CHAPTER

4 Simple Vapour Compression Refrigeration Systems

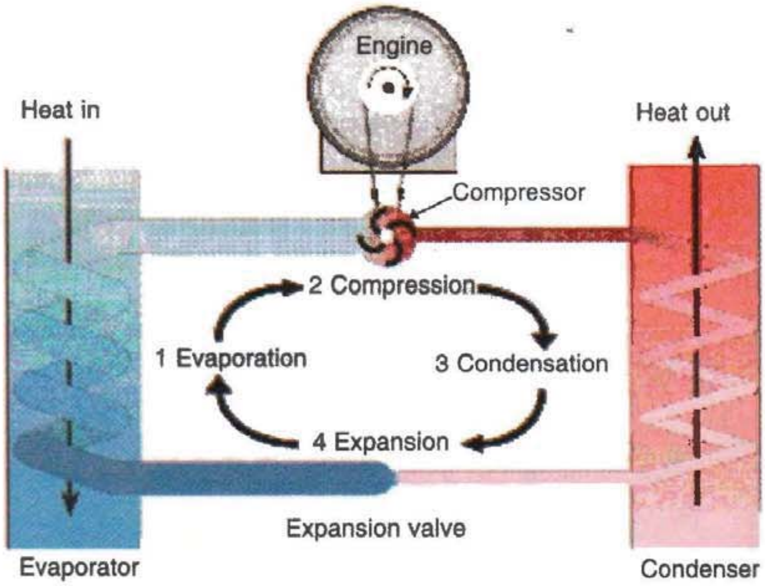
4.1 Introduction

A vapour compression refrigeration system* is an improved type of air refrigeration system in which a suitable working substance, termed as refrigerant, is used. It condenses and evaporates at temperatures and pressures close to the atmospheric conditions. The refrigerants, usually, used for this purpose are ammonia (NH_3), carbon dioxide (CO_2) and sulphur dioxide (SO_2). The refrigerant used, does not leave the system, but is circulated

* Since low pressure vapour refrigerant from the evaporator is changed into high pressure vapour refrigerant in the compressor, therefore it is named as vapour compression refrigeration system.

throughout the system alternately condensing and evaporating. In evaporating, the refrigerant absorbs its latent heat from the brine* (salt water) which is used for circulating it around the cold chamber. While condensing, it gives out its latent heat to the circulating water of the cooler. The vapour compression refrigeration system is, therefore a latent heat pump, as it pumps its latent heat from the brine and delivers it to the cooler.

The vapour compression refrigeration system is now-a-days used for all purpose refrigeration. It is generally used for all industrial purposes from a small domestic refrigerator to a big air conditioning plant.



Engine-driven vapour compression heat pump.

Note: The first vapour compression system was developed in 1834 by Jacob Perkins using hand operation.

4.2 Advantages and Disadvantages of Vapour Compression Refrigeration System over Air Refrigeration System

Following are the advantages and disadvantages of the vapour compression refrigeration system over air refrigeration system :

Advantages

1. It has smaller size for the given capacity of refrigeration.
2. It has less running cost.
3. It can be employed over a large range of temperatures.
4. The coefficient of performance is quite high.

Disadvantages

1. The initial cost is high.
2. The prevention of leakage of the refrigerant is the major problem in vapour compression system.

4.3 Mechanism of a Simple Vapour Compression Refrigeration System

Fig. 4.1 shows the schematic diagram of a simple vapour compression refrigeration system. It consists of the following five essential parts :

1. ***Compressor.*** The low pressure and temperature vapour refrigerant from evaporator is drawn into the compressor through the inlet or suction valve *A*, where it is compressed to a high pressure and temperature. This high pressure and temperature vapour refrigerant is discharged into the condenser through the delivery or discharge valve *B*.

2. ***Condenser.*** The condenser or cooler consists of coils of pipe in which the high pressure and temperature vapour refrigerant is cooled and condensed. The refrigerant, while passing through the condenser, gives up its latent heat to the surrounding condensing medium which is normally air or water.

* Brine is used as it has a very low freezing temperature.

