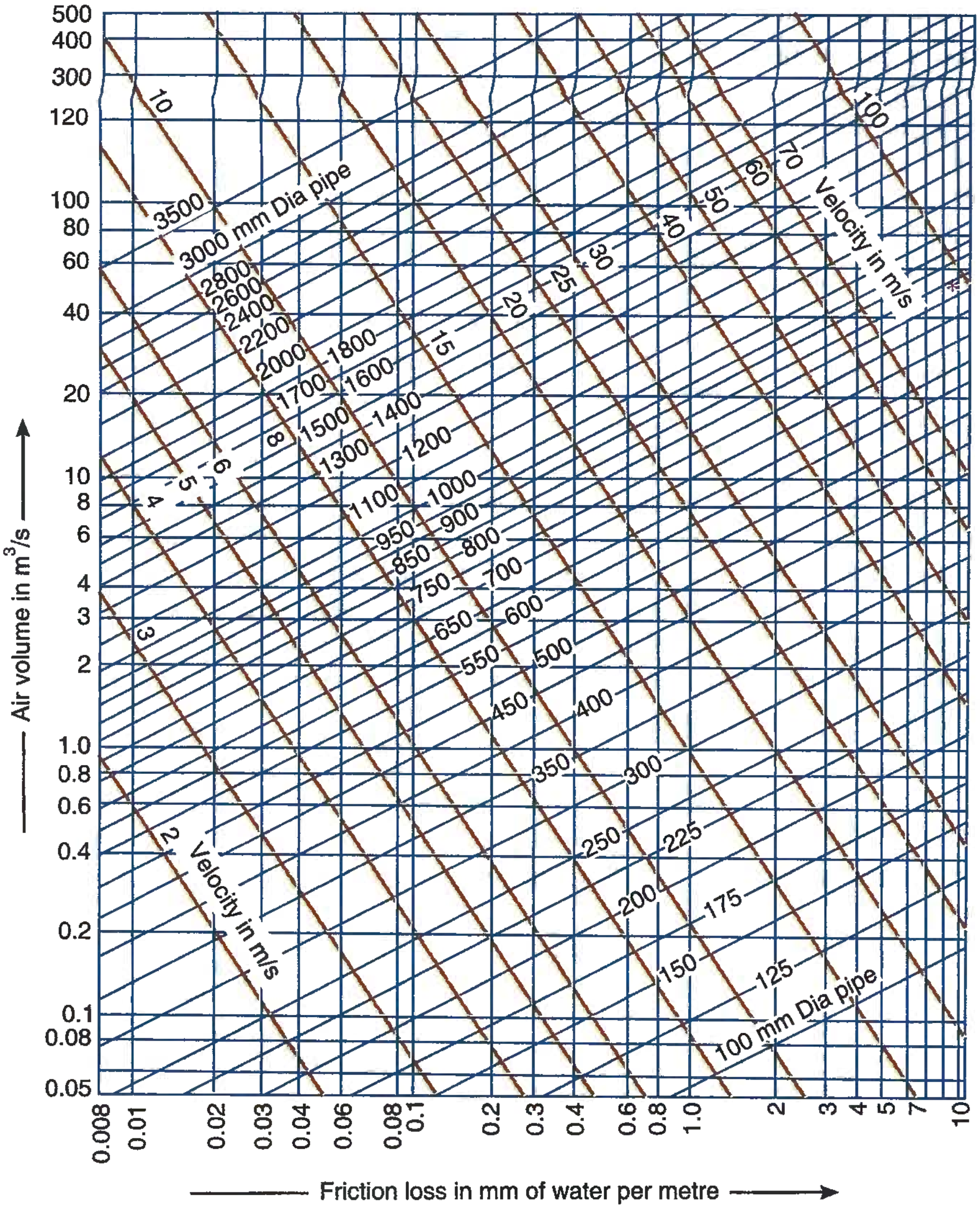


Fig. 20.10. Friction chart for circular ducts.

Notes : 1. If the duct is made of other material, such as plastics, concrete, wood, fibreglass etc., then correction factor should be applied. For small differences in density of air, the correction shall be made according to

$$p_f \propto \rho_a$$

2. If the air is at other temperatures, then the correction shall be made according to

$$p_f \propto \frac{1}{(T)^{0.857}}$$

3. The friction chart, as shown in Fig. 20.10, may be used to determine the pressure loss in rectangular ducts if the equivalent diameter of a circular duct for the rectangular duct is obtained first as discussed in Art. 20.12.

Example 20.9. *A duct of 650 mm diameter and 15 m long carries 144 m³/mm of air at 20°C. Find the pressure loss in the duct by using the friction chart.*

Solution. Given : D = 650 mm ; L = 15 m ; Q = 144 m³/min = 2.4 m³/s

Draw a horizontal line corresponding to 2.4 m³/s intersecting the diagonal line for 650 mm duct diameter at point A, as shown in Fig. 20.11. The pressure loss as read from the bottom of the chart is 0.1 mm of water per metre length of the duct.

Since the length of the duct is 15 m, therefore pressure loss in the duct,

$$= 0.1 \times 15 = 1.5 \text{ mm of water } \textbf{Ans.}$$

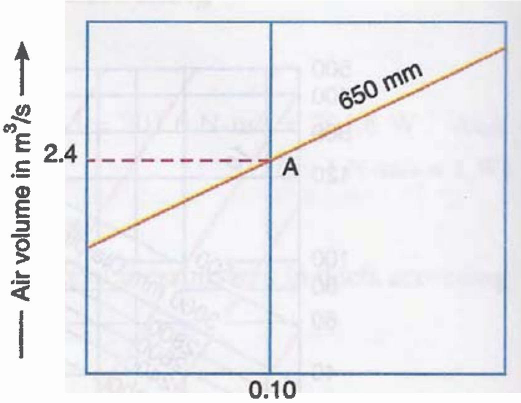


Fig. 20.11

20.14 Dynamic Losses in Ducts

The dynamic losses are caused due to the change in direction or magnitude of velocity of the fluid in the duct. The change in the direction of velocity occurs at bends and elbows. The change in the magnitude of velocity occurs when the area of duct changes. The change in velocity magnitude or direction can be caused only by the accelerating or decelerating forces which may be internal or external. The loss of pressure is due to the loss of the energy of the fluid in overcoming such dynamic forces resisting the changes. The pressure loss due to the change of direction of velocity at elbow is expressed either in terms of velocity pressure head or equivalent additional length which will give frictional loss equal to that caused by the elbow. Thus dynamic pressure loss,

$$p_d = C\,p_v = C\left(\frac{V}{4.04}\right)^2 \text{ in mm of water} \quad \ldots (i)$$

where V is the velocity in m / s and C is the *dynamic loss coefficient* found experimentally.

The dynamic pressure loss expressed in terms of an additional equivalent length (L_e) of the duct is given by

$$p_d = \frac{f\,L_e}{m} \times p_v = \frac{f\,L_e}{m}\left(\frac{V}{4.04}\right)^2 \text{ in mm of water} \quad \ldots (ii)$$

From equations (*i*) and (*ii*), we find that the relationship between the dynamic loss coefficient (C) and equivalent additional length (L_e) is

$$C = \frac{f\,L_e}{m} \quad \text{or} \quad L_e = \frac{C\,m}{f}$$

It is more convenient to use the method of equivalent additional length. It only needs this additional length to be added to the actual total length and the total pressure loss can then be found as frictional loss only. In considering the actual total length, the length of bends and elbows should also be taken in addition to the straight lengths of the ducts, as the equivalent additional length accounts for only the dynamic losses and not the actual frictional losses in length of bends or elbows.

20.15 Pressure Loss due to Enlargement in Area and Static Regain

When the area of a duct changes, the velocity of air flowing through the duct changes. A little consideration will show that when the area increases, the velocity decreases with a rise in pressure and the conversion of velocity head (or velocity pressure) into pressure head (or static pressure) takes place. The increase in static pressure as a result of the conversion from velocity pressure is termed as *static regain.*

Consider a duct *A B C* through which air is flowing and having a sudden or abrupt enlargement at *B*. As a result of this enlargement eddies will be formed in the corner of the duct at *B* as shown in Fig. 20.12. The loss of pressure takes place due to the eddies formed at the suddenly enlarged section.

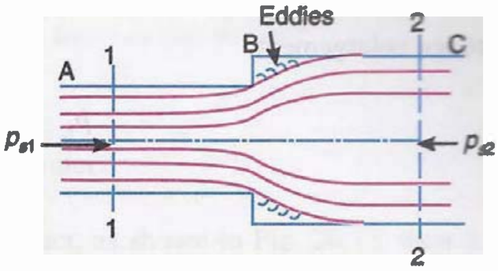


Fig. 20.12. Sudden enlargement.

Let p_{s1} = Static pressure of air at section 1-1,

A_1 = Cross-sectional area of the duct at section 1-1,

V_1 = Velocity of air at section 1-1,

p_{s2}, A_2, V_2 = Corresponding values at section 2-2, and

p_e = Pressure loss due to sudden enlargement.

Applying Bernoulli's equation to sections 1-1 and 2-2, we have

$$p_{s1} + p_{v1} = p_{s2} + p_{v2} + p_L$$

$$p_{s1} + \frac{\rho_a (V_1)^2}{2} = p_{s2} + \frac{\rho_a (V_2)^2}{2} + p_L$$

$$\therefore \quad p_L = [p_{s1} - p_{s2}] + \left[\frac{\rho_a (V_1)^2}{2} - \frac{\rho_a (V_2)^2}{2}\right] \quad \ldots (i)$$

We know that momentum of air per second at section 1-1

$$= \text{Mass} \times \text{Velocity} = (\text{Volume} \times \text{Density}) \times \text{Velocity}$$

$$= (A_1 V_1 \rho_a) V_1 = \rho_a A_1 (V_1)^2 \quad \ldots (\because \text{Volume} = \text{Area} \times \text{velocity})$$

Similarly, momentum of air per second at sections 2-2,

$$= \rho_a A_2 (V_2)^2$$

$\therefore$ Change of momentum per second

$$= \rho_a A_1 (V_1)^2 - \rho_a A_2 (V_2)^2 \quad \ldots (ii)$$

Since the flow between sections 1-1 and 2-2 is continuous, therefore

$$A_1 V_1 = A_2 V_2 \quad \text{or} \quad A_1 = \frac{A_2 V_2}{V_1}$$

Substituting this value of A_1 in equation (*ii*), we have change of momentum per second,

$$= \frac{\rho_a A_2 V_2 (V_1)^2}{V_1} - \rho_a A_2 (V_2)^2$$

$$= \rho_a A_2 V_2 V_1 - \rho_a A_2 (V_2)^2 \quad \ldots (iii)$$

The force responsible for this change of momentum

$$= (p_{s2} - p_{s1}) A_2 \quad \ldots (iv)$$

Equating equations (*iii*) and (*iv*), we have

$$(p_{s2} - p_{s1}) A_2 = \rho_a A_2 V_2 V_1 - \rho_a A_2 (V_2)^2$$

or $$p_{s2} - p_{s1} = \rho_a [V_1 V_2 - (V_2)^2]$$

and $$p_{s1} - p_{s2} = \rho_a [(V_2)^2 - V_1 V_2]$$

Substituting this value of $(p_{s1} - p_{s2})$ in equation (*i*), we find that pressure loss due to sudden enlargement,

$$p_L = \rho_a\left[(V_2)^2 - V_1V_2\right] + \left[\frac{\rho_a(V_1)^2}{2} - \frac{\rho_a(V_2)^2}{2}\right]$$

$$= \rho_a\left[\frac{2(V_2)^2 - 2V_1V_2 + (V_1)^2 - (V_2)^2}{2}\right]$$

$$= \rho_a\left[\frac{(V_1)^2 + (V_2)^2 - 2V_1V_2}{2}\right] = \rho_a\left[\frac{(V_1 - V_2)^2}{2}\right]^{*} \text{ N/m}^2$$

$$= 1.2\left[\frac{(V_1 - V_2)^2}{2}\right] = 0.6\,(V_1 - V_2)^2 \text{ N/m}^2$$

... (Taking $\rho_a = 1.2$ kg / m³)

$$= \frac{0.6}{9.81}(V_1 - V_2)^2 = \left(\frac{V_1 - V_2}{4.04}\right)^2 \text{ mm of water} \quad \ldots (v)$$

... ($\because$ 1 N/m² = $\frac{1}{9.81}$ mm of water)

We know that $A_1V_1 = A_2V_2$

$\therefore$ $V_2 = \frac{A_1V_1}{A_2}$ or $V_1 = \frac{A_2V_2}{A_1}$

Substituting the value of V_2 in equation (*v*), we have

$$p_L = \left[\frac{V_1 - \dfrac{A_1V_1}{A_2}}{4.04}\right]^2 = \left(1 - \frac{A_1}{A_2}\right)^2\left(\frac{V_1}{4.04}\right)^2$$

$$= C_1\left(\frac{V_1}{4.04}\right)^2 = C_1\,p_{v1} \quad \ldots (vi)$$

where $C_1 = \left(1 - \frac{A_1}{A_2}\right)^2$, known as loss coefficient, and

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 \text{ in mm of water}$$

Now substituting the value of V_1 in equation (v),

$$p_L = \left[\frac{\left(\dfrac{A_2V_2}{A_1} - V_2\right)}{4.04}\right]^2 = \left(\frac{A_2}{A_1} - 1\right)^2\left(\frac{V}{4.04}\right)^2$$

$$= C_2\left(\frac{V_2}{4.04}\right)^2 = C_2\,p_{v2} \quad \ldots (vii)$$

* In case the pressure loss due to sudden enlargement (p_L) is required in metres of air, then

$$p_L = \frac{(V_1 - V_2)^2}{2g} \text{ m of air}$$

where $$C_2 = \left(\frac{A_2}{A_1} - 1\right)^2$$, known as loss coefficient, and

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2 \text{ in mm of water}$$

When there is a gradual enlargement in area of the duct, as shown in Fig. 20.13, then the pressure loss due to gradual enlargement is given by

$$p_L = C_r C_1 \left(\frac{V_1}{4.04}\right)^2 = C_r C_2 \left(\frac{V_2}{4.04}\right)^2$$

where C_r is the loss coefficient giving the ratio of the actual loss to the loss for sudden enlargement. The following table shows the values of C_r as a function of included angle θ of the sides.

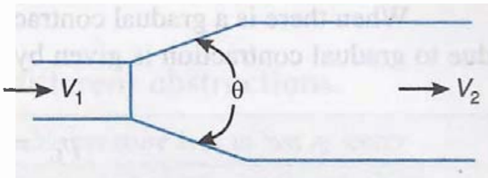


Fig. 20.13. Gradual enlargement.

Table 20.3. Values of loss coefficient.

Condition (θ°)	5	7	10	20	30	40
Loss coefficient (C_r)	0.17	0.22	0.28	0.45	0.59	0.73

Notes : 1. When the enlargement is not accompanied with pressure loss, then there will be full conversion of the velocity pressure into static pressure. In such a case, static pressure regain.

$$SPR = p_{s2} - p_{s1} = p_{v1} - p_{v2}$$

2. When the enlargement is accompanied with pressure loss, the increase in static pressure or static regain is reduced by the amount of the pressure loss. In such a case, static pressure regain,

$$SPR = p_{s2} - p_{s1} = (p_{v1} - p_{v2}) - p_L = R\,(p_{v1} - p_{v2})$$

where R is the static regain factor.

20.16 Pressure Loss due to Contraction in Area

Consider a duct *ABC* through which air is flowing and having a sudden or abrupt contraction at *B*, as shown in Fig. 20.14. It may be noted that when air is flowing through a duct of such a section, the eddies are formed at two places, *i.e.* at the shoulders of the large section and beyond the entry at the smaller section forming a vena contracta at section 1-1. Strictly speaking, the loss of pressure due to sudden contraction is not due to contraction itself, but it is due to the sudden enlargement of flow area from vena contracta (*i.e.* section 1-1) to the section of the smaller duct (*i.e.* section 2-2).

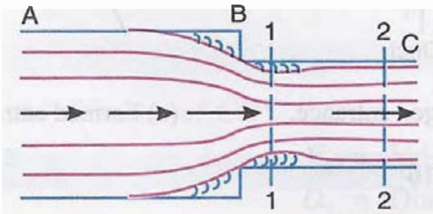


Fig. 20.14. Sudden contraction.

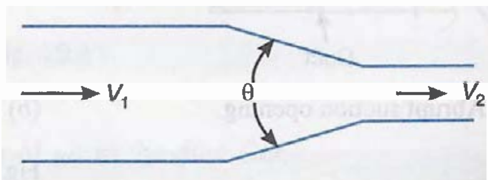


Fig. 20.15. Gradual contraction.

Let A_1 = Area of duct at section 1-1,

V_1 = Velocity of air at section 1-1,

A_2, V_2 = Corresponding values at section 2-2, and

p_L = Pressure loss due to sudden contraction.

Since the pressure loss due to sudden contraction is equal to the pressure loss due to sudden enlargement from section 1-1 to section 2-2, therefore pressure loss due to sudden contraction,

$$p_L = \left(1 - \frac{A_1}{A_2}\right)^2 \left(\frac{V_1}{4.04}\right)^2 = C_1\left(\frac{V_1}{4.04}\right)^2 = C_2\left(\frac{V_2}{4.04}\right)^2$$

When there is a gradual contraction in area of the duct as shown in Fig. 20.15, then the loss due to gradual contraction is given by

$$p_L = C_r C_1\left(\frac{V_1}{4.04}\right)^2 = C_r C_2\left(\frac{V_2}{4.04}\right)^2$$

where C_r is the loss coefficient. The following table shows the values of C_r as a function of included angle θ of the sides.

Table 20.4. Values of loss coefficient.

Condition (θ°)	30°	45°	60°
Loss coefficient (C_r)	0.02	0.04	0.07

20.17 Pressure Loss at Suction and Discharge of a Duct

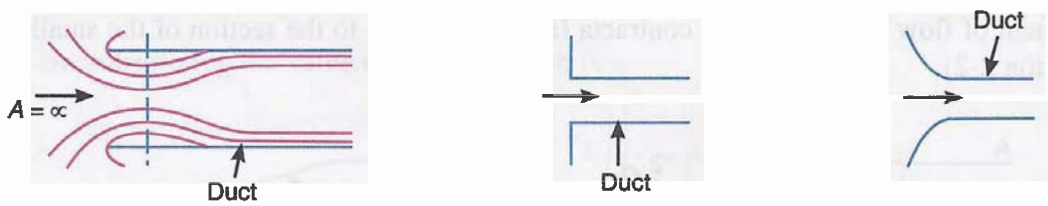
The pressure loss at suction to the duct is given by

$$p_L = \frac{C V^2}{2g} \text{ m of air} = \frac{C V^2}{2} \times \rho_a \text{ N/m}^2$$

$$= C\left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

where V is the velocity of air in the duct in m/s and C is the loss coefficient.

In case of an abrupt suction opening, as shown in Fig. 20.16 (*a*), the air is accelerated as it approaches to the opening, forming a vena contracta inside the duct. The area changes from infinity to the duct area. In such a case, the loss coefficient C is taken as 0.85. By making a flanged entrance as shown in Fig. 20.16 (*b*), the loss coefficient is reduced to 0.34. The loss coefficient can be further reduced to 0.03 by making formed entrance of bell-mouth shape as shown in Fig. 20.16 (*c*).



(*a*) Abrupt suction opening. (*b*) Flanged entrance. (*c*) Formed entrance.

Fig. 20.16

The pressure loss at the discharge of a duct is actually a loss due to the energy of head, which the flowing air has, by virtue of its motion. The value of pressure loss at the discharge or exit of a duct is given by

$$p_L = \frac{V^2}{2g} \text{ m of air}$$

$$= \frac{V^2}{2} \times \rho_a \text{ N/m}^2 = \left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

20.18 Pressure Loss due to an Obstruction in a Duct

In the previous articles, we have discussed the frictional and dynamic losses in ducts. In addition to these losses, the pressure losses also occur due to various obstructions in the path of air flow from the fan to the outlet (*i.e.* air conditioned room). The possible losses for different obstructions are given in the following table.

Table 20.5. Possible pressure loss for different obstructions.

Type of obstruction	*Possible pressure loss in mm of water*
Air heaters or cooler with several rows	5 to 10
Air washers	6.25 to 10
Air filters	5 to 10
Screen grills	2.5 to 5

Note : The actual pressure losses in the above equipments must be obtained from the data of the manufacturer whose equipment is to be used in the air conditioning system under consideration.

Example 20.10. *A length of main circular duct has three branch ducts taking equal air volumes at equal intervals. Each interval duct has a friction loss of 1.3 mm of water and a static pressure of 5 mm of water is necessary at each branch to cope with its friction loss. If the initial velocity in the main duct of 1.2 m diameter is 600 m/min, calculate the velocities and diameters of the second and third lengths, whereby the static pressure regain is sufficient to overcome the friction loss in the succeeding length of main duct up to the next branch. The static pressure regain factor is 0.6.*

Draw a simple sketch of the duct system and identify total, static and velocity pressures at the appropriate points of change.

Solution. Given : $p_{f1} = p_{f2} = p_{f3} = 1.3$ mm of water ; $p_{SB} = p_{SD} = p_{SF} = 5$ mm of water ; $D_1 = 1.2$ m ; $V_1 = 600$ m/min $= 10$ m/s ; $R = 0.6$

The duct system is shown in Fig. 20.17.

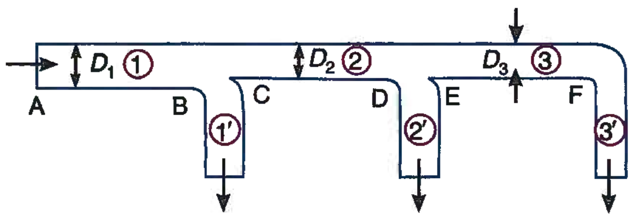


Fig. 20.17

Velocity and diameter of CD

Let V_2 = Velocity of air in the duct *CD*,

D_2 = Diameter of the duct *CD*, and

p_{v2} = Velocity pressure in the duct *CD*.