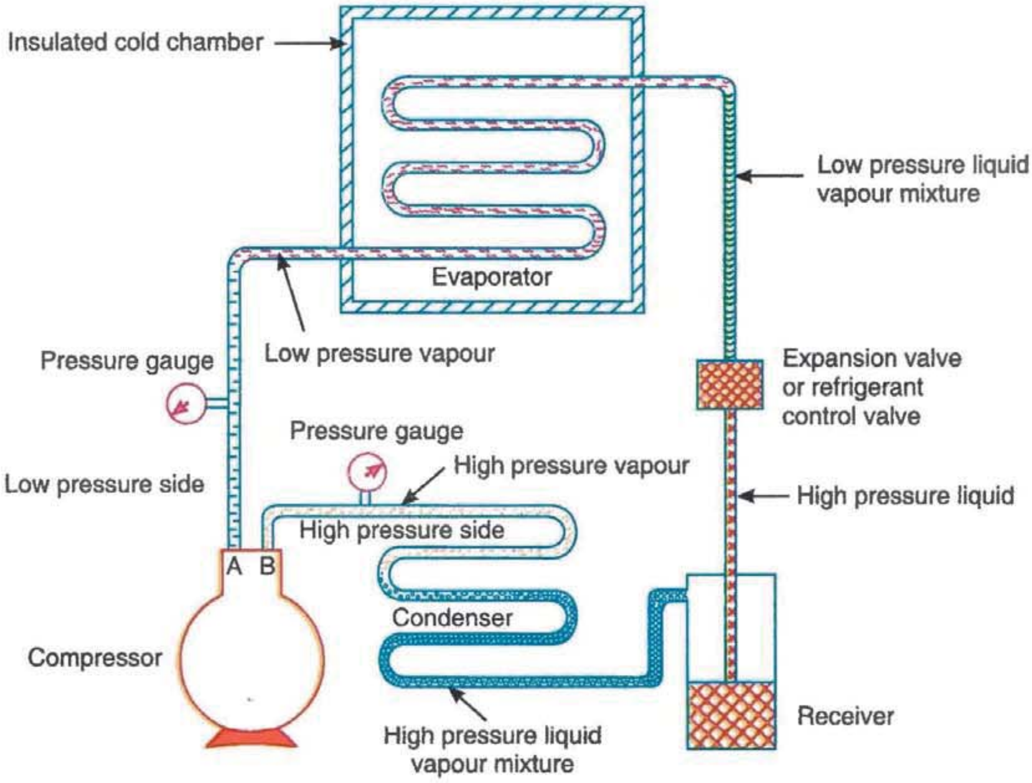


Fig. 4.1. Simple vapour compression refrigeration system.

3. ***Receiver.*** The condensed liquid refrigerant from the condenser is stored in a vessel known as receiver from where it is supplied to the evaporator through the expansion valve or refrigerant control valve.

4. ***Expansion valve.*** It is also called throttle valve or refrigerant control valve. The function of the expansion valve is to allow the liquid refrigerant under high pressure and temperature to pass at a controlled rate after reducing its pressure and temperature. Some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporised in the evaporator at the low pressure and temperature.

5. ***Evaporator.*** An evaporator consists of coils of pipe in which the liquid-vapour refrigerant at low pressure and temperature is evaporated and changed into vapour refrigerant at low pressure and temperature. In evaporating, the liquid vapour refrigerant absorbs its latent heat of vaporisation from the medium (air, water or brine) which is to be cooled.

Note: In any compression refrigeration system, there are two different pressure conditions. One is called the *high pressure side* and other is known as *low pressure side*. The high pressure side includes the discharge line (*i.e.* piping from delivery valve *B* to the condenser), condenser, receiver and expansion valve. The low pressure side includes the evaporator, piping from the expansion valve to the evaporator and the suction line (*i.e.* piping from the evaporator to the suction valve *A*).

4.4 Pressure-Enthalpy (*p-h*) Chart

The most convenient chart for studying the behaviour of a refrigerant is the *p-h* chart, in which the vertical ordinates represent pressure and horizontal ordinates represent enthalpy (*i.e.* total heat). A typical chart is shown in Fig. 4.2, in which a few important lines of the complete chart are drawn. The saturated liquid line and the saturated vapour line merge into one another at the critical point. A saturated liquid is one which has a temperature equal to the saturation temperature corresponding to its pressure.The space to the left of the saturated liquid line will, therefore, be sub-cooled liquid region. The space between the liquid and the vapour lines is called wet vapour region and to the right of the saturated vapour line is a superheated vapour region.

In the following pages, we shall draw the *p-h* chart along with the *T-s* diagram of the cycle.