We know that the quantity of air passing through the duct AB,

$$Q_1 = A_1 V_1 = \frac{\pi}{4}(D_1)^2 V_1 = \frac{\pi}{4}(1.2)^2\, 600 = 678.6 \text{ m}^3/\text{min}$$

Quantity of air passing through each branch,

$$Q_1' = Q_2' = Q_3' = \frac{678.6}{3} = 226.2 \text{ m}^3/\text{min}$$

Velocity pressure in the duct AB,

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{10}{4.04}\right)^2 = 6.13 \text{ mm of water}$$

We know that static pressure regain,

$$SPR = R\,(p_{v1} - p_{v2})$$

Since the static pressure regain is equal to the friction loss in the duct CD, therefore

$$R\,(p_{v1} - p_{v2}) = p_{f2}$$

or $$0.6\,(6.13 - p_{v2}) = 1.3$$

∴ $$p_{v2} = 6.13 - 1.3 / 0.6 = 3.96 \text{ mm of water}$$

Velocity pressure in the duct CD,

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2$$

or $$V_2 = 4.04\sqrt{p_{v2}} = 4.04\sqrt{3.96} = 8.04 \text{ m/s} = 482.4 \text{ m/min}$$ **Ans.**

Quantity of air passing through the duct CD,

$$Q_2 = Q_1 - Q_1' = 678.6 - 226.2 = 452.4 \text{ m}^3/\text{min}$$

We know that $$Q_2 = A_2 V_2 = \frac{\pi}{4}(D_2)^2 V_2$$

∴ $$(D_2)^2 = \frac{4Q_2}{\pi V_2} = \frac{4 \times 452.4}{\pi \times 482.4} = 1.194 \text{ m}^2 \quad \text{or} \quad D_2 = 1.09 \text{ m}$$ **Ans.**

Velocity and diameter of duct EF

Let V_3 = Velocity in the duct EF,

D_3 = Diameter of the duct EF, and

p_{v3} = Velocity pressure in the duct EF.

Static pressure regain,

$$SPR = R\,(p_{v2} - p_{v3})$$

Since the static pressure regain is equal to the friction loss in the duct EF, therefore

$$R\,(p_{v2} - p_{v3}) = p_{f3}$$

or $$0.6\,(3.96 - p_{v3}) = 1.3$$

∴ $$p_{v3} = 3.96 - 1.3 / 0.6 = 1.79 \text{ mm of water}$$

Velocity pressure in the duct EF,

$$p_{v3} = \left(\frac{V_3}{4.04}\right)^2$$

∴ $$V_3 = 4.04\sqrt{p_{v3}} = 4.04\sqrt{1.79} = 5.4 \text{ m/s}$$

$$= 324.3 \text{ m/min}$$ **Ans.**

Quantity of air passing through the duct *EF*,

$$Q_3 = Q_2 - Q_2' = 452.4 - 226.2 = 226.2 \text{ m}^3/\text{min}$$

We know that $$Q_3 = A_3 V_3 = \frac{\pi}{4}(D_3)^2 V_3$$

$$\therefore \quad (D_3)^2 = \frac{4Q_3}{\pi V_3} = \frac{4 \times 226.2}{\pi \times 324.3} = 0.888 \text{ m}^2 \quad \text{or} \quad D_3 = 0.94 \text{ m} \textbf{ Ans.}$$

Total, static and velocity pressures at the appropriate points of change

The total, static and velocity pressures at the appropriate points of change are tabulated in the following table :

Table 20.6

Pressure in mm of water	*Appropriate points of change*				
	B	*C*	*D*	*E*	*F*
Total pressure (p_T)	11.13	10.26	8.96	8.09	6.79
Static pressure (p_S)	5	6.3	5	6.3	5
Velocity pressure (p_V)	6.13	3.96	3.96	1.79	1.79

Fabricated ducts.

Example 20.11. *A circular duct of 100 mm diameter converges gradually to a 75 mm duct. The static pressure just upstream of the reducer is 30 mm of water and the velocity is 450 m/min. The loss of pressure in the reducer is 0.1 of the velocity head in the duct downstream of the reducer. Calculate the total pressures upstream and downstream of the reducer. Also determine the pressure indicated by a U-tube water manometer connected differentially to pressure tappings upstream and downstream of the reducer.*

Solution. Given : $D_1 = 100$ mm ; $D_2 = 75$ mm ; $p_{s1} = 30$ mm of water ; $V_1 = 450$ m / min = 7.5 m /s ; $p_L = 0.1\ p_{v2}$

The cross-section of the duct is shown in Fig. 20.18.

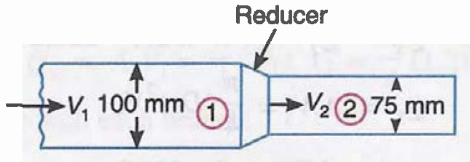


Fig. 20.18

Total pressure upstream and downstream of the reducer

Let p_{T1} = Total pressure upstream of the reducer,

p_{T2} = Total pressure downstream of the reducer, and

V_2 = Velocity downstream of the reducer.

We know that velocity pressure upstream of the reducer,

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{7.5}{4.04}\right)^2 = 3.45 \text{ mm of water}$$

$$\therefore \quad p_{T1} = p_{s1} + p_{v1} = 30 + 3.45 = 33.45 \text{ mm of water } \textbf{Ans.}$$

We know that $A_1 V_1 = A_2 V_2$ or $\frac{\pi}{4}(D_1)^2 V_1 = \frac{\pi}{4}(D_2)^2 V_2$

$$\therefore \quad V_2 = V_1\left(\frac{D_1}{D_2}\right)^2 = 7.5\left(\frac{100}{75}\right)^2 = 13.35 \text{ m/s}$$

Velocity pressure downstream of the reducer,

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{13.35}{4.04}\right)^2 = 10.9 \text{ mm of water}$$

and pressure loss in the reducer,

$$p_L = 0.1\, p_{v2} = 0.1 \times 10.9 = 1.09 \text{ mm of water}$$

We know that $p_{T1} = p_{T2} + p_L$

$$\therefore \quad p_{T2} = p_{T1} - p_L = 33.45 - 1.09 = 32.36 \text{ mm of water } \textbf{Ans.}$$

Pressure indicated by a U-tube water manometer

We know that static prèssure downstream of the reducer,

$$p_{s2} = p_{T2} - p_{v2} \qquad \ldots (\because p_{T2} = p_{s2} + p_{v2})$$

$$= 32.36 - 10.9 = 21.46 \text{ mm of water}$$

The pressure indicated by a U-tube water manometer is the difference between the static pressures at the tapping sections.

$$\therefore \quad p_{s1} - p_{s2} = 30 - 21.46 = 8.54 \text{ mm of water } \textbf{Ans.}$$

Example 20.12. *The duct, as shown in Fig. 20.19, is supplied with air at A and discharges freely to atmosphere at F. The duct AB is 0.6 m in diameter and 45 m long, the duct CD is 1.2 m in diameter and 150 m long and the duct EF is 0.45 m in diameter and 15 m long. The velocity of air in duct AB is 600 m/min.*

The loss in the expander BC is 0.5 times the velocity pressure in duct AB and the loss in the reducer DE is 0.2 times the velocity pressure in the duct EF.

Using the expression

$$p_f = \frac{0.263 V^{1.35}}{D^{1.27}}$$

where p_f is the pressure loss due to friction in mm of water per 100 m length of duct, V is the duct velocity in m/s and D is the diameter of duct in m, calculate the static pressure at point A.

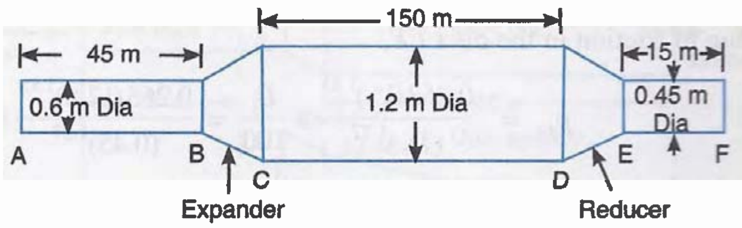


Fig. 20.19

Solution. Given : $D_1 = 0.6$ m ; $L_1 = 45$ m ; $D_2 = 1.2$ m ; $L_2 = 150$ m ; $D_3 = 0.45$ m ; $L_3 = 15$ m ; $V_1 = 600$ m / min = 10 m / s

Let V_2 = Velocity of air in duct *CD*, and

V_3 = Velocity of air in duct *EF*.

We know that $A_1V_1 = A_2V_2 = A_3V_3$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = V_1\left(\frac{D_1}{D_2}\right)^2 = 10\left(\frac{0.6}{1.2}\right)^2 = 2.5 \text{ m/s}$$

and
$$V_3 = \frac{A_1V_1}{A_3} = V_1\left(\frac{D_1}{D_3}\right)^2 = 10\left(\frac{0.6}{0.45}\right)^2 = 17.8 \text{ m/s}$$

Assuming standard air, velocity pressure in the duct *AB*,

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{10}{4.04}\right)^2 = 6.13 \text{ mm of water}$$

Velocity pressure in the duct *CD*,

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{2.5}{4.04}\right)^2 = 0.383 \text{ mm of water}$$

and velocity pressure in the duct *EF*,

$$p_{v3} = \left(\frac{V_3}{4.04}\right)^2 = \left(\frac{17.8}{4.04}\right)^2 = 19.4 \text{ mm of water}$$

Pressure loss in the expander *BC*,

$$p_{L1} = 0.5 \times p_{v1} = 0.5 \times 6.13 = 3.065 \text{ mm of water}$$

Pressure loss in the reducer *DE*,

$$p_{L2} = 0.2 \times p_{v3} = 0.2 \times 19.4 = 3.88 \text{ mm of water}$$

We know that the pressure loss due to friction in the duct *AB*,

$$p_{f1} = \frac{0.263(V_1)^{1.85}}{(D_1)^{1.27}} \times \frac{L_1}{100} = \frac{0.263(10)^{1.85}}{(0.6)^{1.27}} \times \frac{45}{100} \text{ mm of water}$$

$$= \frac{0.263 \times 70.8 \times 45}{0.523 \times 100} = 16.02 \text{ mm of water}$$

Pressure loss due to friction in the duct *CD*,

$$p_{f2} = \frac{0.263(V_2)^{1.85}}{(D_2)^{1.27}} \times \frac{L_2}{100} = \frac{0.263(2.5)^{1.85}}{(1.2)^{1.27}} \times \frac{150}{100} \text{ mm of water}$$

$$= \frac{0.263 \times 5.45 \times 150}{1.26 \times 100} = 1.7 \text{ mm of water}$$

and pressure loss due to friction in the duct EF,

$$p_{f3} = \frac{0.263 (V_3)^{1.85}}{(D_2)^{1.27}} \times \frac{L_3}{100} = \frac{0.263 (17.8)^{1.85}}{(0.45)^{1.27}} \times \frac{15}{100} \text{ mm of water}$$

$$= \frac{0.263 \times 205.7 \times 15}{0.363 \times 100} = 22.35 \text{ mm of water}$$

∴ Total pressure loss in the duct between A and F,

$$\Sigma p_L = p_{f1} + p_{L1} + p_{f2} + p_{L2} + p_{f3}$$

$$= 16.02 + 3.065 + 1.7 + 3.88 + 22.35 = 47.015 \text{ mm of water}$$

Let p_{s1} = Static pressure at point A.

Now applying Bernoulli's equation between A and F,

$$p_{s1} + p_{v1} = p_{s3} + p_{v3} + \Sigma p_L$$

Since the discharge at F is to atmosphere, therefore $p_{s3} = 0$. Substituting the values in the above equation, we have

$$p_{s1} + 6.13 = 0 + 19.4 + 47.015$$

∴ p_{s1} = 60.285 mm of water **Ans.**

Example 20.13. *A centrifugal fan of outlet 0.9 m by 0.7 m is moving standard air at a rate of 680 m³/min through a system which consists of straight inlet and outlet ducts. The inlet duct is 0.9 m diameter and 15 m long and the discharge duct is 1 m diameter and 60 m long. There is a diffuser between the fan discharge and the 1 m diameter duct for which the loss of pressure may be taken as 0.3 times the difference between velocity pressures. The loss at entry to the inlet duct is 0.5 times of the velocity pressure there and the friction factor 'f' for the inlet duct is 0.004 and for the outlet duct is 0.0035. Determine : 1. fan total pressure; 2. static pressures at the fan inlet and outlet; and 3. plot the variation of the total pressure and static pressure along the system.*

Assume that the air is sucked in by the inlet duct and delivered by the outlet duct is at atmospheric pressure.

Solution. Given : $a = 0.9$ m ; $b = 0.7$ m ; $Q = 680$ m³/min ; $D_1 = 0.9$ m ; $L_1 = 15$ m ; $D_4 = 1$ m ; $L_4 = 60$ m ; $f_1 = 0.004$; $f_4 = 0.0035$

The duct system is shown in Fig. 20.20 (*a*),

1. *Fan total pressure*

Cross-sectional area of the inlet duct 1-2,

$$A_1 = A_2 = \frac{\pi}{4}(D_1)^2 = \frac{\pi}{4}(0.9)^2 = 0.636 \text{ m}^2$$

∴ Velocity of air in the inlet duct 1-2,

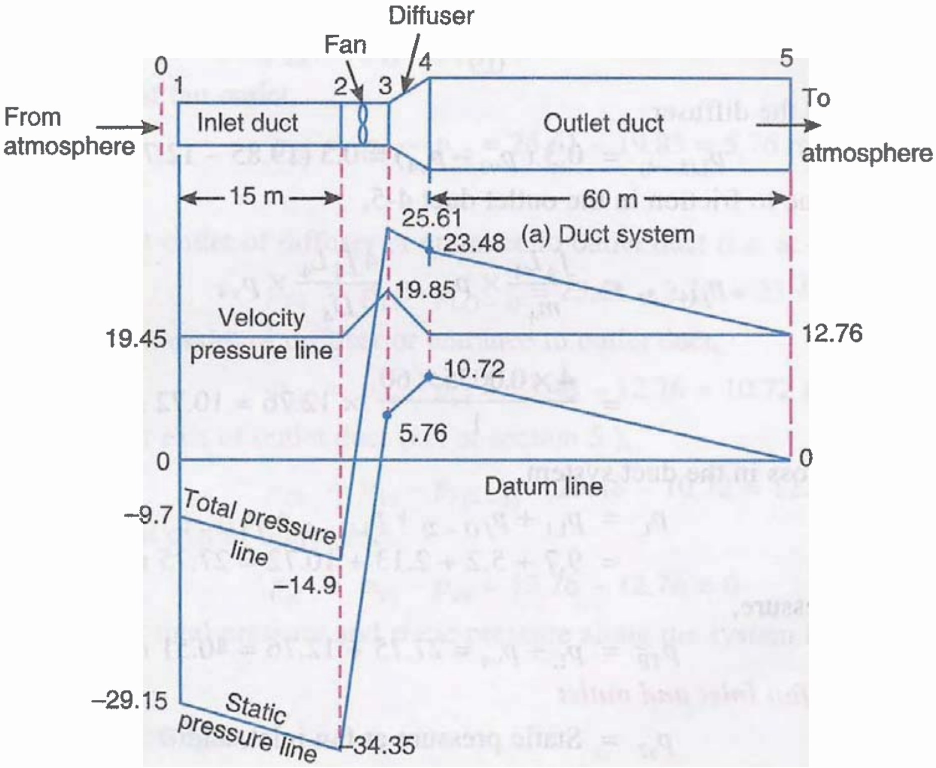
$$V_1 = V_2 = \frac{Q}{A_1} = \frac{680}{0.636} = 1069.2 \text{ m/min} = 17.82 \text{ m/s}$$

and velocity pressure in the inlet duct 1-2,

$$p_{v1} = p_{v2} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{17.82}{4.04}\right)^2 = 19.45 \text{ mm of water}$$

Cross-sectional area at the fan outlet,

$$A_3 = a \times b = 0.9 \times 0.7 = 0.63 \text{ m}^2$$



(*b*) Variation of total pressure and static pressure along the duct system.

Fig. 20.20

∴ Velocity of air at the fan outlet,

$$V_3 = \frac{Q}{A_3} = \frac{680}{0.63} = 1079.4 \text{ m/min} = 18 \text{ m/s}$$

and velocity pressure at the fan outlet,

$$p_{v3} = \left(\frac{V_3}{4.04}\right)^2 = \left(\frac{18}{4.04}\right)^2 = 19.85 \text{ mm of water}$$

Cross-sectional area of the outlet duct 4-5,

$$A_4 = A_5 = \frac{\pi}{4}(D_4)^2 = \frac{\pi}{4} \times 1^2 = 0.7855 \text{ m}^2$$

∴ Velocity of air in the outlet duct 4-5,

$$V_4 = V_5 = \frac{Q}{A_4} = \frac{680}{0.7855} = 865.7 \text{ m/min} = 14.43 \text{ m/s}$$

and velocity pressure in the outlet duct 4-5,

$$p_{v4} = p_{v5} = \left(\frac{V_4}{4.04}\right)^2 = \left(\frac{14.43}{4.04}\right)^2 = 12.76 \text{ mm of water}$$

Pressure loss at entry to the inlet duct at point 1,

$$p_{L1} = 0.5\, p_{v1} = 0.5 \times 19.45 = 9.7 \text{ mm of water}$$

Pressure loss due to friction in the inlet duct 1-2,

$$p_{f(1-2)} = \frac{f_1 L_1}{m_1} \times p_{v1} = \frac{4 f_1 L_1}{D_1} \times p_{v1} \qquad \ldots \left(\because m_1 = \frac{D_1}{4}\right)$$

$$= \frac{4 \times 0.004 \times 15}{0.9} \times 19.45 = 5.2 \text{ mm of water}$$

Pressure loss in the diffuser,

$$p_{L(3-4)} = 0.3\,(p_{v3} - p_{v4}) = 0.3\,(19.85 - 12.76) = 2.13 \text{ mm of water}$$

Pressure loss due to friction in the outlet duct 4-5,

$$p_{f(4-5)} = \frac{f_4 L_4}{m_4} \times p_{v4} = \frac{4 f_4 L_4}{D_4} \times p_{v4} \qquad \ldots \left(\because m_4 = \frac{D_4}{4}\right)$$

$$= \frac{4 \times 0.0035 \times 60}{1} \times 12.76 = 10.72 \text{ mm of water}$$

Total pressure loss in the duct system,

$$p_L = p_{L1} + p_{f(1-2)} + p_{L(3-4)} + p_{f(4-5)}$$

$$= 9.7 + 5.2 + 2.13 + 10.72 = 27.75 \text{ mm of water}$$

∴ Fan total pressure,

$$p_{TF} = p_L + p_{v4} = 27.75 + 12.76 = 40.51 \text{ mm of water}$$ **Ans.**

2. Static pressure at fan inlet and outlet

Let p_{s2} = Static pressure at fan inlet, and

p_{s3} = Static pressure at fan outlet.

Applying Bernoulli's equation to a section at *O* in the atmosphere upstream of the entrance to inlet duct and to a point at fan inlet (*i.e.* at section 2), we have

Total pressure at *O* = Total pressure at 2

$$p_{T0} = p_{T2} = p_{s2} + p_{v2} + p_{L1} + p_{f(1-2)}$$

Since the total pressure in the atmosphere (p_{T0}) is zero, therefore

$$0 = p_{s2} + 19.45 + 9.7 + 5.2 = p_{s2} + 34.35$$

∴ $p_{s2} = -34.35$ mm of water **Ans.**

Again applying Bernoulli's equation to fan inlet (*i.e.* section 2) and fan outlet (*i.e.* section 3), we have

$$p_{T2} = p_{T3}$$

$$p_{s2} + p_{v2} + p_{TF} = p_{s3} + p_{v3}$$

$$-34.35 + 19.45 + 40.51 = p_{s3} + 19.85$$

or $p_{s3} = 5.76$ mm of water **Ans.**

3. Variation of total pressure and static pressure along the system

We know that total pressure at entrance to inlet duct (*i.e.* at section 1),

$$p_{T1} = p_{T0} - p_{L1} = 0 - 9.7 = -9.7 \text{ mm of water}$$

Static pressure at entrance to inlet duct,

$$p_{s1} = p_{T1} - p_{v1} = -9.7 - 19.45 = -29.15 \text{ mm of water}$$

Total pressure at fan inlet (*i.e.* at section 2),

$$p_{T2} = p_{T1} - p_{f(1-2)} = -9.7 - 5.2 = -14.9 \text{ mm of water}$$

Static pressure at fan inlet,

$$p_{s2} = p_{T2} - p_{v2} = -14.9 - 19.45 = -34.35 \text{ mm of water}$$

Total pressure at fan outlet or inlet to diffuser (*i.e.* at section 3),

$$p_{T3} = p_{T2} + p_{TF} = -14.9 + 40.51 = 25.61 \text{ mm of water}$$

Static pressure at fan outlet,

$$p_{s3} = p_{T3} - p_{v3} = 25.61 - 19.85 = 5.76 \text{ mm of water}$$

... (same as before)

Total pressure at outlet of diffuser or entrance to outlet duct (*i.e.* at section 4),

$$p_{T4} = p_{T3} - p_{L(3-4)} = 25.61 - 2.13 = 23.48 \text{ mm of water}$$

Static pressure at outlet of diffuser or entrance to outlet duct,

$$p_{s4} = p_{T4} - p_{v4} = 23.48 - 12.76 = 10.72 \text{ mm of water}$$

Total pressure at exit of outlet duct (*i.e.* at section 5),

$$p_{T5} = p_{T4} - p_{f(4-5)} = 23.48 - 10.72 = 12.76 \text{ mm of water}$$

Static pressure at exit of outlet duct,

$$p_{s5} = p_{T5} - p_{v5} = 12.76 - 12.76 = 0$$

The variation of total pressure and static pressure along the system is shown in Fig. 20.20 (*b*). **Ans.**

20.19 Duct Design

The object of duct design is to determine the dimensions of all ducts in the given system. The ducts should carry the necessary volume of conditioned air from the fan outlet to the conditioned space with minimum frictional and dynamic losses. The duct layout must be made so as to reach the outlet without least number of bends, obstructions and area changes. The area changes must be gradual where possible and limited to not more than 20° for diverging area and 60° for converging area. For rectangular ducts, the aspect ratio of 4 and less is desirable but it should not be greater than 8 in any case. The minimum sheet metal is required with square cross-section for given cross-sectional area.

The velocities in the ducts must be high enough to reduce the size of the ducts but it should be low enough to reduce the noise and pressure losses to economise power requirement. The velocities recommended for various applications are given in the following table :

Table 20.7. Recommended velocities for various applications.

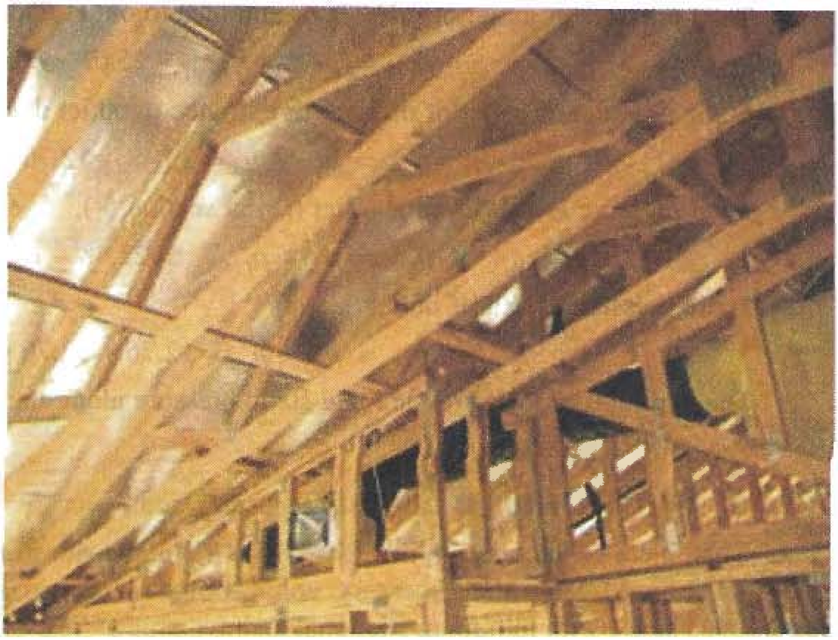
Designation	*Recommended velocities in m/min*		
	Residences	*Schools, theatres and public buildings*	*Industrial buildings*
Outdoor air intakes	150	150	150
Filters	75	90	105
Heating coils	135	150	180
Air washers	150	150	150
Fan outlets	300 – 480	400 – 600	480 – 720
Main ducts	200 – 300	300 – 400	350 – 550
Branch duct	180	180 – 270	240 – 300
Branch risers	150	180 – 210	240

After the layout of the duct is decided and the requirements of air quantities at various outlets are known, then the size of the ducts may be obtained as discussed in the following pages.

20.20 Methods for Determination of Duct Size

The following three methods for determination of duct size are important from the subject point of view :

1. Velocity reduction method. In this method, the velocities in the ducts are assumed such that they progressively decrease as the flow proceeds. The pressure drops are calculated for these velocities for respective branches and the main duct. The duct sizes are determined for assumed velocities and known quantities of air to be supplied through the respective ducts. The pressure at the outlet is adjusted by dampers in the respective ducts. The fan is designed to overcome the pressure losses along any single run including losses of the main duct, branch duct, elbows, enlargements and contractions of areas etc. In case the fan is already selected, the velocities in reducing order are adjusted to consume pressure available in the longest run or the run in which the maximum pressure loss is expected. The pressure in the remaining branches is adjusted by dampers.



Heating and air - conditioning ducts.

This method is the easiest in sizing ducts and the velocities can be adjusted to avoid noise. The major disadvantage of this system is that considerable experience and judgement is required in selecting velocities so as to make the system optimum in economy and power.

The velocity reduction method of designing ducts is usually adopted for very simple systems.

2. Equal pressure drop (or friction loss) method. In this method, the size of duct is decided to give equal pressure drop (or friction loss) per metre length in all ducts. If the layout of the ducts is symmetrical giving the same length of the various runs, this method gives equal pressure loss in various branches and no dampering is required to balance them. In case the runs are of different lengths, then the shortest run will have minimum loss and consequently high pressure at the outlet. It is, therefore, necessary to reduce this high pressure by heavy dampering or modifying this method to provide higher velocities in shorter runs. But the high velocities in short run to reduce high pressure may create objectionable noise. Thus noise absorbing outlets and fittings must be provided. The dampers if provided near the main duct will help in reducing the noise as the branch duct will dissipate some noise.

The velocities, in this method , are automatically reduced in the branch ducts as the flow is decreased. This method does not however balance the pressures at the outlets if the branches are of different lengths and hence dampers are required for balancing the pressure drops in various branches.

A modification of this method is to design the main duct for equal friction and branch ducts for consuming the pressure available at the take-off from the main duct. In such a design, the pressures at the outlet will be same and no dampering is required for balancing the pressure drops in various runs. However, dampers are provided for small adjustment.

3. *Static regain method.* In this method, the size of the duct is decided to give equal pressure at all outlets, for perfect balancing of the air duct layout system. This may be done by equalising the pressure losses in various branches. This is possible if the friction loss in each branch is made equal to the gain in pressure due to reduction in velocity. The gain in pressure (or static pressure regain) due to change in velocity is given by

$$SPR = R\rho_a \left[\frac{(V_1)^2 - (V_2)^2}{2}\right] = R\,(p_{v1} - p_{v2}) \qquad \cdots \left(\because p_v = \frac{\rho_a V^2}{2}\right)$$

where R is the static regain factor.

It may not be possible to design economically very long branches and the branches very near to the fan for complete regain. In such cases, it is sufficient to design the main duct for complete regain and provide same pressure at all outlets from the main duct for branches. The partial regain may be considered a good practice for a few outlets from the main duct, so that same pressure loss is allowed in the beginning.

This method allows for balancing but,reducing velocity increases duct size and it should not be taken beyond the economic limit.

Example 20.14. *In the duct system, as shown in Fig. 20.21, the cross-section of the main duct is such that the width is always twice the depth. The total quantity of air entering at A is 690 m³/min and the static pressure at A is 18 mm of water.*

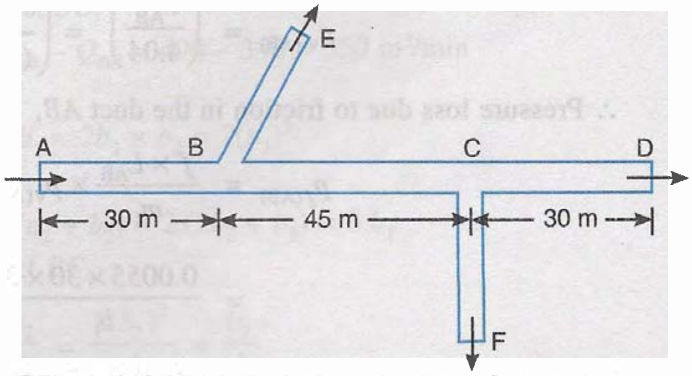


Fig. 20.21

A branch 0.9 m broad and 0.6 m deep is led off at B for a length of 45 m. This discharges at atmospheric pressure and there is a loss of 0.5 velocity head at the conversion piece. Another branch at C is so designed that it discharges 120 m³/min.

Calculate the dimensions of the three sections of the main duct, designing on a basis of uniform pressure drop. Neglect velocity changes in the main duct and assume that the discharge pressure at D is atmospheric. The value of friction factor may be taken as 0.0055.

Solution. Given : Q_{AB} = 690 m³/min ; p_{SA} = 18 mm of water ; a = 0.9 m ; b = 0.6 m ; L_{BE} = 45 m ; $p_{L(BE)} = 0.5\, p_{v\,(BE)}$; Q_{CF} = 120 m³/min ; f = 0.0055 ; L_{AB} = 30 m ; L_{BC} = 45 m ; L_{CD} = 30 m

We know that when the velocity changes in the main duct AD are neglected, then the static pressure at A must be equal to the pressure loss due to friction between A and D.

$$\therefore \quad p_{f\,(AB)} + p_{f\,(BC)} + p_{f\,(CD)} = p_{SA} = 18 \text{ mm of water}$$

Total length of the duct AD,

$$L = L_{AB} + L_{BC} + L_{CD} = 30 + 45 + 30 = 105 \text{ m}$$

∴ Pressure drop per m length of the duct AD

$$= \frac{18}{105} = 0.17 \text{ mm of water}$$

Dimensions of the duct AB

Let b_1 = Depth of the duct AB, and

a_1 = Width of the duct $AB = 2\, b_1$ ... (Given)

Cross-sectional area of the duct AB,

$$A_1 = a_1 b_1 = 2b_1 \times b_1 = 2(b_1)^2$$

Wetted perimeter of the duct AB,

$$P_1 = 2\,(a_1 + b_1) = 2\,(2b_1 + b_1) = 6\, b_1$$

∴ Hydraulic mean depth of the duct AB,

$$m_1 = \frac{A_1}{P_1} = \frac{2(b_1)^2}{6\, b_1} = \frac{b_1}{3}$$

Velocity of air in the duct AB,

$$V_{AB} = \frac{Q_{AB}}{A_1} = \frac{690}{2(b_1)^2} = \frac{345}{(b_1)^2} \text{ m/min} = \frac{5.75}{(b_1)^2} \text{ m/s}$$

Velocity pressure in the duct AB,

$$p_{v\,(AB)} = \left(\frac{V_{AB}}{4.04}\right)^2 = \left(\frac{5.75}{(b_1)^2 \times 4.04}\right)^2 = \frac{2}{(b_1)^4}$$

∴ Pressure loss due to friction in the duct AB,

$$p_{f(AB)} = \frac{f \times L_{AB}}{m_1} \times p_{v(AB)}$$

$$= \frac{0.0055 \times 30 \times 3}{b_1} \times \frac{2}{(b_1)^4} = \frac{0.99}{(b_1)^5} \quad \text{... (i)}$$

Since the pressure drop per m length is 0.17 mm of water, therefore total pressure loss due to friction in the duct AB is

$$p_{f(AB)} = 0.17 \times 30 = 5.1 \text{ mm of water} \quad \text{... (ii)}$$

Equating equations (*i*) and (*ii*), we have

$$\frac{0.99}{(b_1)^5} = 5.1 \quad \text{or} \quad (b_1)^5 = \frac{0.99}{5.1} = 0.194$$

∴ $b_1 = 0.72$ m **Ans.**

and $a_1 = 2b_1 = 2 \times 0.72 = 1.44$ m **Ans.**

Dimensions of the duct BC

Let b_2 = Depth of the duct BC, and

a_2 = Width of the duct $BC = 2b_2$ ... (Given)

First of all, let us find out the quantity of air flowing in the duct BE. We know that cross-sectional area of the duct BE,

$$A = ab = 0.9 \times 0.6 = 0.54 \text{ m}^2$$

Wetted perimeter of the duct BE,

$$P = 2\,(a + b) = 2\,(0.9 + 0.6) = 3 \text{ m}$$

and hydraulic mean depth of the duct *BE*,

$$m = \frac{A}{P} = \frac{0.54}{3} = 0.18 \text{ m}$$

∴ Pressure loss due to friction in the duct *BE*,

$$p_{f\,(BE)} = \frac{f \times L_{BE}}{m} \times p_{v\,(BE)} = \frac{0.0055 \times 45}{0.18} \times p_{v\,(BE)} = 1.375\, p_{v\,(BE)}$$

Pressure loss at the conversion piece

$$= 0.5\, p_{v\,(BE)} \quad \text{... (Given)}$$

∴ Pressure at $B = 1.375\, p_{v\,(BE)} + 0.5\, p_{v\,(BE)} = 1.875\, p_{v\,(BE)}$ ... (*iii*)

Also Pressure at $B = p_{SA} - p_{f\,(AB)} = 18 - 5.1 = 12.9$ mm of water ... (*iv*)

Equating equations (*iii*) and (*iv*),

$$1.875\, p_{v\,(BE)} = 12.9 \quad \text{or} \quad p_{v\,(BE)} = \frac{12.9}{1.875} = 6.88 \text{ mm of water}$$

We know that velocity of air in the duct *BE*,

$$V_{BE} = 4.04\sqrt{p_{v\,(BE)}} = 4.04\sqrt{6.88} = 10.5 \text{ m/s} = 630 \text{ m/min}$$

∴ Quantity of air flowing through the duct *BE*,

$$Q_{BE} = A \times V_{BE} = 0.54 \times 630 = 340 \text{ m}^3\text{/min}$$

and quantity of air flowing through the duct *BC*,

$$Q_{BC} = Q_{AB} - Q_{BE} = 690 - 340 = 350 \text{ m}^3\text{/min}$$

Cross-sectional area of the duct *BC*,

$$A_2 = a_2 b_2 = 2b_2 \times b_2 = 2(b_2)^2$$

Wetted perimeter of the duct *BC*,

$$P_2 = 2\,(a_2 + b_2) = 2\,(2b_2 + b_2) = 6\,b_2$$

∴ Hydraulic mean depth of the duct *BC*,

$$m_2 = \frac{A_2}{P_2} = \frac{2(b_2)^2}{6\,b_2} = \frac{b_2}{3}$$

Velocity of air in the duct *BC*,

$$V_{BC} = \frac{Q_{BC}}{A_2} = \frac{350}{2(b_2)^2} = \frac{175}{(b_2)^2} \text{ m/min} = \frac{2.92}{(b_2)^2} \text{ m/s}$$

Velocity pressure in the duct *BC*,

$$p_{v\,(BC)} = \left[\frac{V_{BC}}{4.04}\right]^2 = \left[\frac{2.92}{(b_2)^2 \times 4.04}\right]^2 = \frac{0.52}{(b_2)^4}$$

∴ Pressure loss due to friction in the duct *BC*,

$$p_{f\,(BC)} = \frac{f \times L_{BC}}{m_2} \times p_{v\,(BC)} = \frac{0.0055 \times 45 \times 3}{b_2} \times \frac{0.52}{(b_2)^4} = \frac{0.386}{(b_2)^5} \quad \text{... (v)}$$

Since the pressure drop per m length is 0.17 mm of water, therefore total pressure loss due to friction in the duct *BC* is

$$p_{f\,(BC)} = 0.17 \times 45 = 7.65 \text{ mm of water} \quad \text{... (vi)}$$

Equating equations (*v*) and (*vi*),

$$\frac{0.386}{(b_2)^5} = 7.65 \quad \text{or} \quad (b_2)^5 = \frac{0.386}{7.65} = 0.05$$

$\therefore \quad b_2 = 0.55$ m **Ans.**

and $\quad a_2 = 2b_2 = 2 \times 0.55 = 1.1$ m **Ans.**

Dimensions of the duct CD

Let $\quad b_3 =$ Depth of the duct *CD*, and

$a_3 =$ Width of the duct $CD = 2\,b_3$ ... (Given)

Cross-sectional area of the duct *CD*,

$$A_3 = a_3 b_3 = 2b_3 \times b_3 = 2\,(b_3)^2$$

Wetted perimeter of the duct *CD*,

$$P_3 = 2\,(a_3 + b_3) = 2\,(2b_3 + b_3) = 6\,b_3$$

$\therefore$ Hydraulic mean depth of the duct *CD*,

$$m_3 = \frac{A_3}{P_3} = \frac{2(b_3)^2}{6\,b_3} = \frac{b_3}{3}$$

We know that the quantity of the air flowing through the duct *CD*,

$$Q_{CD} = Q_{BC} - Q_{CF} = 350 - 120 = 230 \text{ m}^3\text{/min}$$

Velocity of air in the duct *CD*,

$$V_{CD} = \frac{Q_{CD}}{A_3} = \frac{230}{2(b_3)^2} = \frac{115}{(b_3)^2} \text{ m/min} = \frac{1.92}{(b_3)^2} \text{ m/s}$$

Velocity pressure in the duct *CD*,

$$p_{v\,(CD)} = \left(\frac{V_{CD}}{4.04}\right)^2 = \left[\frac{1.92}{(b_3)^2 \times 4.04}\right]^2 = \frac{0.226}{(b_3)^4} \text{ mm of water}$$

$\therefore$ Pressure loss due to friction in the duct *CD*,

$$p_{f\,(CD)} = \frac{f \times L_{CD}}{m_3} \times p_{v\,(CD)} = \frac{0.0055 \times 30 \times 3}{b_3} \times \frac{0.226}{(b_3)^4}$$

$$= \frac{0.112}{(b_3)^5} \text{ mm of water} \qquad \text{... (vii)}$$

Also pressure loss due to friction in the duct *CD*,

$$p_{f\,(CD)} = 0.17 \times 30 = 5.1 \text{ mm of water} \qquad \text{... (viii)}$$

Equating equations (*vii*) and (*viii*), we have

$$\frac{0.112}{(b_3)^5} = 5.1 \quad \text{or} \quad (b_3)^5 = \frac{0.112}{5.1} = 0.022$$

$\therefore \quad b_3 = 0.466$ m **Ans.**

and $\quad a_3 = 2b_3 = 2 \times 0.466 = 0.932$ m **Ans.**

Carbon fibre air duct system.

Example 20.15. *An air conditioning system has volume flow rate of 7.5 m³/s and fan outlet velocity is 10 m/s. The duct has four branches with 90° elbows. The first branch is 10 m from fan. The distance between branches is 10 m and the main duct has 90° elbow 10m after the fourth branch. The volume flow rate in each branch is 1.5 m³/s. The main duct runs 10 m after the 90° bend.*

1. Using equal friction method, determine the equivalent diameter of duct and dimensions of rectangular duct if one side of the duct is 0.5 m.

2. Determine the total pressure drop. Given:

$$\frac{p_f}{L} = \frac{0.002\ 268\ (Q)^{1.852}}{(D)^{4.973}} \text{ mm of water ; } p_v = \left(\frac{V}{4.04}\right)^2 \text{ mm of water ;}$$

$$D = \frac{1.3(ab)^{0.625}}{(a+b)^{0.25}} \text{ ; Elbow loss} = 0.25\ p_v$$

Fitting losses where changes in area occur = 0.25 × Difference of velocity pressures

Dynamic loss in branch = 0.2 p_v + Elbow loss

Solution. Given : $Q_{AB} = 7.5$ m³/s ; $V_{AB} = 10$ m/s; $L_{AB} = L_{BC} = L_{CD} = L_{DE} = L_{EF} = 10$ m

The duct system is shown in Fig. 20.22.

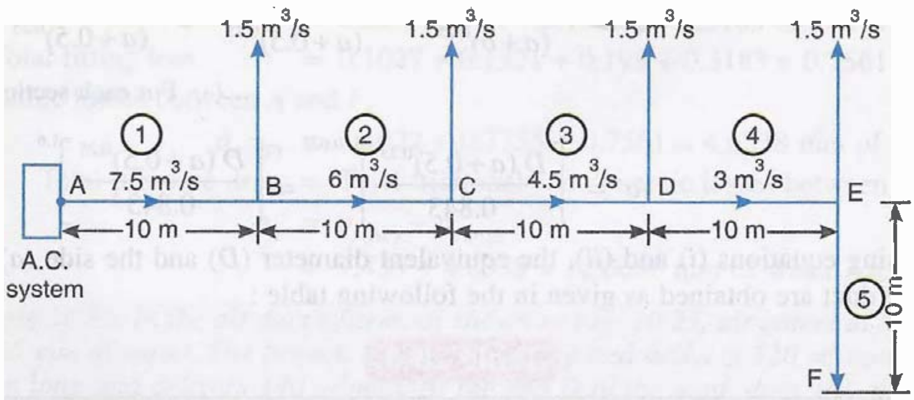


Fig. 20.22

Since the volume flow rate in each branch at *B*, *C*, *D* and *E* is 1.5 m³/s, therefore $Q_{BC} = 7.5 - 1.5 = 6$ m³/s ; $Q_{CD} = 6 - 1.5 = 4.5$ m³/s ; $Q_{DE} = 4.5 - 1.5 = 3$ m³/s

(a) Equivalent diameter of the duct and dimensions of the rectangular duct

Let a = Longer side of the duct *AB* in metres, and

b = Shorter side of the duct *AB* = 0.5 m ...(Given)

∴ Cross-sectional area of the duct B,

$$A_{AB} = a \times b = a \times 0.5 = 0.5\,a \text{ m}^2$$

We know that quantity of air passing through the duct *AB*,

$$Q_{AB} = A_{AB} \times V_{AB} = 0.5a \times 10 = 5a$$

∴ $a = Q_{AB} / 5 = 7.5 / 5 = 1.5$ m **Ans.**

and equivalent diameter for the duct *AB*,

$$D_{AB} = \frac{1.3\,(ab)^{0.625}}{(a+b)^{0.25}} = \frac{1.3\,(1.5\times0.5)^{0.625}}{(1.5+0.5)^{0.25}} = \frac{1.3\times0.8354}{1.1892}$$

$$= 0.9132 \text{ m } \textbf{Ans.}$$

We know that pressure loss due to friction in the duct *AB* per metre length,

$$\frac{p_{f(AB)}}{L_{AB}} = \frac{0.002\ 268\ (Q_{AB})^{1.852}}{(D_{AB})^{4.973}} = \frac{0.002\ 268(7.5)^{1.852}}{(0.9132)^{4.973}} = \frac{0.094\ 68}{0.636\ 64}$$

$$= 0.1487 \text{ mm of water}$$

This is constant for all sections.

∴ Equivalent diameter for each section,

$$D = \left[\frac{0.002\ 268(Q)^{1.852}}{0.1487}\right]^{1/4.973} = 0.4312\ (Q)^{0.3724} \qquad \ldots(i)$$

It is given that equivalent diameter,

$$D = \frac{1.3\,(ab)^{0.625}}{(a+b)^{0.25}} = \frac{1.3\,(a\times0.5)^{0.625}}{(a+0.5)^{0.25}} = \frac{0.843\,(a)^{0.625}}{(a+0.5)^{0.25}}$$

...(∵ For each section, b = 0.5 m)

$$\therefore \qquad a = \left[\frac{D\,(a+0.5)^{0.25}}{0.843}\right]^{1/0.625} = \left[\frac{D\,(a+0.5)^{0.25}}{0.843}\right]^{1.6} \qquad \ldots(ii)$$

Now using equations (*i*) and (*ii*), the equivalent diameter (*D*) and the side (*a*) for various sections of the duct are obtained as given in the following table :

Table 20.8.