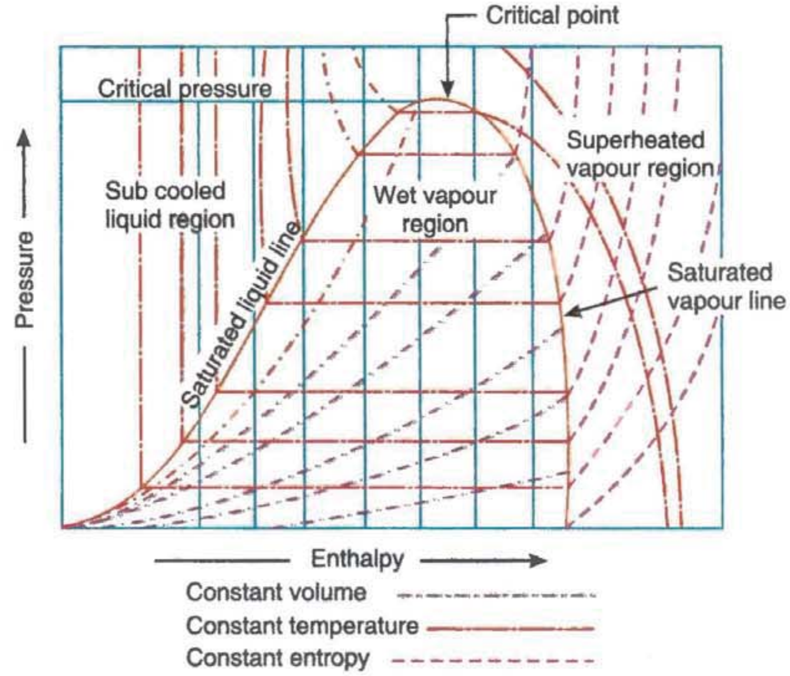


Fig. 4.2. Pressure - enthalpy (*p-h*) chart.

4.5 Types of Vapour Compression Cycles

We have already discussed that vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles, yet the following are important from the subject point of view :

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

Now we shall discuss all the above mentioned cycles, one by one, in the following pages.

4.6 Theoretical Vapour Compression Cycle with Dry Saturated Vapour after Compression

A vapour compression cycle with dry saturated vapour after compression is shown on *T-s* and *p-h* diagrams in Fig. 4.3 (*a*) and (*b*) respectively. At point 1, let T_1, p_1 and s_1, be the temperature, pressure and entropy of the vapour refrigerant respectively. The four processes of the cycle are as follows :

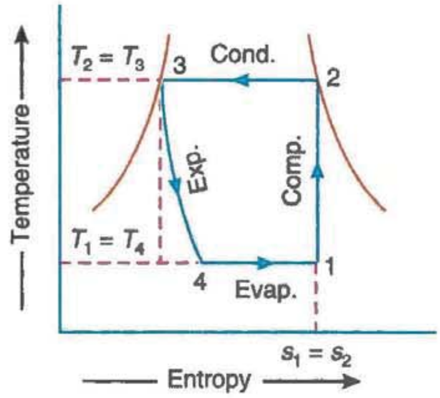
1. ***Compression process.*** The vapour refrigerant at low pressure p_1 and temperature T_1 is compressed isentropically to dry saturated vapour as shown by the vertical line 1-2 on *T-s* diagram and by the curve 1-2 on *p-h* diagram. The pressure and temperature rises from p_1 to p_2 and T_1 to T_2 respectively.

The work done during isentropic compression per kg of refrigerant is given by

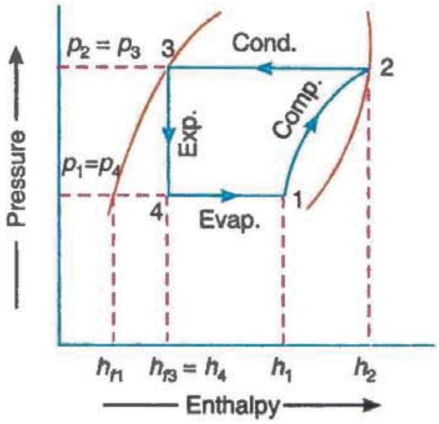
$$w = h_2 - h_1$$

where h_1 = Enthalpy of vapour refrigerant at temperature T_1, *i.e.* at suction of the compressor, and

h_2 = Enthalpy of the vapour refrigerant at temperature T_2, *i.e.* at discharge of the compressor.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.3. Theoretical vapour compression cycle with dry saturated vapour after compression.

2. *Condensing process.* The high pressure and temperature vapour refrigerant from the compressor is passed through the condenser where it is completely condensed at constant pressure p_2 and temperature T_2, as shown by the horizontal line 2-3 on *T-s* and *p-h* diagrams. The vapour refrigerant is changed into liquid refrigerant. The refrigerant, while passing through the condenser, gives its latent heat to the surrounding condensing medium.