Section	*Length (L) m*	*Flow rate (Q) m³/s*	*Equivalent diameter (D) m from equaton (i)*	**Side (a) m from equation (ii)*	*Cross-sectional area (A) m² A = a × b = a × 0.5*	*Velocity (V)m/s V = Q/A*	*Velocity pressure (p_v) mm of water* $p_v = \left[\frac{V}{4.04}\right]^2$
AB	10	7.5	0.9132	1.5	0.75	10	6.1268
BC	10	6	0.8404	1.2424	0.6212	9.6587	5.7158
CD	10	4.5	0.7550	0.980	0.49	9.1837	5.1674
DE	10	3	0.6492	0.710	0.355	8.4507	4.3754
EF	10	1.5	0.5015	0.4216	0.2108	7.1157	3.1022

From the above table, we find that *equivalent diameter for various sections are :*

$D_{AB} = 0.9132$ m ; $D_{BC} = 0.8404$ m ; $D_{CD} = 0.7550$ m ; $D_{DE} = 0.6492$ m ; and $D_{EF} = 0.5015$ m. **Ans.**

* The side a is obtained by iteration from equation (*ii*)

and the dimensions of rectangular duct for various sections are:

Section AB = 1.5 m × 0.5 m ; Section BC = 1.2424 m × 0.5 m ;

Section CD = 0.980 m × 0.5 m ; Section DE = 0.7110 m × 0.5 m ; and

Section EF = 0.4216 m × 0.5 m **Ans.**

2. *Total pressure drop*

We know that total pressure loss due to friction between AF,

$$p_{f(AF)} = 0.1487 \times L_{AF} = 0.1487 \times 50 = 7.435 \text{ mm of water}$$

... ($\because$ p_f per m length = 0.1487 mm of water)

Now let us find the dynamic losses between A and F as discussed below :

(*a*) Loss in discharge opening

$$= p_{v(EF)} = 3.1022 \text{ mm of water}$$

(*b*) Elbow loss $= 0.25\, p_{v(EF)} = 0.25 \times 3.1022 = 0.7755$ mm of water

(*c*) Fitting loss $= 0.25 \times$ Difference of velocity pressures

$\therefore$ Fitting loss at B $= 0.25\,[p_{v(AB)} - p_{v(BC)}]$

$= 0.25\,(6.1268 - 5.7158) = 0.1027$ mm of water

Fitting loss at C $= 0.25\,[p_{v(BC)} - p_{v(CD)}]$

$= 0.25\,[5.7158 - 5.1674] = 0.1371$ mm of water

Fitting loss at D $= 0.25\,[p_{v(CD)} - p_{v(DE)}]$

$= 0.25\,[5.1674 - 4.3754] = 0.198$ mm of water

Fitting loss at E $= 0.25\,[p_{v(DE)} - p_{v(EF)}]$

$= 0.25\,[4.3754 - 3.1022] = 0.3183$ mm of water

$\therefore$ Total fitting loss $= 0.1027 + 0.1371 + 0.198 + 0.3183 = 0.7561$ mm of water

and total dynamic losses between A and F,

$$p_{d(AF)} = 3.1022 + 0.7755 + 0.7561 = 4.6338 \text{ mm of water}$$

$\therefore$ Total pressure drop = Total frictional and dynamic losses between A and F

$$p_T = p_{f(AF)} + p_{d(AF)}$$

$= 7.435 + 4.6338 = 12.0688$ mm of water **Ans.**

Example 20.16. *In the air duct system, as shown in Fig. 20.23, air enters at A with a static pressure of 7.5 mm of water. The branch at B is 15 m long and delivers 120 m³/min. The branch at C is 22.5 m long and delivers 140 m³/min. At the end D of the main duct, the air delivered is 200 m³/min.*

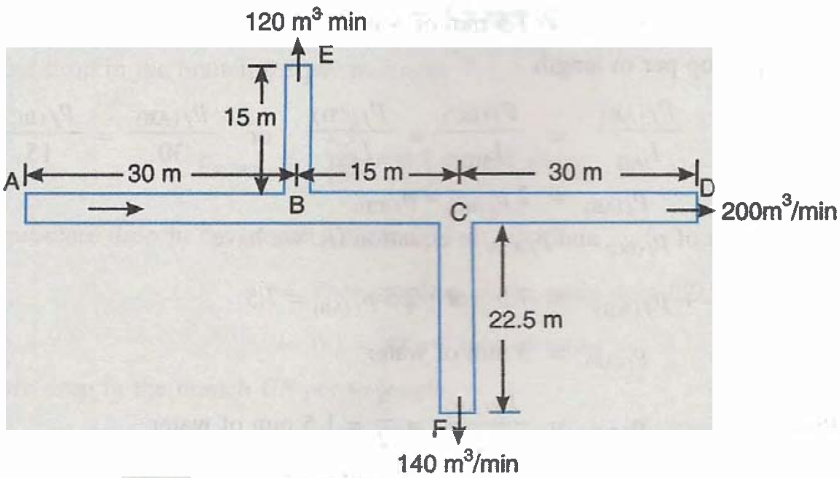


Fig. 20.23

Using friction chart and equal pressure drop method, determine the correct diameter and velocity pressures in lengths, AB, BC, CD, branch BE and branch CF. Consider friction losses only.

Solution. Given : p_{SA} = 7.5 mm of water ; L_{BE} = 15 m ; Q_{BE} = 120 m³/min ; L_{CF} = 22.5 m ; Q_{CF} = 140 m³/min ; Q_{CD} = 200 m³/min ; L_{AB} = 30 m ; L_{BC} = 15 m ; L_{CD} = 30 m

Diameter of lengths AB, BC, CD, branch BE and branch CF

According to Bernoulli's equation, the pressure losses between the main duct *AD* plus velocity pressure at *D* must be equal to the total pressure at *A*. Thus

$$p_{f\,(AB)} + p_{f\,(BC)} + p_{f\,(CD)} + p_{v\,(D)} = p_{SA} + p_{v\,(AB)}$$

Neglecting velocity pressure in the main duct, total pressure losses between the main duct *AD*,

$$p_f = p_{f\,(AB)} + p_{f\,(BC)} + p_{f\,(CD)} = 7.5 \text{ mm of water}$$

Total length of the main duct *AD*,

$$L = L_{AB} + L_{BC} + L_{CD} = 30 + 15 + 30 = 75 \text{ m}$$

Pressure drop per metre length of the duct

$$= 7.5 / 75 = 0.1 \text{ mm of water}$$

∴ Total pressure loss due to friction in the duct *AB*,

$${}^{*}p_{f\,(AB)} = 0.1 \times 30 = 3 \text{ mm of water}$$

Total pressure loss due to friction in the duct *BC*,

$$p_{f\,(BC)} = 0.1 \times 15 = 1.5 \text{ mm of water}$$

and total pressure loss due to friction in the duct *CD*,

$$p_{f\,(CD)} = 0.1 \times 30 = 3 \text{ mm of water}$$

We know that the quantity of air passing through the duct *AB*,

$$Q_{AB} = Q_{BE} + Q_{CF} + Q_{CD}$$
$$= 120 + 140 + 200 = 460 \text{ m}^3/\text{min} = 7.67 \text{ m}^3/\text{s}$$

---

* The total pressure loss due to friction in the ducts *AB*, *BC* and *CD* may be calculated as discussed below :

We know that the total pressure loss due to friction in the main duct *AD* is

$$p_{f\,(AB)} + p_{f\,(BC)} + p_{f\,(CD)} = 7.5 \text{ mm of water} \quad \text{... } (i)$$

For equal pressure drop per m length

$$\frac{p_{f\,(AB)}}{L_{AB}} = \frac{p_{f\,(BC)}}{L_{BC}} = \frac{p_{f\,(CD)}}{L_{CD}} \quad \text{or} \quad \frac{p_{f\,(AB)}}{30} = \frac{p_{f\,(BC)}}{15} = \frac{p_{f\,(CD)}}{30}$$

∴ $$p_{f\,(AB)} = 2\,p_{f\,(BC)} = p_{f\,(CD)}$$

Substituting the value of $p_{f\,(BC)}$ and $p_{f\,(CD)}$ in equation (*i*), we have

$$p_{f\,(AB)} + \frac{p_{f\,(AB)}}{2} + p_{f\,(AB)} = 7.5 \quad \text{or} \quad 2.5\,p_{f\,(AB)} = 7.5$$

∴ $$p_{f\,(AB)} = 3 \text{ mm of water}$$

We know that $$p_{f\,(BC)} = \frac{p_{f\,(AB)}}{2} = \frac{3}{2} = 1.5 \text{ mm of water}$$

and $$p_{f\,(CD)} = p_{f\,(AB)} = 3 \text{ mm of water}$$

Quantity of air passing through the duct BC,

$$Q_{BC} = Q_{CF} + Q_{CD} = 140 + 200 = 340 \text{ m}^3/\text{min} = 5.67 \text{ m}^3/\text{s}$$

and quantity of air passing through the duct CD,

$$Q_{CD} = 200 \text{ m}^3/\text{min} = 3.33 \text{ m}^3/\text{s} \quad \text{... (Given)}$$

From the friction chart, as shown in Fig. 20.10, we find that for 7.67 m^3/s and 0.1 mm of water, the diameter of duct AB,

$$D_{AB} = 0.95 \text{ m}$$ **Ans.**

Similarly, for 5.67 m^3/s and 0.1 mm of water, the diameter of duct BC,

$$D_{BC} = 0.85 \text{ m}$$ **Ans.**

and for 3.33 m^3/s and 0.1 mm of water, the diameter of duct CD,

$$D_{CD} = 0.72 \text{ m}$$ **Ans.**

We know that static pressure at B,

$$p_{SB} = p_{SA} - p_{f(AB)}$$
$$= 7.5 - 3 = 4.5 \text{ mm of water}$$

Since the branch BE is 15 m long, therefore pressure drop required per metre length

$$^*p_{f(BE)} = \frac{4.5}{15} = 0.3 \text{ mm of water}$$

Now from the friction chart, we find that for Q_{BE} = 120 m^3/min (or 2 m^3/s) and 0.3 mm of water, the diameter of branch BE,

$$D_{BE} = 0.46 \text{ m}$$ **Ans.**

---

* The pressure drop in the branches BE and CF may be obtained as discussed below:

We know that pressure drop in the main duct per m length

$$= 0.1 \text{ mm of water}$$

Since the pressure drops are equal, therefore

Pressure drop in the branch BE for 15 m length

= Pressure drop in the main duct BD of 45 m length

$$= 0.1 \times 45 = 4.5 \text{ mm of water}$$

∴ Pressure drop in the branch BE per m length

$$p_{f(BE)} = \frac{4.5}{15} = 0.3 \text{ mm of water}$$

Similarly, pressure drop in the branch CF for 22.5 m length

= Pressure drop in the main duct CD for 30 m length

$$= 0.1 \times 30 = 3 \text{ mm of water}$$

∴ Pressure drop in the branch CF per m length,

$$p_{f(CF)} = \frac{3}{22.5} = 0.133 \text{ mm of water}$$

We know that static pressure at *C*,

$$p_{SC} = p_{SB} - p_{f\,(BC)} = 4.5 - 1.5 = 3 \text{ mm of water}$$

Since the branch *CF* is 22.5 m, therefore pressure drop required per metre length

$$p_{f\,(CF)} = \frac{3}{22.5} = 0.133 \text{ mm of water}$$

From the friction chart, we find that for Q_{CD} = 140 m³/min (or 2.33 m³/s) and 0.133 mm of water, the diameter of branch *CF*,

$$D_{CF} = 0.6 \text{ m}$$ **Ans.**

Velocity pressure in lengths AB, BC, CD, branch BE and branch CF

The velocity pressure in lengths *AB*, *BC*, *CD* and branch *BE* and branch *CF* are calculated in the following table :

Table 20.9

Length	*Diameter (D) m*	*Cross-sectional area* $\left(A = \frac{\pi}{4} D^2\right) m^2$	*Quantity of air (Q) m³/min*	*Velocity* $\left(V = \frac{Q}{A}\right)$ *m/min*	*Velocity pressure* $*p_v = \left(\frac{V}{242.4}\right)^2$ *mm of water*
AB	0.95	0.709	460	648.8	7.16
BC	0.85	0.567	340	599.6	6.12
CD	0.72	0.407	200	491.4	4.11
BE	0.46	0.166	120	722.9	8.89
CF	0.6	0.283	140	494.7	4.16

Example 20.17. *An air duct system is provided as shown in Fig 20.24.*

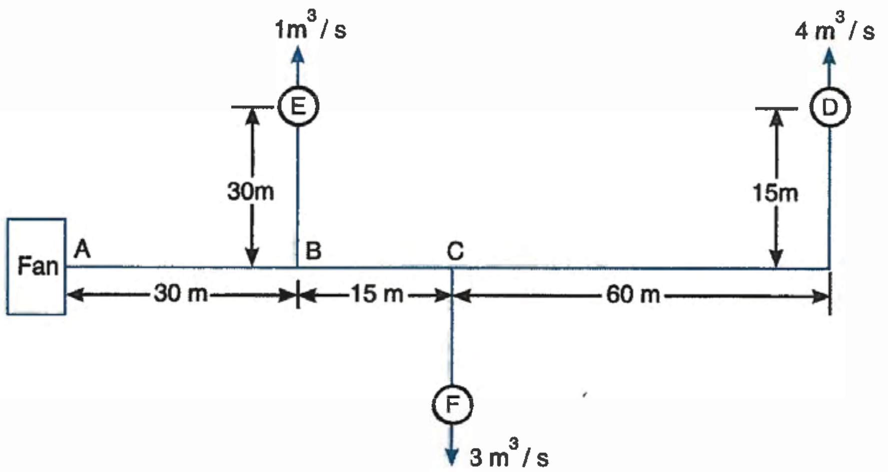


Fig. 20.24

1. *Determine the dimensions of AB, BC and CD using the equal friction method. Choose a friction rate of 0.08 mm of water per metre length of duct. Use the following formula for friction rate :*

---

* When velocity (*V*) is in m / min, then velocity pressure,

$$p_v = \left(\frac{V}{60 \times 4.04}\right)^2 = \left(\frac{V}{242.4}\right)^2 \text{ mm of water}$$

$$\frac{p_f}{L} = \frac{2.268 \times 10^{-3} (Q)^{1.852}}{(D)^{4.973}} \text{ mm of water/m}$$

where Q is m^3/s and D and L are in metres.

2. *Determine the total and static pressures at point A. Assume free exit at each outlet.*
   Losses are given by :
   For elbow: 0.25 p_v; Branch : 0.2 p_v + Elbow loss
   For straight-through section : 0.25 ($p_{v1} - p_{v2}$)
3. *Find the diameter of BE so that no dampering is required in the section.*

Solution. Given : L_{AB} = 30 m ; L_{BC} = 15 m ; L_{CD} = 60 + 15 = 75 m ; Q_{BE} = 1 m^3/s; Q_{CF} = 3 m^3/s ; Q_{CD} = 4 m^3/s ; p_f / L = 0.08 mm of water / m

1. Dimensions of AB, BC and CD

We know that quantity of air passing through the duct *AB*,

$$Q_{AB} = Q_{BE} + Q_{CF} + Q_{CD} = 1 + 3 + 4 = 8 \text{ m}^3/\text{s}$$

Quantity of air passing through the duct *BC*,

$$Q_{BC} = Q_{CF} + Q_{CD} = 3 + 4 = 7 \text{ m}^3/\text{s}$$

and quantity of air passing through the duct *CD*,

$$Q_{CD} = 4 \text{ m}^3/\text{s}$$

We know that pressure loss due to friction per metre length of duct,

$$\frac{p_f}{L} = \frac{2.268 \times 10^{-3} (Q)^{1.852}}{(D)^{4.973}} = 0.08 \text{ mm of water} \qquad \text{...(Given)}$$

$$\therefore \qquad D = \left[\frac{2.268 \times 10^{-3} (Q)^{1.852}}{0.08}\right]^{1/4.973} = 0.488\ 46\ (Q)^{0.3724}$$

The diameter and velocity pressure in lengths *AB, BC* and *CD* are calculated in the following table :

Table 20.10

Section	*Length (L)* m	*Quantity of air (Q)* m^3/s	*Diameter (D)* $0.488\ 46(Q)^{0.3724}$ m	*Cross-sectional area (A)* $\left[\frac{\pi}{4} D^2\right]$ m^2	*Velocity* $V = \frac{Q}{A}$ m / s	*Velocity pressure* $p_v = \left(\frac{V}{4.04}\right)^2$ mm of water
AB	30	8	1.0596	0.8819	9.071	5.041
BC	15	7	1.0082	0.7984	8.767	4.709
CD	75	4	0.8185	0.5262	7.602	3.541

From the above table, we find that diameter of duct *AB*,

$$D_{AB} = 1.0596 \text{ m} \quad \textbf{Ans.}$$

Diameter of duct *BC*, D_{BC} = 1.0082 m **Ans.**

and diameter of duct *CD*, D_{CD} = 0.8185 m **Ans.**

2. Total and static pressure at point A

We know that pressure loss due to friction per metre length of duct,

$$\frac{p_f}{L} = 0.08 \text{ mm of water}$$

∴ Total pressure loss due to friction in the duct *AB*,

$$p_{f\,(AB)} = 0.08 \times L_{AB} = 0.08 \times 30 = 2.4 \text{ mm of water}$$

Total pressure loss due to friction in the duct *BC*,

$$p_{f(BC)} = 0.08 \times L_{BC} = 0.08 \times 15 = 1.2 \text{ mm of water}$$

Total pressure loss due to friction in the duct *CD*,

$$p_{f(CD)} = 0.08 \times L_{CD} = 0.08 \times 75 = 6 \text{ mm of water}$$

and total pressure loss due to friction from *A* to *D*,

$$p_f = p_{f\,(AB)} + p_{f\,(BC)} + p_{f\,(CD)}$$
$$= 2.4 + 1.2 + 6 = 9.6 \text{ mm of water}$$

The dynamic losses between *A* and *D* are as follows :

(*a*) Loss in discharge opening $= p_{v\,(CD)} = 3.541$ mm of water

(*b*) Elbow loss $= 0.25\, p_{v\,(CD)} = 0.25 \times 3.541 = 0.885$ mm of water

(*c*) Fitting loss at *B* $= 0.25\,[\,p_{v(AB)} - p_{v(BC)}\,]$

$= 0.25\,(5.041 - 4.709) = 0.083$ mm of water

and fitting loss at *C* $= 0.25\,[\,p_{v(BC)} - p_{v(CD)}\,]$

$= 0.25\,[4.709 - 3.541] = 0.292$ mm of water

∴ Total dynamic loss, $p_d = 3.541 + 0.885 + 0.083 + 0.292 = 4.801$ mm of water

We know that total friction and dynamic losses between *A* and *D*

= Total pressure at fan exit at *A*

$$p_T = p_{TA} = p_f + p_d = 9.6 + 4.801 = 14.401 \text{ mm of water}$$ **Ans.**

and static pressure at *A*, $p_{sA} = p_{TA} - p_{v\,(AB)} = 14.401 - 5.041 = 9.36$ mm of water **Ans.**

3. Diameter of BE so that no dampering is required

Let D_{BE} = Diameter of branch *BE*.

We know that total pressure at *B*,

$$p_{TB} = p_{TA} - p_{f(AB)} = 14.401 - 2.4 = 12.001 \text{ mm of water}$$

and the total pressure loss in the branch *BE*,

$$p_{T\,(BE)} = 0.25\, p_v + 0.2\, p_v + p_v + p_{f\,(BE)}$$
$$= 1.45\, p_v + p_{f(BE)}$$

The first term on the right hand side represents the sum of dynamic losses in the elbow at *B*, the area change in the branch and the velocity pressure at discharge. Equating it to the total pressure at *B* (p_{TB}) for complete balancing, we have

$$1.45\, p_v + p_{f\,(BE)} = p_{TB} = 12.001$$

This equation can only be solved by trial and error.

Asuming $D_{BE} = 0.4$ m

Pressure loss due to friction in the branch BE,

$$p_{f\,(BE)} = \frac{2.268\times10^{-3}(Q_{BE})^{1.852}}{(D_{BE})^{4.973}}\times L_{BE}$$

$$= \frac{2.268\times10^{-3}(1)^{1.852}}{(0.4)^{4.973}}\times30 = 6.48 \text{ mm of water}$$

and velocity of air in the branch BE,

$$V_{BE} = \frac{Q_{BE}}{A_{BE}} = \frac{4Q_{BE}}{\pi\,(D_{BE})^2} = \frac{4\times1}{\pi\,(0.4)^2} = 7.957 \text{ m / s}$$

∴ Velocity pressure in the branch BE,

$$p_{v(BE)} = \left(\frac{V_{BE}}{4.04}\right)^2 = \left(\frac{7.957}{4.04}\right)^2 = 3.88 \text{ mm of water}$$

We know that total pressure loss in the branch BE,

$$p_{T(BE)} = 1.45\,p_v + p_{f(BE)}$$

$$= 1.45 \times 3.88 + 6.48 = 12.106 \text{ mm of water}$$

... $[\because p_v = p_{v(BE)}]$

This is approximately equal to the total pressure at B (p_{TB}). Hence an assumed diameter for branch BE (*i.e.* D_{BE}) of 0.4 m is satisfactory. **Ans.**

20.21 System Resistance

A single duct-line having lengths of different cross-sections, bends, expanders, reducers, dampers and registers etc. constitutes a system of resistances in series, in which all the pressure losses occurring at a given volumetric flow rate are added together to give the system resistance or total pressure loss at the given volumetric flow rate.

We know that the pressure loss (frictional or dynamic) is directly proportional to the velocity pressure, therefore total pressure loss in a duct system,

$$p_L = C_1 p_{v1} + C_2 p_{v2} + \ldots \qquad \ldots (i)$$

We also know that $p_{v1} \propto (V_1)^2$, $p_{v2} \propto (V_2)^2$, and so on. Thus, the equation (*i*) may be written as

$$p_L = C_1 (V_1)^2 + C_2 (V_2)^2 + \ldots \qquad \ldots (ii)$$

But for a system in series, the volumetric flow rate (Q) is same. Therefore from the continuity equation,

$$Q = A_1 V_1 = A_2 V_2 = \ldots$$

i.e. $V_1 \propto Q$, $V_2 \propto Q$ and so on.

∴ Equation (*ii*) may be written as

$$p_L = K_1 Q^2 + K_2 Q^2 + \ldots$$

$$= (K_1 + K_2 + \ldots)\, Q^2 = KQ^2$$

This expression represents the relationship between the pressure loss (or resistance) in the system and the volume flow rate. Thus, system resistance,

$$R = KQ^2$$

where K is a constant of the system.

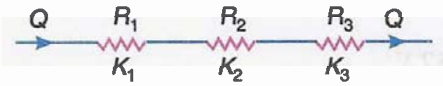
20.22 Systems in Series

If a number of systems R_1, R_2, R_3 etc. having constants K_1, K_2, K_3 etc. are connected in series, as shown in Fig. 20.25(*a*), it can be reduced to a single equivalent system as shown in Fig. 20.25(*b*). The resistance of a single equivalent system or the overall system resistance for the given flow rate is obtained by adding the individual system resistances, *i.e.*

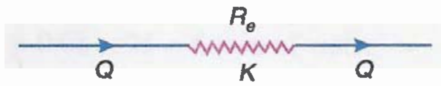
$$R_e = R_1 + R_2 + R_3$$

or
$$K\,Q^2 = K_1 Q^2 + K_2 Q^2 + K_3 Q^2$$

and
$$K = K_1 + K_2 + K_3$$



(*a*) Systems in series.

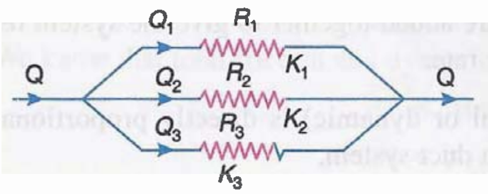


(*b*) Single equivalent system.

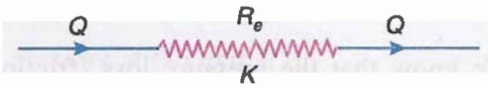
Fig. 20.25

20.23 Systems in Parallel

If a number of systems R_1, R_2, R_3 etc. having constants K_1, K_2, K_3 etc. are connected in parallel, as shown in Fig. 20.26 (*a*), it can be reduced to a single equivalent system as shown in Fig. 20.26 (*b*). The resistance of a single equivalent system or the overall system resistance for the given flow rate may be obtained by calculating the constant K for the equivalent system from the constants K_1, K_2, K_3 etc. of the parallel system.



(*a*) Systems in parallel.



(*b*) Single equivalent system.

Fig. 20.26

We know that
$$Q = Q_1 + Q_2 + Q_3$$

$$\sqrt{\frac{R_e}{K}} = \sqrt{\frac{R_1}{K_1}} + \sqrt{\frac{R_2}{K_2}} + \sqrt{\frac{R_3}{K_3}}$$

For the systems in parallel, the pressure loss is same, *i.e.*

$$R_e = R_1 = R_2 = R_3$$

$\therefore$
$$\sqrt{\frac{1}{K}} = \sqrt{\frac{1}{K_1}} + \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}}$$

Example 20.18. *A duct system is represented diagramatically as shown in Fig. 20.27, where*

R_1 = 60 mm of water at 180 m^3/min,

R_2 = 18 mm of water at 60 m³/min, and

R_3 = 30 mm of water at 75 m³/min.

Calculate the constant of an equivalent resistance of the above system and hence the pressure loss if the volume flow rate through R_1 is 120 m³/min.

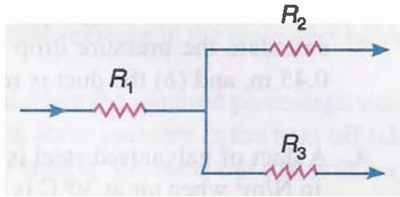


Fig. 20.27

Solution. Given : R_1 = 60 mm of water ; Q_1 = 180 m³/min; R_2 = 18 mm of water ; Q_2 = 60 m³/min ; R_3 = 30 mm of water ; Q_3 = 75 m³/min ; Q_1' = 120 m³/min

We know that the constant for the resistance R_1,

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{60}{(180)^2} = 1.85 \times 10^{-3}$$

Now, let us find the constant for the single equivalent resistance (R_e) for the parallel resistances R_2 and R_3. We know that the constant for the resistance R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{18}{(60)^2} = 5 \times 10^{-3}$$

Similarly, constant for the resistance R_3,

$$K_3 = \frac{R_3}{(Q_3)^2} = \frac{30}{(75)^2} = 5.3 \times 10^{-3}$$

Let K = Constant for the single equivalent resistance R_e.

$$\therefore \quad \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{K_2}} + \frac{1}{\sqrt{K_3}} = \frac{1}{\sqrt{5 \times 10^{-3}}} + \frac{1}{\sqrt{5.3 \times 10^{-3}}}$$

$$= 14.14 + 13.7 = 27.84$$

or

$$K = \frac{1}{(27.84)^2} = 1.3 \times 10^{-3}$$

Since the resistance R_1 of constant K_1 is connected with single equivalent resistance R_e of constant K in series, therefore constant for the whole system,

$$K' = K_1 + K$$

$$= 1.85 \times 10^{-3} + 1.3 \times 10^{-3} = 3.15 \times 10^{-3}$$

and the equivalent system resistance,

$$R_{e'} = K' (Q_{1'})^2 = 3.15 \times 10^{-3} (Q_{1'})^2 \text{ Ans.}$$

Since the system resistance means the pressure loss, therefore pressure loss for a volume flow rate of Q_1' = 120 m³/min

$$= 3.15 \times 10^{-3} (120)^2 = 45.36 \text{ mm of water Ans.}$$

EXERCISES

1. A main duct 0.6 m by 0.6 m carries air at the rate of 280 m³/min. It is divided into two ducts, one being 0.6 m by 0.3 m and the other is 0.6 m by 0.45 m. If the mean velocity in the larger branch is 9 m/s, calculate the mean velocity in the main duct and the other branch. Also find the mean velocity pressure in each duct assuming standard air density.
   [Ans. 12.96 m/s ; 12.4 m/s ; 10.3 mm of water ; 9.4 mm of water ; 4.96 mm of water]
2. A grill has 15 m³/min of air passing through it and a duct connecting to it has 0.1 m² cross-sectional area. The static pressure behind the grill is 2.5 mm of water. Find the effective area of the grill in square metres.
   [Ans. 0.036 m²]

3. A duct of 25 m length passes air at the rate of 120 m^3/min. Assuming friction factor as 0.006, calculate the pressure drop in the duct in mm of water when (*a*) the duct is circular of diameter 0.45 m, and (*b*) the duct is rectangular of sides 0.6 m and 0.45 m.

[**Ans.** 13 mm of water ; 3.93 mm of water]

4. A duct of galvanised steel is of 300 mm diameter and 15 m long. Find the pressure drop in the duct in N/m^2 when air at 30°C is flowing with a mean velocity of 480 m/min. Use Moody chart to find the value of friction factor '*f*'. [**Ans.** 33.4 N/m^2]

5. A duct of 0.45 m diameter and 90 m long leads from a fan discharge chamber where the pressure is 15 mm of water to a plenum chamber where the pressure is 10 mm of water.

   In order to increase the flow, two alternatives are considered. One is to lay a duct of 0.3 m diameter and 90 m long in parallel with the duct of 0.45 m diameter. The other is to increase the diameter of 0.45 m diameter duct for the last 60 m length. Calculate the increased diameter so that this method gives the same flow as the 0.45 m and 0.3 m ducts in parallel. Assume that the pressures in the fan chamber and plenum chamber are unaffected by changes in the flow and consider duct friction losses only. The friction factor may be taken as 0.0055. [**Ans.** 0.567 m]

6. A 0.3 m diameter circular duct carries standard air at a velocity of 360 m/min. It is replaced by a rectangular duct having the same pressure loss per unit length due to friction. Determine the dimensions of the rectangular duct if the aspect ratio is to be 1.5 for (*a*) the same volume flow through the two ducts, and (*b*) the same velocity in the two ducts.

[**Ans.** 0.336 m , 0.224 m ; 0.375 m, 0.25 m]

7. A fan delivers air at 8 m/s to the main line of air conditioning duct. After a straight run of 10 m, a branch of 4 m length delivers air to a room with 2 m^3/s. Another branch after a 5 m further run is taken from the main branch. This branch is 5 m in length and delivers 3 m^3/s. The main branch runs straight for another 5 m and then turns by 90° and runs for another 5 m length and delivers 3 m^3/s of air. The losses are given as

   For elbow : 0.25 p_v where $p_v = \left(\dfrac{V}{4.04}\right)^2$ mm of water.

   For branch : 0.2 p_v + Elbow loss

   For straight through section : 0.25 × Difference of velocity pressure

   Fitting losses where change in area occurs.

   Find the size of all the ducts and determine the static pressure requirement. Use the following formula for friction rate :

$$\frac{p_f}{L} = \frac{0.002\,268\,(Q)^{1.852}}{(D)^{4.973}}$$

   where Q is in m^3/s and D is in m.

   Find the friction rate in main line and then assume same friction rate for whole of duct work.

[**Ans.** 1.128 m , 1.01 m ; 0.783 m ; 0.905 mm of water ; 0.0586 mm of water / m length]

8. A main air duct of constant rectangular cross-section 2 m by 0.6 m has several branch pieces leading off it. One of these branches is shown in Fig. 20.28. The branch leaves the main at right angles and has a 90° bend in it of radius 0.6 m, as well as a damper for flow regulation and an exit grill. The branch is 0.6 m by 0.3 m and 3 m long. The flow through the main duct before this off take is 750 m^3/min and the flow required in the branch is 90 m^3/min. Calculate the

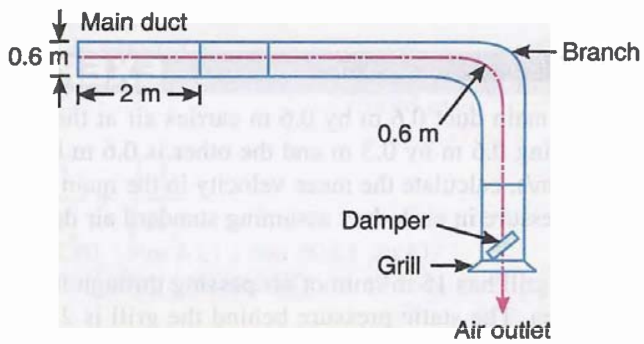


Fig. 20.28

damper resistance necessary to give the required flow when the static pressure in the main duct before off-take is 10 mm of water.

If the branch pieces are placed 12 m apart along the main duct, calculate the required percentage static regain in the main duct across the off take shown in order that the static pressure in the next off take will be 10 mm of water also. The dynamic loss coefficient for the branch, 90° bend and exit grill may be taken as 0.65, 0.3 and 0.8 respectively. **[Ans.** 9.94 mm of water ; 66.2%]

9. An air duct system is provided as shown in Fig. 20.29. The standard air enters at point *A* with a static pressure of 12 mm of water.

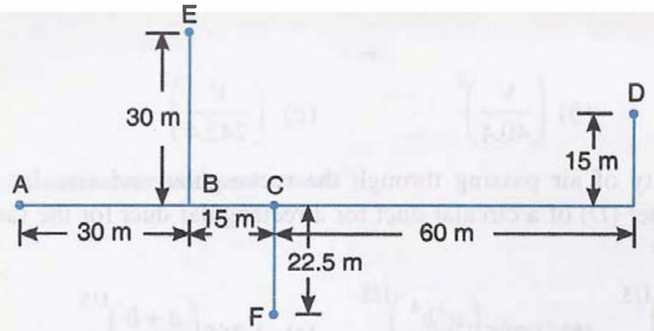


Fig. 20.29

The branch *BE* delivers 60 m^3/min at *E*, branch *CF* delivers 180 m^3/min at *F* and the main duct delivers 240 m^3/min at *D*.

Using equal pressure drop method, find the duct dimensions. Assume free exit at each outlet.

[Ans. D_{AB} = 1 m ; D_{BC} = 0.9 m ; D_{CD} = 0.75 m ; D_{BE} = 0.375 m ; D_{CF} = 0.55 m]

10. An air conditioning duct runs straight from fan over 60 m length. It has four equally spaced outlet diffusers mounted on duct, the last one being at the end of duct. The volume flow rate through each diffuser is 1 m^3/s. The velocity at duct inlet is 15 m/s. Carry out the duct design by static regain method if static regain factor is 0.75 at each transition and frictional pressure drop is given by

$$\frac{p_f}{L} = \frac{0.002\,268\,(Q)^{1.852}}{(D)^{4.973}} \text{ mm of water}$$

and $$p_v = \left(\frac{V}{4.04}\right)^2 \text{ mm of water}$$

where Q is in m^3/s and V is in m /s.

[Ans. D_1 = 0.583 m ; V_2 = 9.15 m / s, D_2 = 0.646 m ; V_3 = 5.82 m /s, D_3 = 0.661 m ; V_4 = 3.6 m / s, D_4 = 0.595 m]

[Hint : Solve this question by using the procedure given in Example 20.10]

QUESTIONS

1. Why the ducts are used in an air conditioning system ?
2. Which material is commonly used for making ducts in the air conditioning systems?
3. What do you understand by static and velocity pressure in a duct ?
4. Derive an expression for the equivalent diameter of circular duct corresponding to a rectangular duct of sides *a* and *b*, for the same pressure loss per unit length, when (*i*) the quantity of air passing through both the ducts is same, and (*ii*) the velocity of air flowing through both the ducts is same. The friction factor remains the same for both the ducts.
5. Describe the different methods of air conditioning duct design. Why are dampers required in some systems ?

OBJECTIVE TYPE QUESTIONS

1. A duct is said to be a low velocity duct if the velocity of air in the duct is up to
   (*a*) 600 m / min (*b*) 800 m / min (*c*) 1200 m / min (*d*) 1600 m / min
2. The duct is made of
   (*a*) galvanised iron (*b*) aluminium (*c*) fibreglass (*d*) any one of these
3. The fibreglass ducts may be used in .... velocity applications.
   (*a*) high (*b*) low
4. If the velocity of air flowing through the duct is V m / min., then the velocity pressure (p_v) in mm of water is given by
   (*a*) $\left(\frac{V}{4.04}\right)^2$ (*b*) $\left(\frac{V}{40.4}\right)^2$ (*c*) $\left(\frac{V}{242.4}\right)^2$ (*d*) $\left(\frac{V}{2424}\right)^2$
5. When the quantity of air passing through the rectangular and circular ducts is same, then the equivalent diameter (D) of a circular duct for a rectangular duct for the same pressure loss per unit length is given by
   (*a*) $1.265\left(\frac{a^3b^3}{a+b}\right)^{1/5}$ (*b*) $1.265\left(\frac{a^4b^4}{a+b}\right)^{1/5}$ (*c*) $1.265\left(\frac{a+b}{a^3b^3}\right)^{1/5}$ (*d*) $1.265\left(\frac{a+b}{a^4b^4}\right)^{1/5}$

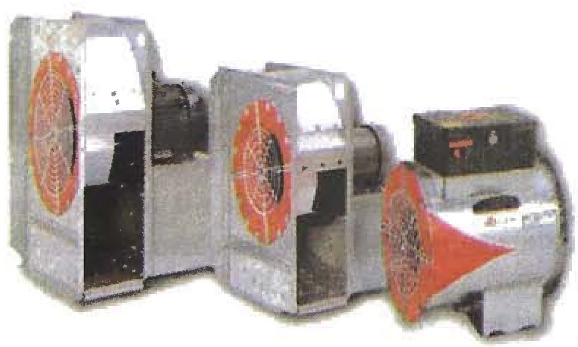
   where a and b = Longer and shorter sides of the rectangular duct respectively.
6. When the velocity of air passing through the rectangular and circular ducts is same, then the equivalent diameter (D) of a circular duct for a rectangular duct for the same pressure loss per unit length is given by
   (*a*) $\frac{a+b}{ab}$ (*b*) $\frac{2ab}{a+b}$ (*c*) $\frac{2a}{a-b}$ (*d*) $\frac{2b}{a+b}$
7. For rectangular ducts, the aspect ratio is equal to
   (*a*) sum of longer and shorter sides (*b*) difference of longer and shorter sides
   (*c*) product of longer and shorter sides (*d*) ratio of longer and shorter sides
8. The aspect ratio, for rectangular ducts, should not be greater than .... in any case.
   (*a*) 8 (*b*) 10 (*c*) 12 (*d*) 16
9. In designing ducts, the equal friction method is ideal
   (*a*) only for return ducts (*b*) when the system is balanced
   (*c*) when the system is not balanced (*d*) none of these
10. The static regain method of designing the ducts as compared to equal friction method
    (*a*) increases balancing problems
    (*b*) increases the cost of sheet metal for the duct
    (*c*) decreases the cost of sheet metal for the duct
    (*d*) none of the above

ANSWERS

1. (*a*)	2. (*d*)	3. (*b*)	4. (*c*)	5. (*a*)
6. (*b*)	7. (*d*)	8. (*a*)	9. (*b*)	10. (*b*)

CHAPTER

21 Fans



21.1 Introduction

A fan is a kind of pump which is used for pumping or circulating the air through the entire duct system and the conditioned space. It is usually located at the inlet of the air conditioner. A fan, essentially, consists of a rotating wheel (called impeller) which is surrounded by a stationary member known as housing. The energy is transmited to the air by the power driven wheel and a pressure difference is created to provide flow of air. The air may be moved by either creating an above-atmospheric pressure (*i.e.* positive pressure) or a below-atmospheric pressure (*i.e.* negative pressure). All fans produce both the conditions. The air at inlet to the fan is below atmospheric pressure while at the exhaust or outlet of the fan is above atmospheric pressure. The air feed into a fan is called *induced draft* while the air exhaust from a fan is called *forced draft.*

The fans, irrespective of their type of construction, may function as either blowers or exhausters. The blowers discharge air against a pressure at their outlet whereas exhausters remove gases from a space by suction.

21.2 Types of Fans

The following two types of fans may be used for the transmission of air :

1. Centrifugal or radial flow fans, and 2. Axial flow fans.

When the air enters the impeller axially and is discharged radially from the impeller, it is called a *centrifugal* or *radial flow fan.*

When the air flows parallel to the axis of impeller, it is called an *axial flow fan.*

21.3 Centrifugal Fans

The centrifugal fans are widely used for duct air conditioning system, because they can efficiently move large or small quantities of air over a greater range of operating pressures. All centrifugal fans have an impeller or wheel mounted in a scroll type of housing, as shown in Fig. 21.1. The impeller is turned either by the direct drive or more frequently by an electric motor employing pulleys and belt. The centrifugal force created by the rotating impeller moves the air outward along the blade channels. The outward moving air streams are combined by the scroll into a single large air stream. This air stream leaves the fan through the discharge outlet.

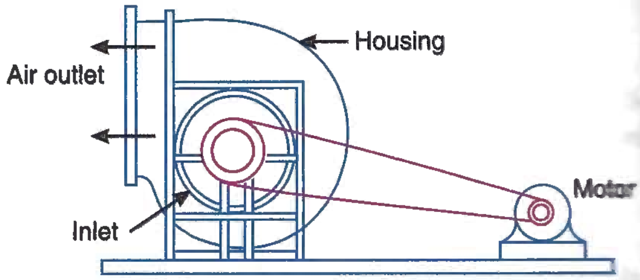
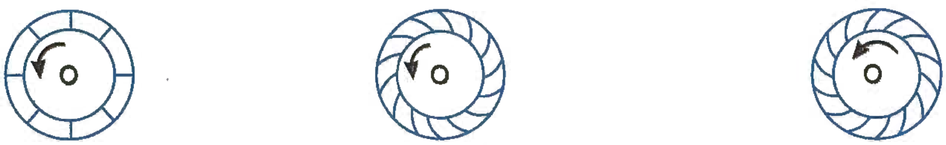


Fig. 21.1. Centrifugal fan.

The fan impeller may have the following three types of blades :

1. Radial or straight blades, 2. Forward curved blades, and 3. Backward curved blades.

The centrifugal fans with *radial blades,* as shown in Fig. 21.2 (*a*), have simple impeller construction. The blades run straight out from a central hub. Some fans of this type have heavy steel blades with high structural strength. These fans provide very high pressure at high speeds.



(*a*) Radial. (*b*) Forward curved. (*c*) Backward curved.

Fig. 21.2. Three types of fan impeller blades.

A large number of centrifugal fans installed in air conditioning systems have impellers with *forward curved blades,* as shown in Fig. 21.2 (*b*). Since the blades are very shallow in depth, therefore the diameter of the housing air-inlet opening more nearly approaches to that of the impeller. The ample inlet opening, together with stream-lined hub of the wheel, promotes a smooth flow of air into the rotating blades. This increases the efficiency of the fan and reduces its noise. The forward curved blades are more capable of overcoming the attached duct system resistance when their operation is at low speeds.

The centrifugal fan impeller may have *backward curved blades,* as shown in Fig. 21.2 (*c*). The backward curved blades must be operated at a much higher speed of rotation than the forward curved blades, if the same static pressure is to be produced in each case. In some cases, the higher speed may be an advantage because of a possible direct connection to the driving motor. The fan impellers having backward curved blades operate at high efficiency and have no overloading power characteristic. They also offer the advantage of wide ranges of capacity at constant speed with small changes in the power requirements.

Radial centrifugal fans.

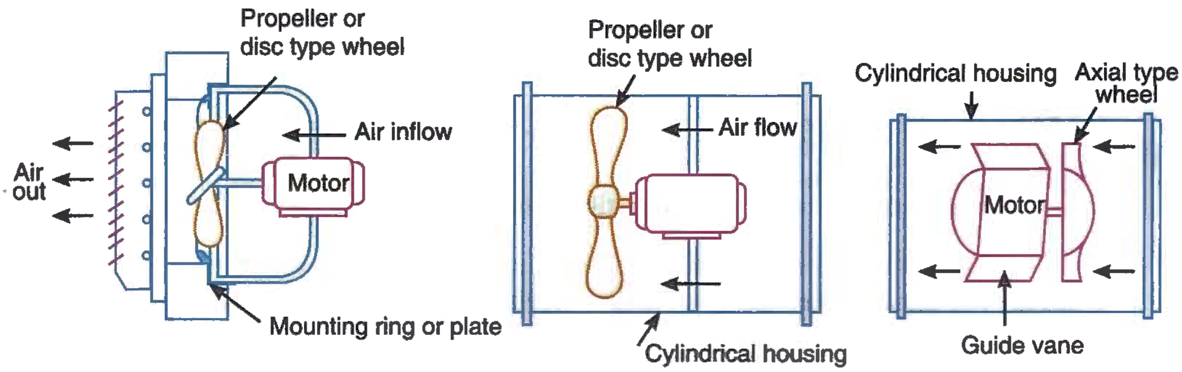
Centrifugal blowers.

Note : The number of impeller blades varies in centrifugal fans. The radial blade impellers seldom have more than 8 or 10 blades. The forward curved impellers usually have 24 to 64 blades whereas the backward curved impeller usually have 10 to 16 blades.

21.4 Axial Flow Fans

The axial flow fans are divided into the following three groups :

1. ***Propeller fan.*** A propeller type of axial flow fan consists of a propeller or disc type wheel which operates within a mounting ring as shown in Fig. 21.3 (*a*). The design of the ring surrounding the wheel is important because it prevents the air discharged from being drawn backward into the wheel around its periphery. The propeller fans are used only when the resistance to air movement is small. They are useful for the ventilation of attic spaces, lavatories and bathrooms, removal of cooking odours from kitchens and many other applications where little or no duct work is involved.



(*a*) Propeller fan. (*b*) Tube axial fan. (*c*) Vane axial fan.

Fig. 21.3. Types of axial flow fans.

2. ***Tube axial fan.*** A tube axial fan, consists of a propeller wheel housed in a simple cylinder as shown in Fig. 21.3 (*b*). The wheel may be driven either from an electric motor within the cylinder directly connected to its shaft or may be driven through a belt arrangement from a motor mounted outside the housing. These fans are easily installed in round ducts. They are more efficient than propeller fans. The air discharge from tube axial fan follows a spiral path as it leaves the cylindrical housing.

3. ***Vane axial fan.*** A vane axial fan combines a tube axial fan wheel mounted in a cylinder with a set of air guide vanes, as shown in Fig. 21.3 (*c*). This fan eliminates spiral flow of the

discharge air and reduces the turbulence of flow. The efficiency of operation and the pressure characteristics are better than those of tube axial fan. The straight line flow leaving the fan assures quiet operation.

Note : The axial flow fans are never used for duct air conditioning system because they are incapable of developing high pressures. These fans are particularly suitable for handling large volumes of air at relatively low pressures.

Axial flow fans.

21.5 Total Pressure Developed by a Fan

We have already discussed the static pressure, velocity pressure and total pressure of air in ducts. In case of a fan, the *fan static pressure* (p_{SF}) is the pressure increase produced by a fan. The *fan velocity pressure* (p_{vF}) is the velocity pressure corresponding to the mean velocity of air at the fan outlet based on the total outlet area without any deductions for motors, fairings, or other bodies. The total pressure created by a fan or the *fan total pressure* (p_{TF}) is the algebraic difference between the total pressure at the fan outlet and the total pressure at the fan inlet. Mathematically, fan total pressure,

$$p_{TF} = p_{T2} - p_{T1}$$

where p_{T2} = Total pressure at fan outlet

= Static pressure at fan outlet + Velocity pressure at fan outlet

$= p_{S2} + p_{v2}$, and

p_{T1} = Total pressure at fan inlet

= Static pressure at fan inlet + Velocity pressure at fan inlet

$= p_{S1} + p_{v1}$

We know that the total pressure at a point is the sum of static pressure and velocity pressure at that point. Thus, for a fan,

Fan total pressure = Fan static pressure + Fan velocity pressure

i.e. $$p_{TF} = p_{SF} + p_{vF}$$

Since the fan velocity pressure (p_{vF}) is the velocity pressure at the fan outlet (p_{v2}), therefore

$$p_{TF} = p_{SF} + p_{v2}$$

Notes : 1. If the fan has no suction duct, the entry losses to the fan housing are considered as part of the fan losses and are reflected in the mechanical efficiency of the fan.