3. *Expansion process.* The liquid refrigerant at pressure $p_3 = p_2$ and temperature $T_3 = T_2$ is expanded by *throttling process through the expansion valve to a low pressure $p_4 = p_1$ and temperature $T_4 = T_1$, as shown by the curve 3-4 on *T-s* diagram and by the vertical line 3-4 on *p-h* diagram. We have already discussed that some of the liquid refrigerant evaporates as it passes through the expansion valve, but the greater portion is vaporised in the evaporator. We know that during the throttling process, no heat is absorbed or rejected by the liquid refrigerant.

Notes: (*a*) In case an expansion cylinder is used in place of throttle or expansion valve to expand the liquid refrigerant, then the refrigerant will expand isentropically as shown by dotted vertical line on *T-s* diagram in Fig. 4.3 (*a*). The isentropic expansion reduces the external work being expanded in running the compressor and increases the refrigerating effect. Thus, the net result of using the expansion cylinder is to increase the coefficient of performance.

Since the expansion cylinder system of expanding the liquid refrigerant is quite complicated and involves greater initial cost, therefore its use is not justified for small gain in cooling capacity. Moreover, the flow rate of the refrigerant can be controlled with throttle valve which is not possible in case of expansion cylinder which has a fixed cylinder volume.

(*b*) In modern domestic refrigerators, a capillary (small bore tube) is used in place of an expansion valve.

4. *Vaporising process.* The liquid-vapour mixture of the refrigerant at pressure $p_4 = p_1$ and temperature $T_4 = T_1$ is evaporated and changed into vapour refrigerant at constant pressure and temperature, as shown by the horizontal line 4-1 on *T-s* and *p-h* diagrams. During evaporation, the liquid-vapour refrigerant absorbs its latent heat of vaporisation from the medium (air, water or brine) which is to be cooled. This heat which is absorbed by the refrigerant is called *refrigerating effect* and it is briefly written as R_E. The process of vaporisation continues upto point 1 which is the starting point and thus the cycle is completed.

* The throttling process is an irreversible process.

We know that the refrigerating effect or the heat absorbed or extracted by the liquid-vapour refrigerant during evaporation per kg of refrigerant is given by

$$R_E = h_1 - h_4 = h_1 - h_{f3} \qquad \ldots (\because h_{f3} = h_4)$$

where h_{f3} = Sensible heat at temperature T_3, *i.e.* enthalpy of liquid refrigerant leaving the condenser.

It may be noticed from the cycle that the liquid-vapour refrigerant has extracted heat during evaporation and the work will be done by the compressor for isentropic compression of the high pressure and temperature vapour refrigerant.

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note: The ratio of C.O.P. of vapour compression cycle to the C.O.P. of Carnot cycle is known as *refrigeration efficiency* (η_R) or performance index (P.I.).

Example 4.1. *In an ammonia vapour compression system, the pressure in the evaporator is 2 bar. Ammonia at exit is 0.85 dry and at entry its dryness fraction is 0.19. During compression, the work done per kg of ammonia is 150 kJ. Calculate the C.O.P. and the volume of vapour entering the compressor per minute, if the rate of ammonia circulation is 4.5 kg/min. The latent heat and specific volume at 2 bar are 1325 kJ/kg and 0.58 m³/kg respectively.*

Solution. Given : $p_1 = p_4 = 2$ bar ; $x_1 = 0.85$; $x_4 = 0.19$; $w = 150$ kJ/kg ; $m_a = 4.5$ kg/min ; $h_{fg} = 1325$ kJ/kg ; $v_g = 0.58$ m³/kg

C.O.P.

The *T-s* and *p-h* diagrams are shown in Fig. 4.3 (*a*) and (*b*) respectively.