In an actual system, the fan has a suction duct and apparatus such as filters and coils. In such a system, the total pressure at the fan inlet is always equal to the total frictional resistance in that part of the system. Also, the total pressure at the fan inlet in such a system is always *negative* and it is numerically less than the static pressure at the fan inlet.

2. If the fan has no discharge duct (*i.e.* the fan delivers air directly into a free open space), the discharge static pressure is zero (*i.e.* $p_{s2} = 0$). Thus the total pressure at the fan outlet is equal to the velocity pressure (*i.e.* $p_{T2} = p_{v2}$). In an actual system, the fan has a discharge duct. In such a system, the total pressure at the fan outlet is equal to the velocity pressure at the point of discharge plus all pressure losses in the path taken by air to reach that point.

21.6 Fan Air Power

The power output of a fan is expressed in terms of air power and represents the work done by the fan. Mathematically, total fan air power (based on fan total pressure, p_{TF}),

$$P_{at} = \frac{9.81\,Q \times p_{TF} \times K_P}{60} \text{ (in watts)}$$

where Q = Total quantity of air flowing at the fan inlet in m^3/min,

p_{TF} = Fan total pressure in mm of water, and

K_P = Compressibility coefficient.

Similarly, static fan air power based on the fan static pressure (p_{SF}) ,

$$P_{as} = \frac{9.81\,Q \times p_{SF} \times K_P}{60} \text{ (in watts)}$$

Note : If Q is expressed in m^3/s and p_{TF} and p_{SF} are in N/m^2, then total fan air power (in watts),

$$P_{at} = Q \times p_{TF} \times K_P$$

and static fan air power, $P_{as} = Q \times p_{SF} \times K_P$

21.7 Fan Efficiencies

The ratio of the total fan air power to the driving power (or brake power) required at the fan shaft is known as *total fan efficiency*. It is also called *mechanical efficiency* of the fan. Mathematically, total fan efficiency,

$$\eta_{TF} = \frac{\text{Total fan air power } (P_{at})}{\text{Input or brake power } (B.P.)}$$

Similarly, static fan efficiency,

$$\eta_{SF} = \frac{\text{Static fan air power } (P_{as})}{\text{Input or brake power } (B.P.)}$$

Example 21.1. *A centrifugal fan has a circular inlet duct of 0.45 m diameter and a rectangular outlet duct of 0.45 m by 0.375 m. The static pressure at the fan inlet is –12.5 mm of water and the static pressure at the fan outlet is 25 mm of water when the fan delivers 115 m^3/min and absorbs 1 kW.*

Assuming standard air density in both ducts and compressibility factor as 1, determine (a) total pressure at fan inlet and outlet, (b) fan total pressure and fan static pressure, and (c) fan efficiency and fan static efficiency.

Solution. Given : D = 0.45 m ; a = 0.45 m ; b = 0.375 m ; p_{S1} = – 12.5 mm of water ; p_{S2} = 25 mm of water ; Q = 115 m^3/min ; $B.P.$ = 1 kW ; K_P = 1

(a) Total pressure at fan inlet and outlet

Let p_{T1} = Total pressure at fan inlet, and

p_{T2} = Total pressure at fan outlet.

Cross-sectional area of circular inlet duct,

$$A_1 = \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(0.45)^2 = 0.16 \text{ m}^2$$

∴ Velocity of air in the inlet duct,

$$V_1 = \frac{Q}{A_1} = \frac{115}{0.16} = 718.7 \text{ m}^3\text{/min} = 11.98 \text{ m}^3\text{/s}$$

We know that velocity pressure in the inlet duct,

$$p_{v1} = \left(\frac{V_1}{4.04}\right)^2 = \left(\frac{11.98}{4.04}\right)^2 = 8.8 \text{ mm of water}$$

∴ Total pressure at fan inlet,

$$p_{T1} = p_{S1} + p_{v1} = -12.5 + 8.8 = -3.7 \text{ mm of water} \quad \textbf{Ans.}$$

Cross-sectional area of rectangular outlet duct,

$$A_2 = a \times b = 0.45 \times 0.375 = 0.17 \text{ m}^2$$

∴ Velocity of air in the outlet duct,

$$V_2 = \frac{Q}{A_2} = \frac{115}{0.17} = 676.5 \text{ m}^3\text{/min} = 11.3 \text{ m}^3\text{/s}$$

We know that velocity pressure in the outlet duct,

$$p_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{11.3}{4.04}\right)^2 = 7.8 \text{ mm of water}$$

∴ Total pressure at fan outlet,

$$p_{T2} = p_{S2} + p_{v2} = 25 + 7.8 = 32.8 \text{ mm of water} \quad \textbf{Ans.}$$

(b) Fan total pressure and fan static pressure

We know that fan total pressure,

$$p_{TF} = p_{T2} - p_{T1} = 32.8 - (-3.7) = 36.5 \text{ mm of water} \quad \textbf{Ans.}$$

and fan static pressure,

$$p_{SF} = p_{TF} - p_{v2} = 36.5 - 7.8 = 28.7 \text{ mm of water} \quad \textbf{Ans.}$$

(c) Fan total efficiency and fan static efficiency

We know that fan air power,

$$P_{at} = \frac{9.81\,Q \times p_{TF} \times K_P}{60} = \frac{9.81 \times 115 \times 36.5 \times 1}{60} \text{ W}$$

$$= 686.3 \text{ W} = 0.6863 \text{ kW}$$

and static fan air power,

$$P_{as} = \frac{9.81\,Q \times p_{SF} \times K_P}{60} = \frac{9.81 \times 115 \times 28.7 \times 1}{60} \text{ W}$$

$$= 539.6 \text{ W} = 0.5396 \text{ kW}$$

∴ Fan total effciency,

$$\eta_{TF} = \frac{P_{at}}{B.P.} = \frac{0.6863}{1} = 0.6863 \text{ or } 68.63\% \quad \textbf{Ans.}$$

and fan static efficiency,

$$\eta_{SF} = \frac{P_{as}}{B.P.} = \frac{0.5396}{1} = 0.5396 \text{ or } 53.96\% \quad \textbf{Ans.}$$

21.8 Fan Performance Curves

A fan performance curve is a graph of a fan's volume rate plotted against pressure, power, or efficiency. The performance curves for the various types of fans are shown in Fig. 21.4 to Fig. 21.7.

In all the figures, the abscissas represent the range of air flow capacity expressed as a percentage of the amount of air delivered when the fan is discharging freely into an open space. The ordinates represent the percentages of efficiency, power at free delivery and static pressure with outlet closed.

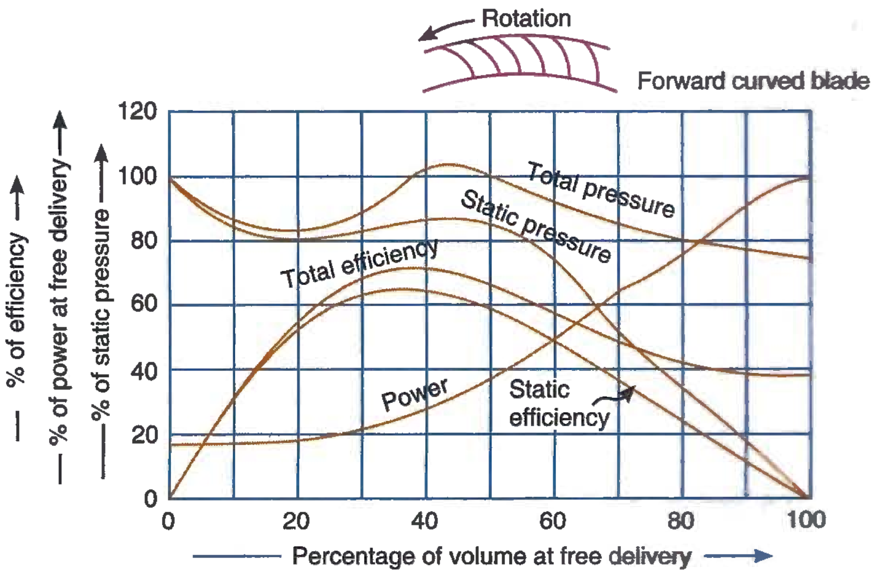


Fig. 21.4. Performance curves for a centrifugal fan with forward curved blades.

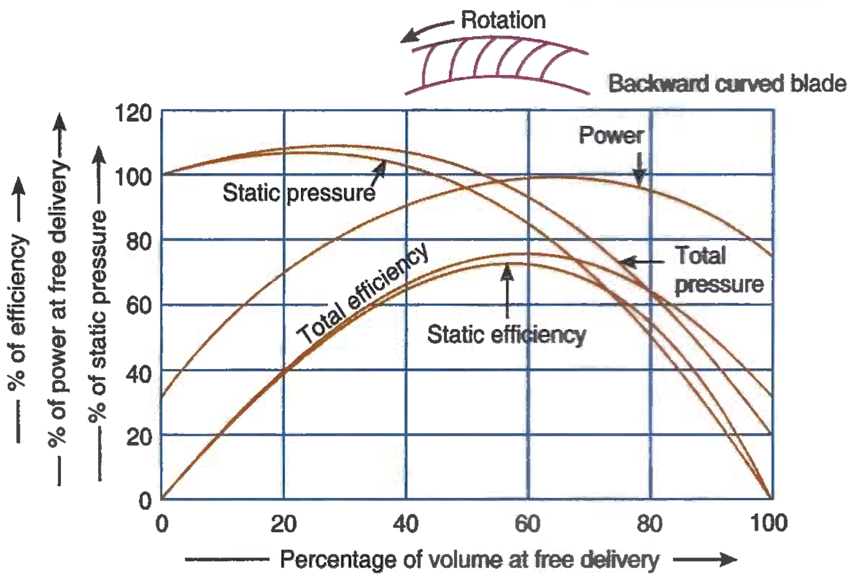


Fig. 21.5. Performance curves for a centrifugal fan with backward curved blades.

From Fig. 21.4, we see that the centrifugal fans with forward curved blades require an ever-increasing amount of power as the air volume is increased. However, this type of centrifugal fan provides greater static pressure for a given blade-tip velocity than the other types and it is commonly used in air conditioning systems inspite of this disadvantage.

From Fig. 21.5, we see that centrifugal fans with backward curved blades require maximum power. The operating condition which requires the maximum power is close to the combination of volume and static pressure under which the fan operates most efficiently. The fans of this type are said to have non- overloading power characteristic which means that the driving motor cannot be overloaded if the fan and motor are properly selected.

The axial flow fans, as shown in Fig. 21.7, also have non over-loading power characteristics.

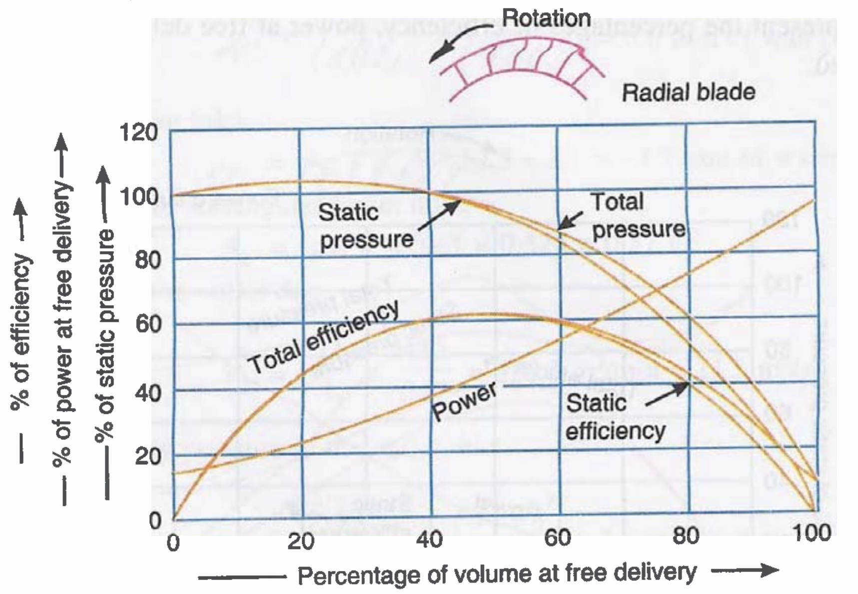


Fig. 21.6. Performance curves for a centrifugal fan with radial blades.

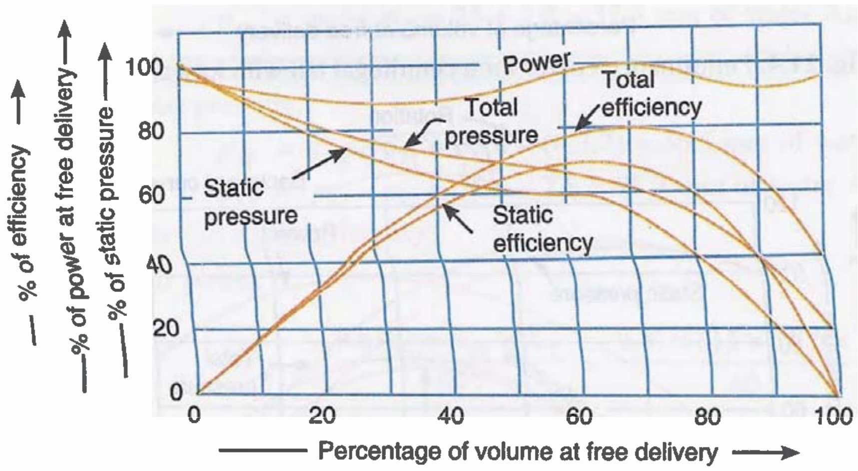


Fig. 21.7. Performance curves for an axial flow air foil-type fan .

21.9 Velocity Triangles for Moving Blades of a Centrifugal Fan

Consider a stream of air entering the backward curved blade at C and leaving it at D, as shown in Fig. 21.8 (*a*). The velocity triangles at the inlet and outlet tips of the blade are shown in Fig. 21.8 (*b*).

Let V_{b1} = Linear or tangential velocity of the moving blade at inlet (BA).

V_1 = Absolute velocity of air entering the blade ($A'C$).

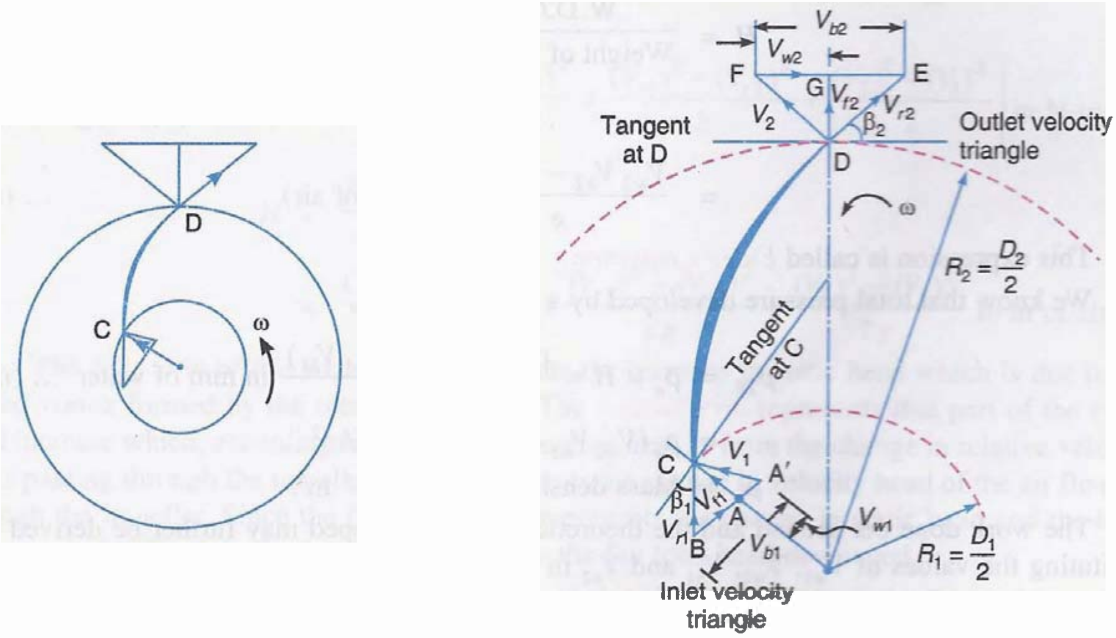
V_{f1} = Velocity of flow at inlet (AC). It is the radial component of V_1.

V_{r1} = Relative velocity of air to the moving blade at inlet (*BC*). It is vectorial difference between V_{b1} and V_1.

V_{w1} = Velocity of whirl at inlet. It is the tangential component of V_1.

β_1 = Blade angle at inlet. It is the angle which the relative velocity (V_{r1}) makes with the tangent at the blade inlet. It is equal to the angle between V_{r1} and V_{b1} (*i.e.* angle *CBA*).

$V_{b2}, V_2, V_{f2}, V_{r2}, V_{w2}, \beta_2$ = Corresponding values at outlet of the blade tip.



(*a*) (*b*)

Fig. 21.8. Velocity triangles for a centrifugal fan.

It may be seen from the above that the suffix 1 stands for the blade inlet and the suffix 2 stands for blade outlet. A little consideration will show that as the air enters and leaves the blades without any shock (or in other words tangentially), therefore the shape of the blades will be such that V_{r1} and V_{r2} are along the tangents to the blades at inlet and outlet respectively.

Let m = Mass of air flowing through the impeller in kg per second,

R_1 = Internal radius of the impeller = $D_1/2$

R_2 = External radius of the impeller = $D_2/2$, and

ω = Angular velocity of the impeller in radians per second

$$= \frac{V_{b1}}{R_1} \text{ or } \frac{V_{b2}}{R_2}$$

We know that the angular momentum entering the impeller per second

= Mass of air flowing per second × Velocity of whirl × Radius of impeller

$$= m\, V_{w1} R_1$$

Similarly, angular momentum leaving the impeller per second

$$= m\, V_{w2} R_2$$

According to Newton's second law of angular motion, the torque in the direction of motion of blades is equal to the rate of change of angular momentum.

∴ Torque in the direction of motion of blades

$$= mV_{w2}R_2 - mV_{w1}R_1 = m\,(V_{w2}R_2 - V_{w1}R_1)$$

and work done per second in the direction of motion of blades,

$$\text{W.D./second} = \text{Torque} \times \text{Angular velocity} = m\,(V_{w2}R_2 - V_{w1}R_1)\,\omega$$

Since $V_{b1} = \omega R_1$ and $V_{b2} = \omega R_2$, therefore

$$\text{W.D./second} = m\,(V_{w2}V_{b2} - V_{w1}V_{b1}) \qquad \ldots (i)$$

∴ Theoretical total head developed by a centrifugal fan,

$$H = \frac{\text{W.D./second}}{\text{Weight of air / second}} = \frac{m\,(V_{w2}V_{b2} - V_{w1}V_{b1})}{mg}$$

... (∵ Weight = *mg*)

$$= \frac{V_{w2}V_{b2} - V_{w1}V_{b1}}{g} \text{ (in m of air)} \qquad \ldots (ii)$$

This expression is called *Euler's equation.*

We know that total pressure developed by a centrifugal fan,

$$p_{TF} = \rho_a \times H = \frac{\rho_a(V_{w2}V_{b2} - V_{w1}V_{b1})}{g} \text{ in mm of water} \qquad \ldots (iii)$$

$$= \rho_a(V_{w2}V_{b2} - V_{w1}V_{b1}) \text{ in N/m}^2$$

where ρ_a = Mass density of air in kg / m³.

The work done per second and the theoretical head developed may further be derived by substituting the values of V_{w1}, V_{w2}, V_{b1} and V_{b2} in equation (*i*) and (*ii*) from the inlet and outlet velocity triangles. From the inlet velocity triangle, we find that

$$V_{b1} = AA' + AB = V_{w1} + V_{r1}\cos\beta_1$$

From the outlet velocity triangle,

$$V_{b2} = FG + GE = V_{w2} + V_{r2}\cos\beta_2$$

or

$$V_{w2} = V_{b2} - V_{r2}\cos\beta_2 \qquad \ldots (iv)$$

We know that

$$(V_2)^2 = (V_{b2})^2 + (V_{r2})^2 - 2(V_{b2})(V_{r2})\cos\beta_2$$

$$\therefore \quad \cos\beta_2 = \frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}\,V_{r2}}$$

Substituting the value of $\cos\beta_2$ in equation (*iv*), we get

$$V_{w2} = V_{b2} - V_{r2}\left[\frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}\,V_{r2}}\right]$$

$$= V_{b2} - \left[\frac{(V_{b2})^2 + (V_{r2})^2 - (V_2)^2}{2\,V_{b2}}\right]$$

$$= \frac{2\,(V_{b2})^2 - (V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2\,V_{b2}} = \frac{(V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2\,V_{b2}}$$

Similarly for the inlet velocity triangle,

$$V_{w1} = \frac{(V_{b1})^2 - (V_{r1})^2 + (V_1)^2}{2\,V_{b1}}$$

Substituting the value of V_{w1} and V_{w2} in equation (*i*), we get

$$\text{W.D./second} = m\left[\frac{(V_{b2})^2 - (V_{r2})^2 + (V_2)^2}{2\,V_{b2}} \times V_{b2} - \frac{(V_{b1})^2 - (V_{r1})^2 + (V_1)^2}{2\,V_{b1}} \times V_{b1}\right]$$

$$= m\left[\frac{(V_{b2})^2 - (V_{b1})^2}{2} + \frac{(V_{r1})^2 - (V_{r2})^2}{2} + \frac{(V_2)^2 - (V_1)^2}{2}\right] \text{in N-m or J}$$

and

$$H = \frac{\text{W.D / second}}{mg}$$

$$= \frac{(V_{b2})^2 - (V_{b1})^2}{2g} + \frac{(V_{r1})^2 - (V_{r2})^2}{2g} + \frac{(V_2)^2 - (V_1)^2}{2g} \text{ in m of air}$$

The *first term* of this expression represents the increase in static head which is due to the forced vortex formed by the rotating impeller. The *second term* represents that part of the static head increase which, according to Bernoulli's equation, results from the change in relative velocity of air passing through the impeller. The *third term* is the change in velocity head of the air flowing through the impeller. Since the first two terms represents the change in static head and the third term represents the velocity head, therefore H is the fan total head developed.

Note : If the speed of impeller is N r.p.m., then the blade velocity at inlet or outlet (V_{b1} or V_{b2}) may be obtained by the relations :

$$V_{b1} = \frac{\pi D_1 N}{60}, \quad \text{and} \quad V_{b2} = \frac{\pi D_2 N}{60}$$

where D_1 and D_2 are the internal and external diameters of the impeller respectively.

21.10 Work Done and Theoretical Total Head Developed by a Centrifugal Fan for Radial Entry of Air

We have seen in the previous article that the work done per second by a centrifugal fan,

$$\text{W.D./second} = m\,(V_{w2} \times V_{b2} - V_{w1} \times V_{b1}) \text{ in N-m or J}$$

and theoretical total head developed by a centrifugal fan,

$$H = \frac{1}{g}(V_{w2} \times V_{b2} - V_{w1} \times V_{b1}) \text{ in m of air}$$

When the air enters the blades at right angles (*i.e.* radially) to the direction of motion of the blade, then

$$V_1 = V_{f1} \quad \text{and} \quad V_{w1} = 0$$

∴ For radial entry of air, workdone per second,

$$\text{W.D./second} = m\,(V_{w2} \times V_{b2}) \text{ in N-m or J} \quad \text{... (i)}$$

and theoretical total head developed,

$$H = \frac{1}{g}(V_{w2} \times V_{b2}) \text{ in m of air} \quad \text{... (ii)}$$

Let Q = Quantity of air flowing through the fan in m³/s,

D_1 = Internal diameter of impeller in metres,

D_2 = External diameter of impeller in metres,

b_1 = Width of impeller at inlet in metres, and

b_2 = Width of impeller at outlet in metres.

Since the quantity of air flowing through the impeller is constant, therefore

$$Q = \pi D_1 b_1 V_{f1} = \pi D_2 b_2 V_{f2}$$

or

$$V_{f1} = \frac{Q}{\pi D_1 b_1} \text{ ; and } V_{f2} = \frac{Q}{\pi D_2 b_2}$$

Now from the outlet velocity triangle as shown in Fig. 21.8,

$$\cot \beta_2 = \frac{GE}{GD} = \frac{V_{b2} - V_{w2}}{V_{f2}} \quad \ldots (iii)$$

$$\therefore \quad V_{b2} - V_{w2} = V_{f2} \cot \beta_2 = \frac{Q \cot \beta_2}{\pi D_2 b_2}$$

and

$$V_{w2} = V_{b2} - \frac{Q \cot \beta_2}{\pi D_2 b_2}$$

Substituting the value of V_{w2} in equations (*i*) and (*ii*), we get

$$\text{W.D./second} = m \left[V_{b2} - \frac{Q \cot \beta_2}{\pi D_2 b_2} \right] V_{b2}$$

$$= m \left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2} \right] \text{ in N-m or J} \quad \ldots (iv)$$

and theoretical total head developed,

$$H = \frac{1}{g} \left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2} \right] \text{ in m of air} \quad \ldots (v)$$

Notes : 1. We know that total pressure developed by a centrifugal fan,

$$p_{TF} = \rho_a \times H = \frac{\rho_a \times V_{w2} \times V_{b2}}{g} \text{ in mm of water}$$

... [From equation (*ii*)]

$$= \rho_a \times V_{w2} \times V_{b2} \text{ in N/m}^2$$

$$= \frac{\rho_a}{g} \left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2} \right] \text{ in mm of water}$$

... [From equation (*v*)]

$$= \rho_a \left[(V_{b2})^2 - \frac{Q \cot \beta_2 \times V_{b2}}{\pi D_2 b_2} \right] \text{ in N/m}^2$$

where ρ_a = Mass density of air in kg / m^3.

2. From equation (*iii*),

$$\cot \beta_2 = \frac{V_{b2} - V_{w2}}{V_{f2}} \quad \text{or} \quad V_{w2} = V_{b2} - V_{f2} \cot \beta_2$$

It may be noted that

(*a*) For radial blades, $\beta_2 = 90°$. Thus $V_{w2} = V_{b2}$.

(*b*) For forward curved blades, β_2 is greater than 90°. Thus V_{w2} is greater than V_{b2}.

(*c*) For backward curved blades, β_2 is less than 90°. Thus V_{w2} is less than V_{b2}.

We know that the total pressure developed by a centrifugal fan is

$$p_{TF} = \rho_a \times V_{w2} \times V_{b2} \text{ in N/m}^2$$

Thus, for a given blade velocity (V_{b2}), the total pressure developed by a centrifugal fan having forward curved blades is greatest.

Example 21.2. *A centrifugal fan delivers 120 m³/min when running at 960 r.p.m. The impeller diameter is 0.7 m and the diameter at the blade inlet is 0.48 m. The air enters the impeller with a small whirl component in the direction of impeller rotation, but the relative velocity meets the blade tangentially. The impeller width at inlet is 160 mm and at outlet is 110 mm. The blades are backward curved making angles of 22.5° and 50° with the tangents at inlet and outlet respectively. Draw the inlet and outlet velocity triangles and determine the theoretical total head developed by the impeller.*

Assuming that the losses at inlet, in the impeller and in the casing amount to 70 per cent of the velocity head at impeller outlet and the velocity head at the fan discharge is 10 per cent of the velocity head at impeller outlet, calculate the fan static pressure in mm of water. Take the mass density of air to be 1.2 kg / m³ and neglect the effect of blade thickness and interblade circulation.

Solution. Given : Q = 120 m³/min = 2 m³/s ; N = 960 r.p.m. ; D_2 = 0.7 m ; D_1 = 0.48 m ; b_1 = 160 mm = 0.16 m ; b_2 = 110 mm = 0.11 m ; β_1 = 22.5° ; β_2 = 50° ; ρ_a = 1.2 kg / m³

Theoretical total head developed by the impeller

Let H = Theoretical total head developed by the impeller.

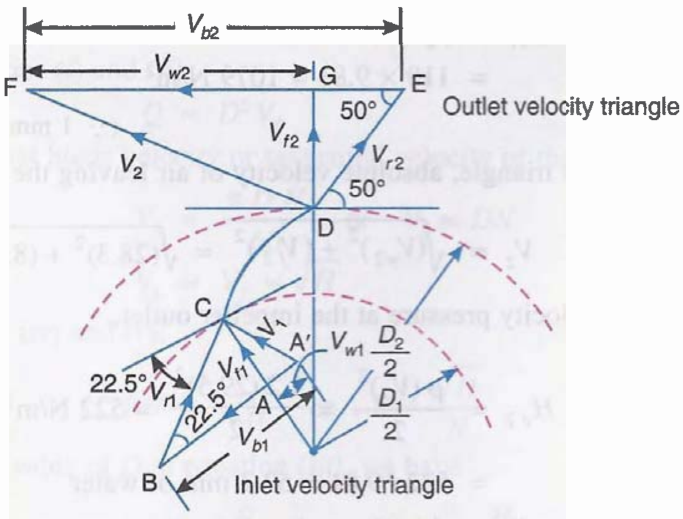


Fig. 21.9

The inlet and outlet velocity triangles are shown in Fig. 21.9. We know that blade velocity at inlet,

$$V_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.48 \times 960}{60} = 24 \text{ m / s}$$

Blade velocity at outlet,

$$V_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.7 \times 960}{60} = 35.2 \text{ m / s}$$

Quantity of air delivered

$$Q = \pi D_1 b_1 V_{f1} = \pi D_2 b_2 V_{f2}$$

∴ Velocity of flow at inlet,

$$V_{f1} = \frac{Q}{\pi D_1 b_1} = \frac{2}{\pi \times 0.48 \times 0.16} = 8.3 \text{ m/s}$$

and velocity of flow at outlet,

$$V_{f2} = \frac{Q}{\pi D_2 b_2} = \frac{2}{\pi \times 0.7 \times 0.11} = 8.27 \text{ m/s}$$

From the inlet velocity triangle,

$$AB = AC \cot 22.5° \quad \text{or} \quad V_{b1} - V_{w1} = V_{f1} \cot 22.5°$$

$$\therefore \quad 24 - V_{w1} = 8.3 \times 2.4142 = 20 \quad \text{or} \quad V_{w1} = 24 - 20 = 4 \text{ m/s}$$

From the outlet velocity triangle,

$$GE = GD \cot 50° \quad \text{or} \quad V_{b2} - V_{w2} = V_{f2} \cot 50°$$

$$\therefore \quad 35.2 - V_{w2} = 8.27 \times 0.8391 = 6.9 \quad \text{or} \quad V_{w2} = 35.2 - 6.9 = 28.3 \text{ m/s}$$

We know that the theoretical total head developed by the impeller,

$$H = \frac{1}{g}(V_{w2} \times V_{b2} - V_{w1} \times V_{b1})$$

$$= \frac{1}{9.81}(28.3 \times 35.2 - 4 \times 24) = 91.76 \text{ m of air}$$ **Ans.**

Fan static pressure

We know that the total pressure developed by the impeller or fan,

$$p_{TF} = \rho_a H = 1.2 \times 91.76 = 110 \text{ mm of water}$$

$$= 110 \times 9.81 = 1079 \text{ N/m}^2$$

... ($\because$ 1 mm of water = 9.81 N/m²)

From the outlet velocity triangle, absolute velocity of air leaving the blade,

$$V_2 = \sqrt{(V_{w2})^2 + (V_{f2})^2} = \sqrt{(28.3)^2 + (8.27)^2} = 29.5 \text{ m/s}$$

∴ *Velocity head or velocity pressure at the impeller outlet,

$$H_{v2} = \frac{\rho (V_2)^2}{2} = \frac{1.2 (29.5)^2}{2} = 522 \text{ N/m}^2$$

$$= 522 / 9.81 = 53.2 \text{ mm of water}$$

Losses at inlet, in the impeller and in the casing

= 70% of velocity head at outlet ... (Given)

$$= 0.7\, H_{v2} = 0.7 \times 53.2 = 37.24 \text{ mm of water}$$

---

* The velocity head (in mm of water) may be calculated by using the relation as discussed in Chapter 20 on 'Ducts'. We know that the velocity head or velocity pressure,

$$H_{v2} = \left(\frac{V_2}{4.04}\right)^2 = \left(\frac{29.5}{4.04}\right)^2 = 53.2 \text{ mm of water}$$

and velocity head or velocity pressure at the fan discharge

$$= 10\% \text{ of velocity head at outlet}$$

$$= 0.1\, H_{v2} = 0.1 \times 53.2 = 5.32 \text{ mm of water}$$

Now applying Bernoulli's equation to fan inlet and outlet, we have

Total pressure at fan inlet + Total pressure developed by impeller

= Losses + Total pressure at fan outlet

Since the total pressure at the fan inlet is zero and the total pressure at the fan outlet (p_{TF}) is the sum of static pressure at fan outlet or fan static pressure (p_{S2}) and the velocity pressure at fan outlet, therefore

$$0 + 110 = 37.24 + p_{S2} + 5.32$$

$$\therefore \quad p_{S2} = 110 - 37.24 - 5.32 = 67.44 \text{ mm of water}$$ **Ans.**

21.11 Specific Speed of a Centrifugal Fan

The specific speed of a centrifugal fan is defined as the speed of a geometrically similar fan which would deliver 1 m^3 of air per second against a head of 1 m of air. It is usually denoted by N_S.

Let Q = Total quantity of air flowing through the fan,

D = Diameter of impeller,

b = Width of impeller,

V_f = Velocity of flow,

N = Speed of impeller, and

H = Head developed by the fan.

We know that $Q = \pi D b V_f$ or $Q \propto DbV_f$ ... (*i*)

Also $D \propto b$ ... (*ii*)

∴ From equations (*i*) and (*ii*),

$$Q \propto D^2 V_f \qquad \text{... }(iii)$$

We also know that blade velocity or tangential velocity of the impeller,

$$V_b = \frac{\pi D N}{60} \quad \text{or} \quad V_b \propto DN \qquad \text{... }(iv)$$

Also $V_b \propto V_f \propto \sqrt{H}$ ... (*v*)

From equations (*iv*) and (*v*),

$$DN \propto \sqrt{H} \quad \text{or} \quad D \propto \frac{\sqrt{H}}{N} \qquad \text{... }(vi)$$

Substituting the value of D in equation (*iii*), we have

$$Q \propto \frac{H}{N^2} \times V_f \quad \text{or} \quad Q \propto \frac{H}{N^2} \times \sqrt{H} \propto \frac{(H)^{3/2}}{N^2}$$

$$\text{... } (\because V_f \propto \sqrt{H})$$

$$\therefore \quad Q = \frac{K(H)^{3/2}}{N^2} \qquad \text{... }(vii)$$

where K is constant of proportionality. According to the definition, if $Q = 1$ m^3/s and $H = 1$ m, then $N = N_S$. Substituting these values in equation (*vii*), we have

$$1 = \frac{K \times 1^{3/2}}{(N_S)^2} \quad \text{or} \quad K = (N_S)^2$$

Now from equation (*vii*),

$$Q = \frac{(N_S)^2 (H)^{3/2}}{N^2} \quad \text{or} \quad N_S = \frac{N\sqrt{Q}}{H^{3/4}}$$

21.12 Fan Similarity Laws

The two fans are said to be geometrically similar when all of their wheel dimensions have the same proportionate ratios.

For any series of geometrically similar fans and for any point on their characteristic curves, the following fan laws hold :

1. The volume flow rate or capacity (Q) of a fan is directly proportional to the fan speed (N) and cube of the impeller diameter (D). In other words

$$Q \propto N \propto D^3$$

or
$$\frac{Q_1}{N_1} = \frac{Q_2}{N_2} \quad \text{and} \quad \frac{Q_1}{(D_1)^3} = \frac{Q_2}{(D_2)^3}$$

It may also be written as

$$\frac{Q_1}{N_1 (D_1)^3} = \frac{Q_2}{N_2 (D_2)^3} = \text{Constant}$$

where suffix 1 represents the actual fan and suffix 2 represents the geometrically similar fan.

2. The total pressure developed by a fan (p_{TF}) is directly proportional to the square of the fan speed (N^2), square of the impeller diameter (D^2), and density of the air (ρ_a). In other words,

$$p_{TF} \propto N^2 \propto D^2 \propto \rho_a$$

or
$$\frac{p_{TF1}}{(N_1)^2} = \frac{p_{TF2}}{(N_2)^2} \quad \text{and} \quad \frac{p_{TF1}}{(D_1)^2} = \frac{p_{TF2}}{(D_2)^2}$$

Also
$$\frac{p_{TF1}}{\rho_{a1}} = \frac{p_{TF2}}{\rho_{a2}}$$

It may also be written as

$$\frac{p_{TF1}}{(N_1)^2 (D_1)^2 \rho_{a1}} = \frac{p_{TF2}}{(N_2)^2 (D_2)^2 \rho_{a2}} = \text{Constant}$$

This is also applicable to fan static pressure (p_{SF}) and fan velocity pressure (p_{vF}).

We know that the total pressure developed by the fan,

$$p_{TF} = \rho_a H$$

where H is the total head developed by the fan,

$$\therefore \quad \frac{\rho_{a1} H_1}{(N_1)^2 (D_1)^2 \rho_{a1}} = \frac{\rho_{a2} H_2}{(N_2)^2 (D_2)^2 \rho_{a2}} \quad \text{or} \quad \frac{H_1}{(N_1)^2 (D_1)^2} = \frac{H_2}{(N_2)^2 (D_2)^2} = \text{Constant}$$

3. The power (P) of a fan is directly proportional to the cube of the fan speed (N^3), fifth power of the impeller diameter (D^5) and density of the air (ρ_a). In other words,

$$P \propto N^3 \propto D^5 \propto \rho_a$$

or
$$\frac{P_1}{(N_1)^3} = \frac{P_2}{(N_2)^3} \quad \text{and} \quad \frac{P_1}{(D_1)^5} = \frac{P_2}{(D_2)^5}$$

Also $$\frac{P_1}{\rho_{a1}} = \frac{P_2}{\rho_{a2}}$$

It may also be written as

$$\frac{P_1}{(N_1)^3 (D_1)^5 \rho_{a1}} = \frac{P_2}{(N_2)^3 (D_2)^5 \rho_{a2}} = \text{Constant}$$

4. The efficiency (η) is constant. In other words,

$$\eta_1 = \eta_2 = \text{Constant}$$

Example. 21.3. *A fan for the ventilation plant is to be exported to an area where the air density is 0.96 kg / m³ and is scheduled to deliver 6 m³/s against a static pressure of 50 mm of water, with a static efficiency of 65 per cent.*

If it is driven by a constant speed motor, calculate the static pressure and shaft power in the maker's works where the air density is 1.2 kg / m³.

Solution. Given : $\rho_1 = 0.96$ kg / m³ ; $Q_1 = 6$ m³/s ; $p_{SF1} = 50$ mm of water ; $\eta_S = 65\%$ $= 0.65$; $\rho_2 = 1.2$ kg / m³

Static pressure in the maker's works

Let p_{SF2} = Static pressure in the maker's works.

We know that $$\frac{p_{SF1}}{\rho_1} = \frac{p_{SF2}}{\rho_2}$$

$$\therefore \quad p_{SF2} = p_{SF1} \times \frac{\rho_2}{\rho_1} = 50 \times \frac{1.2}{0.96} = 62.5 \text{ mm of water}$$

$$= 62.5 \times 9.81 = 613 \text{ N/m}^2 \text{ Ans.}$$

... ($\because$ 1 mm of water = 9.81 N/m²)

Shaft power in the maker's works

Since the speed (N) and diameter of impeller (D) is constant, therefore from the relation

$$\frac{Q_1}{N_1 (D_1)^3} = \frac{Q_2}{N_2 (D_2)^3}, \text{ we get } Q_1 = Q_2 = 6 \text{ m}^3/\text{s}$$

We know that static fan air power,

$$P_{aS2} = Q \times p_{SF2} = 6 \times 613 = 3678 \text{ N-m / s or W}$$

... ($\because$ 1 N-m/s = 1 W)

$\therefore$ Shaft power in the maker's works,

$$P_{S2} = \frac{P_{aS2}}{\eta_S} = \frac{3678}{0.65} = 5660 \text{ W} = 5.66 \text{ kW Ans.}$$

Example 21.4. *A fan of diameter 0.7 m running at 1500 r.p.m. delivers 140 m³/min of air at 15°C against 75 mm of water of total pressure when its total efficiency is 86 per cent. Determine the volume of air delivered, total pressure developed and power consumed, if*

(a) the air temperature is 50°C,

(b) the air temperature is 50°C and the fan speed is increased to 1700 r.p.m., and

(c) the conditions are same as in (b) but a 0.6 m diameter, geometrically similar fan is used.

Solution. Given : $D_1 = 0.7$ m ; $N_1 = 1500$ r.p.m ; $Q_1 = 140$ m³/min ; $t_1 = 15$°C ; $p_{TF1} = 75$ mm of water ; $\eta_T = 86\% = 0.86$; $t_2 = 50$°C

First of all, let us find the ratio of the densities at temperatures 15°C and 50°C.

Let ρ_{a1} = Density of air at 15°C, and

ρ_{a2} = Density of air at 50°C.

We know that for the constant barometric pressure, the density of air at 50°C,

$$\rho_{a2} = \rho_{a1}\left(\frac{273+t_1}{273+t_2}\right)$$

$$\therefore \quad \frac{\rho_{a1}}{\rho_{a2}} = \frac{273+t_2}{273+t_1} = \frac{273+50}{273+15} = 1.12$$

(a) Volume of air delivered, total pressure developed and power consumed when the air temperature is 50°C

Inline centrifugal fans.

Let Q_2 = Volume of air delivered,

p_{TF2} = Total pressure developed, and

P_2 = Power consumed.

We know that

$$\frac{Q_1}{N_1(D_1)^3} = \frac{Q_2}{N_2(D_2)^3}$$

Since there is no change in speed or diameter, therefore

$Q_2 = Q_1 = 140$ m³/min **Ans.**

Now
$$\frac{p_{TF1}}{p_{TF2}} = \frac{\rho_{a1}}{\rho_{a2}}$$

$$\therefore \quad p_{TF2} = p_{TF1} \times \frac{\rho_{a2}}{\rho_{a1}}$$

$$= 75 \times \frac{1}{1.12} = 67 \text{ mm of water}$$

$= 67 \times 9.81 = 657.3$ N/m² **Ans.**

We know that power consumed,

$$P_2 = Q_2 \times p_{TF2} \times \frac{1}{\eta} = 140 \times 657.3 \times \frac{1}{0.86}$$

$= 107\,002$ N-m/min $= 107\,002 / 60 = 1783.4$ W **Ans.**

(b) Volume of air delivered, total pressure developed and power consumed when air temperature is 50°C and speed is increased to 1700 r.p.m.

Let Q_3 = Volume of air delivered,

p_{TF3} = Total pressure developed,

P_3 = Power consumed, and

N_3 = Increased speed = 1700 r.p.m. ... (Given)

We know that
$$\frac{Q_3}{N_3} = \frac{Q_2}{N_2}$$

$$\therefore \quad Q_3 = Q_2 \times \frac{N_3}{N_2} = 140 \times \frac{1700}{1500} = 158.7 \text{ m}^3\text{/min}$$ **Ans.**

... (Here $N_2 = N_1$)

Now
$$\frac{p_{TF3}}{(N_3)^2} = \frac{p_{TF2}}{(N_2)^2}$$

$$\therefore \quad p_{TF3} = p_{TF2}\left(\frac{N_3}{N_2}\right)^2 = 67\left(\frac{1700}{1500}\right)^2 = 86 \text{ mm of water}$$ **Ans.**

and $$\frac{P_3}{(N_3)^3} = \frac{P_2}{(N_2)^3}$$

$$\therefore \quad P_3 = P_2\left(\frac{N_3}{N_2}\right)^3 = 1783.4\left(\frac{1700}{1500}\right)^3 = 2596 \text{ W} \quad \textbf{Ans.}$$

(c) Volume of air delivered, total pressure developed and power consumed for the conditions as in (b) and when diameter is 0.6 m

Let Q_4 = Volume of air delivered,

p_{TF4} = Total pressure developed,

P_4 = Power consumed, and

D_4 = New impeller diameter = 0.6 m . . . (Given)

We know that $$\frac{Q_4}{(D_4)^3} = \frac{Q_3}{(D_3)^3}$$

$$\therefore \quad Q_4 = Q_3\left(\frac{D_4}{D_3}\right)^3 = 158.7\left(\frac{0.6}{0.7}\right)^3 = 99.94 \text{ m}^3\text{/min} \quad \textbf{Ans.}$$

... (Here $D_3 = D_1$)

Now $$\frac{p_{TF4}}{(D_4)^2} = \frac{p_{TF3}}{(D_3)^2}$$

$$\therefore \quad p_{TF4} = p_{TF3}\left(\frac{D_4}{D_3}\right)^2 = 86\left(\frac{0.6}{0.7}\right)^2 = 63.2 \text{ mm of water} \quad \textbf{Ans.}$$

and $$\frac{P_4}{(D_4)^5} = \frac{P_3}{(D_3)^5}$$

$$\therefore \quad P_4 = P_3\left(\frac{D_4}{D_3}\right)^5 = 2596\left(\frac{0.6}{0.7}\right)^5 = 1201 \text{ W} \quad \textbf{Ans.}$$

21.13 Fan and System Characteristic

We have already discussed that all of the duct work elements such as elbows, tees, registers, dampers etc., offer resistance to the flow of air and cause loss in pressure. The change in pressure loss or resistance with the change in flow rate is called *system characteristic*. Any air-conditioning or ventilating system that has a duct work, heating and cooling coils, dampers, registers etc. has a definite system characteristic. The system characteristic is independent of the fan used in that system.

We have seen in the previous chapter that the system resistance or pressure loss of any fixed system varies as the square of the flow rate, *i.e.*

$$R \text{ or } p_L = KQ^2$$

If the resistance of a system is plotted against the varying amounts of flow rates, a

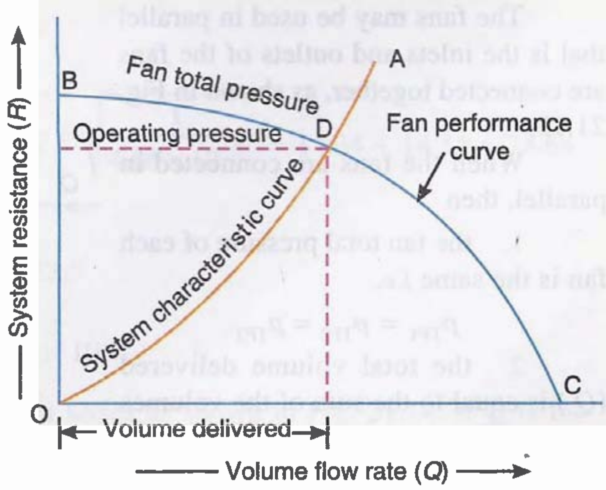


Fig. 21.10. Fan and system characteristic.

curve such as *OA* is obtained as shown in Fig. 21.10. This curve is a parabola and is usually known as a *system characteristic curve.*

When a fan operates in conjunction with a particular system, then the loss of total pressure in the system at a given volume flow must be equal to the total pressure developed by the fan (*i.e.* fan total pressure) at the same volume. This condition is satisfied by the point of intersection (point *D*) of system characteristic curve *OA* and the fan performance curve *BC*. This point of intersection is called the *operating point* of that particular fan in that particular system, as it indicates the volume and pressure at which the fan operates.

21.14 Fans in Series

Some times it is necessary to use more than one fan in conjunction with a given system. The fans may be used in series that is the outlet of the first fan is connected to the inlet of second fan and outlet of the second fan is connected to the inlet of the third fan and so on, as shown in Fig. 21.11.

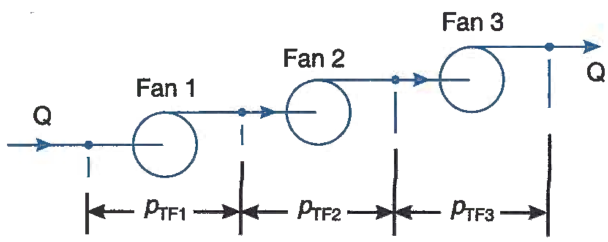


Fig. 21.11. Fans in series.

Series centrifugal fans.

When the fans are connected in series, then

1. the volume flow rate (Q) through each fan is same, *i.e.*

$$Q = Q_1 = Q_2 = Q_3$$

2. the overall fan total pressure (p_{TF}) is equal to the sum of the fan total pressures developed by the individual fans, *i.e.*

$$p_{TF} = p_{TF1} + p_{TF2} + p_{TF3}$$

21.15 Fans in Parallel

The fans may be used in parallel that is the inlets and outlets of the fans are connected together, as shown in Fig. 21.12.

When the fans are connected in parallel, then

1. the fan total pressure of each fan is the same *i.e.*

$$p_{TF1} = p_{TF2} = p_{TF3}$$

2. the total volume delivered (Q) is equal to the sum of the volumes delivered by the individual fans, *i.e.*

$$Q = Q_1 + Q_2 + Q_3$$

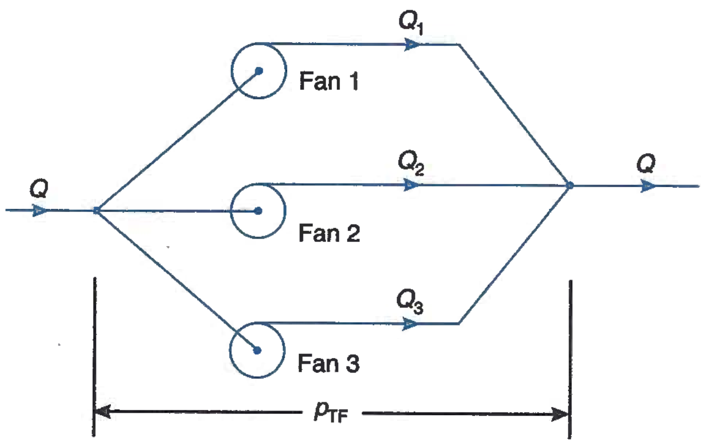


Fig. 21.12. Fans in parallel.

Example 21.5. *A fan delivers air to a system as shown in Fig. 21.13, where R_1 = 50 mm of water for a volume flow of 180 m³/min and $R_2 = R_3$ = 17.5 mm of water for a volume flow of 60 m³/min. The fan performance is as follows :*

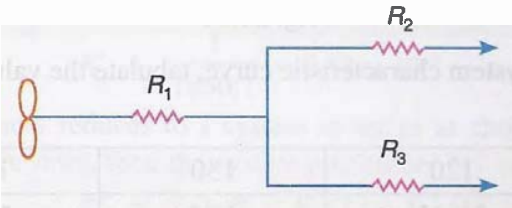


Fig. 21.13

Fan volume flow, m³/min	*120*	*150*	*180*	*210*
Fan total pressure, mm of water	*70*	*55*	*40*	*18*

Find the volume of air handled by the fan. By what percentage would it increase if a third branch of resistance $R_4 = R_2$ is connected in parallel with the existing two branches.

Solution. Given : R_1 = 50 mm of water ; Q_1 = 180 m³/min ; $R_2 = R_3$ = 17.5 mm of water ; $Q_2 = Q_3$ = 60 m³/min

Volume of air handled by the fan

First of all, let us find the resistance of the single equivalent system for the parallel resistances R_2 and R_3. Let R_e be the resistance of the equivalent system and K_e the equivalent constant.

We know that constant for R_1,

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{50}{(180)^2} = 1.54 \times 10^{-3}$$

Similarly, constant for R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{17.5}{(60)^2} = 4.86 \times 10^{-3}$$

and constant for R_3,

$$K_3 = \frac{R_3}{(Q_3)^2} = \frac{17.5}{(60)^2} = 4.86 \times 10^{-3}$$

We know that

$$\sqrt{\frac{1}{K_e}} = \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}}$$

or

$$\sqrt{\frac{1}{K_e}} = \sqrt{\frac{1}{4.86 \times 10^{-3}}} + \sqrt{\frac{1}{4.86 \times 10^{-3}}} = 14.34 + 14.34 = 28.68$$

Squaring both sides, we have

$$\frac{1}{K_e} = (28.68)^2 = 822.5$$

or

$$K_e = \frac{1}{822.5} = 1.2 \times 10^{-3}$$

The whole system now reduces to a system in series as shown in Fig. 21.14. If Q is the volume flow rate of air in m³/min, then the system resistance,

$$R' = R_1 + R_e = K_1 Q^2 + K_e Q^2$$
$$= 1.54 \times 10^{-3} Q^2 + 1.2 \times 10^{-2} Q^2 = 2.74 \times 10^{-3} Q^2$$

Fig. 21.14

In order to draw the system characteristic curve, tabulate the values of R' for different values of Q as given below :

Q (m^3/min)	120	130	140	150
R' (mm of water)	39.45	46.3	53.7	61.65

From the above values of Q and R', draw the system characteristic curve as shown in Fig. 21.15. This curve intersects the fan performance curve plotted for the given values, at point P as shown in Fig. 21.15. The point P is the operating point of the fan.

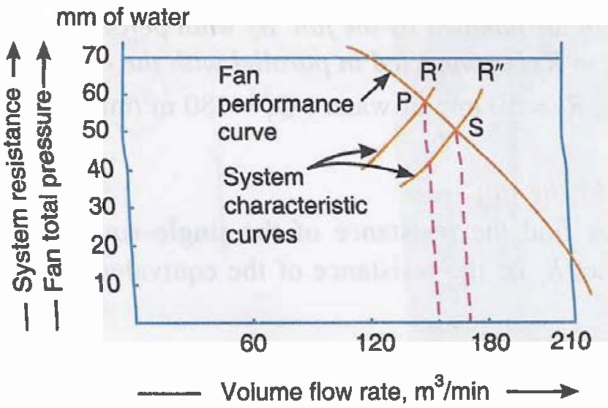


Fig. 21.15

∴ Volume of air handled by the fan

$= \text{Volume of air at } P = 145 \text{ m}^3\text{/min}$ **Ans.**

Percentage increase in volume

When a third branch of resistance $R_4 = R_2$ is connected in parallel, with the existing two branches as shown in Fig. 21.16, then the constant for the equivalent resistance R_e' is given by

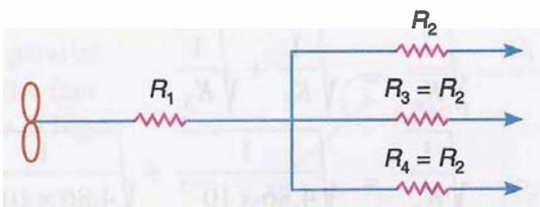


Fig. 21.16

$$\sqrt{\frac{1}{K_e'}} = \sqrt{\frac{1}{K_2}} + \sqrt{\frac{1}{K_3}} + \sqrt{\frac{1}{K_4}}$$

Since $R_2 = R_3 = R_4$, therefore $K_2 = K_3 = K_4 = 4.86 \times 10^{-3}$

$$\therefore \quad \sqrt{\frac{1}{K_e'}} = 3 \times \sqrt{\frac{1}{K_2}} = 3 \times \sqrt{\frac{1}{4.86 \times 10^3}} = 43.02$$

Squaring both sides, we have

$$\frac{1}{K'_e} = (43.02)^2 = 1850.7$$

or
$$K'_e = \frac{1}{1850.7} = 0.54 \times 10^{-3}$$

The whole system now reduces to a system in series as shown in Fig. 21.17. If Q is the volume flow rate of air in m³/min, then the system resistance,

$$R'' = R_1 + R'_e = K_1 Q^2 + K'_e Q^2$$
$$= 1.54 \times 10^{-3} Q^2 + 0.54 \times 10^{-3} Q^2 = 2.04 \times 10^{-3} Q^2$$

Fig. 21.17

Now tabulate the values of R'' for different values of Q as given below :

Q (m³/min)	140	150	160	170	180
R'' (mm of water)	40	45.9	52.2	58.9	66.1

From these values, draw the system characteristic curve as shown in Fig. 21.15. This curve intersects the fan performance curve at point S. At this point, volume of air handled by the fan

$$= 158 \text{ m}^3/\text{min}$$

∴ Percentage increase in volume

$$= \frac{158 - 145}{145} \times 100 = 8.96\% \textbf{ Ans.}$$

Example 21.6. *The fans A and B supply equal volumes of air to a system as shown in Fig. 21.18.*

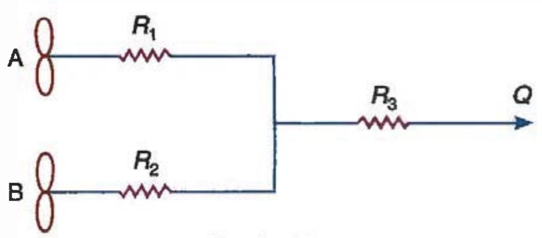


Fig. 21.18

The performance of the two fans is given below :

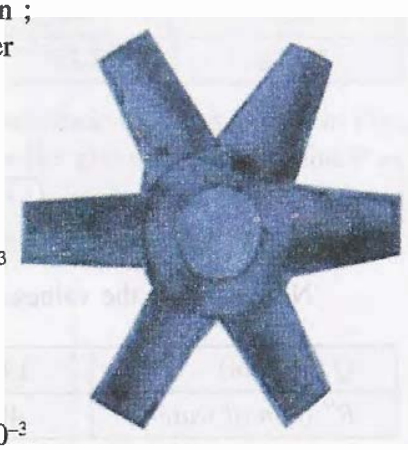
Volume flow (m³/min)	*Fan total pressure (mm of water)*	
	Fan A	*Fan B*
60	*70*	*76.5*
90	*69.5*	*65.5*
120	*67*	*48*
150	*60*	*26*
180	*48*	—

The resistances of the system are

$$R_1 = 15 \text{ mm of water at } 90 \text{ m}^3\text{/min} ;$$
$$R_2 = 7.5 \text{ mm of water at } 120 \text{ m}^3\text{/min, and}$$
$$R_3 = 2.5 \text{ mm of water at } 60 \text{ m}^3\text{/min.}$$

Determine the operating points of each of the two fans and the volume flowing through the common branch.

Solution. Given : R_1 = 15 mm of water ; Q_1 = 90 m³/min ; R_2 = 7.5 mm of water ; Q_2 = 120 m³/min ; R_3 = 2.5 mm of water Q_3 = 60 m³/min



Tube axial flow fans impeller.

Operating points of each of the two fans

We know that constant for R_1,

$$K_1 = \frac{R_1}{(Q_1)^2} = \frac{15}{(90)^2} = 1.85 \times 10^{-3}$$

Similarly, constant for R_2,

$$K_2 = \frac{R_2}{(Q_2)^2} = \frac{7.5}{(120)^2} = 0.52 \times 10^{-3}$$

and constant for R_3, $$K_3 = \frac{R_3}{(Q_3)^2} = \frac{2.5}{(60)^2} = 0.7 \times 10^{-3}$$

Let Q_A and Q_B be the volume of air supplied by the fans A and B. Since the volume of air supplied by the fans A and B is equal, therefore volume of air flowing through the common duct,

$$Q = 2Q_A = 2Q_B \quad \ldots (i)$$

We know that system resistance with fan A,

$$R_A = R_1 + R_3 = K_1 (Q_A)^2 + K_3 Q^2$$
$$= K_1 (Q_A)^2 + K_3 (2Q_A)^2 = (K_1 + 4K_3)(Q_A)^2 \quad \ldots [\because Q = Q_A]$$
$$= (1.85 \times 10^{-3} + 4 \times 0.7 \times 10^{-3})(Q_A)^2 = 4.65 \times 10^{-3} (Q_A)^2$$

Similarly, system resistance with fan B,

$$R_B = R_2 + R_3 = K_2 (Q_B)^2 + K_3 Q^2$$
$$= K_2 (Q_A)^2 + K_3 (2Q_A)^2$$
$$= (K_2 + 4K_3)(Q_A)^2 \quad \ldots \text{[From equation (i)]}$$
$$= (0.52 \times 10^{-3} + 4 \times 0.7 \times 10^{-3})(Q_A)^2$$
$$= 3.32 \times 10^{-3} (Q_A)^2$$

Now tabulate the values of R_A and R_B for different values of Q_A, as given below :

Q_A *(m³/min)*	100	110	120	130
R_A *(mm of water)*	46.5	56.25	67	78.6
R_B *(mm of water)*	33.2	40.17	48	56.1

From these values, draw the system characteristic curves as shown in Fig. 21.19. The curve for R_A intersects the fan performance curve for fan *A*, at point *P* and the curve for R_B intersects the fan performance curve for fan *B*, at point *S*. The points *P* and *S* are the operating points of fans *A* and *B* respectively. From Fig. 21.19, we find that the operating point *P* for fan *A* lies at 120 m^3/min and 67 mm of water. The operating point *S* for fan *B* lies at 120 m^3/min and 48 mm of water. **Ans.**

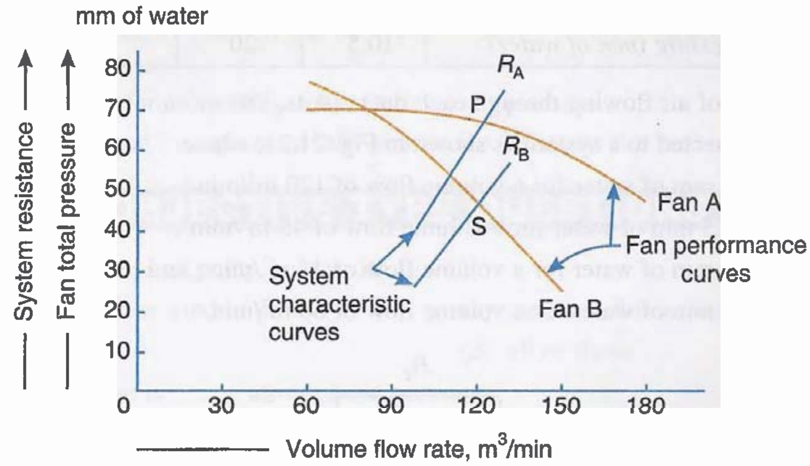


Fig. 21.19

Volume flowing through the common branch

Volume flowing through the common branch,

$$Q = \text{Volume at } P + \text{Volume at } Q$$
$$= 120 + 120 = 240 \text{ m}^3\text{/min} \quad \textbf{Ans.}$$

EXERCISES

1. A fan draws in air freely and discharges through a test duct of cross-section 0.07 m^2 in which the static pressure is 20 per cent of the velocity pressure. If the total efficiency of the fan is 65 percent and the input power is 1 kW, find the quantity of air being delivered in m^3/min. **[Ans.** 99 m^3/min]

2. A centrifugal fan having 0.75 m diameter impeller and rotating at 960 r.p.m. delivers 150 m^3/min of air at 75 mm of total water column. If the air enters the impeller at outlet is 120 mm, determine the required blade angle at outlet. Assume that 45 per cent of the theoretical head is dissipated as impeller and casing losses and take the mass density of air as 1.2 kg / m^3. **[Ans.** 23.5°]

3. A fan is to deliver 500 m^3/min at a static pressure of 25 mm of water when running at 250 r.p.m. and requiring 5 kW. If the fan speed is changed to 300 r.p.m., find the capacity, static pressure and the power required. **[Ans.** 600 m^3/min ; 36 mm of water ; 8.64 kW]

4. A fan delivers air to a system as shown in Fig. 21.20, where

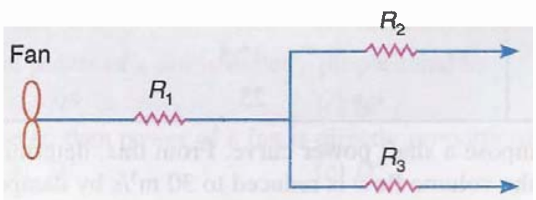


Fig. 21.20

R_1 = 12.5 mm of water for a volume flow of 300 m³/min ;

R_2 = 12.5 mm of water for a volume flow of 180 m³/min ; and

R_3 = 17.5 mm of water for a volume flow of 120 m³/min.

The fan performance is as follows :

Volume flow (m³/min)	300	285	270	255	240
Fan total pressure (mm of water)	10.5	20	26	30	35.5

Find the quantity of air flowing through each duct. **[Ans.** 280 m³/min ; 178 m³/min ; 102 m³/min **]**

5. A fan is connected to a system as shown in Fig. 21.21, where

R_1 = 50 mm of water for a volume flow of 120 m³/min ;

R_2 = 22.5 mm of water for a volume flow of 45 m³/min ;

R_3 = 15 mm of water for a volume flow of 45 m³/min ; and

R_4 = 10 mm of water for a volume flow of 60 m³/min.

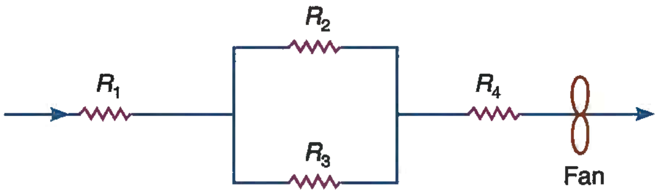


Fig. 21.21

The fan performance is as follows :

Volume flow (m³/min)	60	90	120	150
Fan total pressure (mm of water)	94.5	92.5	87.5	80

Determine the fan operating point and the volumes of air flowing in R_2 and R_3.

[Ans. 103.5 m³/min at 90.5 mm of water ; 46.7 m³/min ; 56.8 m³/min **]**

6. The performance for a centrifugal fan driven by a constant speed motor is given below :

Volume flow (m³/s)	*Static pressure (mm of water)*	*Efficiency (%)*
0	85	0
10	92.5	46
20	95	66
30	90	70
40	80	67
50	65	60
60	47.5	48
70	25	32

Plot these and superimpose a shaft power curve. From this, determine the shaft power at 50 m³/s. Also determine the power if the volume flow is reduced to 30 m³/s by damper regulation.

If instead of using damper regulation, the fan speed is reduced approximately by a hydraulic coupling of constant torque and zero slip, calculate the reduction in power input to the fan shaft.

[Ans. 53 kW ; 37.8 kW ; 18.7 kW **]**

QUESTIONS

1. What is the function of a fan in an air-conditioning system ?
2. Describe a centrifugal fan with the help of a neat sketch.
3. Explain the various types of axial flow fans.
4. Define the following :
   (*a*) Fan total pressure, (*b*) Fan air power, and (*c*) Fan total efficiency.
5. Write in brief explanatory note on the comparative study of the characteristics of backward and forward curved blade fans.
6. Define specific speed for a centrifugal fan. Derive its expression.
7. What do you understand by a geometrically similar fan ? Discuss the various fan similarity laws.

OBJECTIVE TYPE QUESTIONS

1. A fan may be considered as a pump, because it
   (*a*) looks like most other kind of pumps (*b*) circulate fluids, like other pumps
   (*c*) rotates (*d*) all of these
2. The air at inlet of a fan is .... atmospheric pressure.
   (*a*) above (*b*) below
3. In axial flow fans,
   (*a*) the air flows parallel to the axis of impeller
   (*b*) the air flows perpendicular to the axis of impeller
   (*c*) the air may flow either parallel or perpendicular to the axis of impeller
   (*d*) none of the above
4. The air guide vanes are sometimes installed in axial flow fans in order to
   (*a*) increase the static pressure (*b*) eliminate spiral flow of discharge air
   (*c*) reduce high frequency sound generation (*d*) all of these
5. The axial flow fans are particularly suitable for handling
   (*a*) large volumes of air at relatively low pressures (*b*) small volumes of air at relatively low pressures
   (*c*) large volumes of air at relatively high pressures (*d*) small volumes of air at relatively high pressures
6. The fan total pressure is the algebraic .... between the total pressure at the fan outlet and the total pressure at the fan inlet.
   (*a*) sum (*b*) difference
7. Two fans that are of different sizes, but have the same basic shape to their performance curves, are called
   (*a*) base fans (*b*) tubeaxial fans
   (*c*) geometrically similar fans (*d*) multi-stage fans
8. The capacity of a fan is .... cube of the impeller diameter.
   (*a*) directly proportional to (*b*) inversely proportional to
9. If N is the fan speed, then power of a fan is directly proportional to
   (*a*) N (*b*) N^2 (*c*) N^3 (*d*) N^4.
10. If D is the impeller diameter, then power of a fan is directly proportional to
   (*a*) D^2 (*b*) D^3 (*c*) D^4 (*d*) D^5

ANSWERS

1. (*a*)	2. (*b*)	3. (*a*)	4. (*b*)	5. (*a*)
6. (*b*)	7. (*c*)	8. (*a*)	9. (*c*)	10. (*d*)