Since the ammonia vapour at entry to the evaporator (*i.e.* at point 4) has dryness fraction (x_4) equal to 0.19, therefore enthalpy at point 4,

$$h_4 = x_4 \times h_{fg} = 0.19 \times 1325 = 251.75 \text{ kJ/kg}$$

Similarly, enthalpy of ammonia vapour at exit *i.e.* at point 1,

$$h_1 = x_1 \times h_{fg} = 0.85 \times 1325 = 1126.25 \text{ kJ/kg}$$

∴ Heat extracted from the evaporator or refrigerating effect,

$$R_E = h_1 - h_4 = 1126.25 - 251.75 = 874.5 \text{ kJ/kg}$$

We know that work done during compression,

$$w = 150 \text{ kJ/kg}$$

$$\therefore \quad \text{C.O.P.} = R_E / w = 874.5 / 150 = 5.83 \textbf{ Ans.}$$

Volume of vapour entering the compressor per minute

We know that volume of vapour entering the compressor per minute

= Mass of refrigerant / min × Specific volume

$= m_a \times v_g = 4.5 \times 0.58 = 2.61$ m³/min **Ans.**

Example 4.2. *The temperature limits of an ammonia refrigerating system are 25°C and –10°C. If the gas is dry at the end of compression, calculate the coefficient of performance of the cycle assuming no undercooling of the liquid ammonia. Use the following table for properties of ammonia :*

Temperature (°C)	*Liquid heat (kJ/kg)*	*Latent heat (kJ/kg)*	*Liquid entropy (kJ/kg K)*
25	*298.9*	*1166.94*	*1.1242*
–10	*135.37*	*1297.68*	*0.5443*

Solution. Given : $T_2 = T_3 = 25°C = 25 + 273 = 298$ K ; $T_1 = T_4 = -10°$ C $= -10 + 273 =$ 263 K ; $h_{f3} = h_4 = 298.9$ kJ/kg ; $h_{fg2} = 1166.94$ kJ/kg ; $s_{f2} = 1.1242$ kJ/kg K ; $h_{f1} = 135.37$ kJ/ kg ; $h_{fg1} = 1297.68$ kJ/kg ; $s_{f1} = 0.5443$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.4 (*a*) and (*b*) respectively.

Let x_1 = Dryness fraction at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + \frac{x_1 h_{fg1}}{T_1} = 0.5443 + \frac{x_1 \times 1297.68}{263}$$

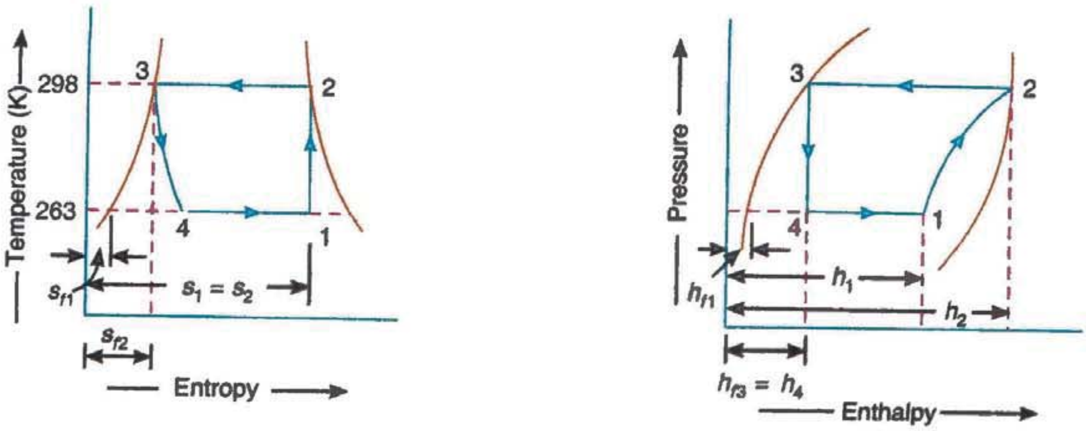
$$= 0.5443 + 4.934\, x_1 \quad \ldots (i)$$

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{h_{fg2}}{T_2} = 1.1242 + \frac{1166.94}{298} = 5.04 \quad \ldots (ii)$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.5443 + 4.934\, x_1 = 5.04 \qquad \text{or} \qquad x_1 = 0.91$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.4

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = 135.37 + 0.91 \times 1297.68 = 1316.26 \text{ kJ/kg}$$

and enthalpy at point 2, $h_2 = h_{f2} + h_{fg2} = 298.9 + 1166.94 = 1465.84$ kJ/kg

∴ Coefficient of performance of the cycle

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1316.26 - 298.9}{1465.84 - 1316.26} = 6.8 \text{ Ans.}$$

Example 4.3. *A vapour compression refrigerator works between the pressure limits of 60 bar and 25 bar. The working fluid is just dry at the end of compression and there is no under-cooling of the liquid before the expansion valve. Determine : 1. C.O.P. of the cycle ; and 2. Capacity of the refrigerator if the fluid flow is at the rate of 5 kg/min.*

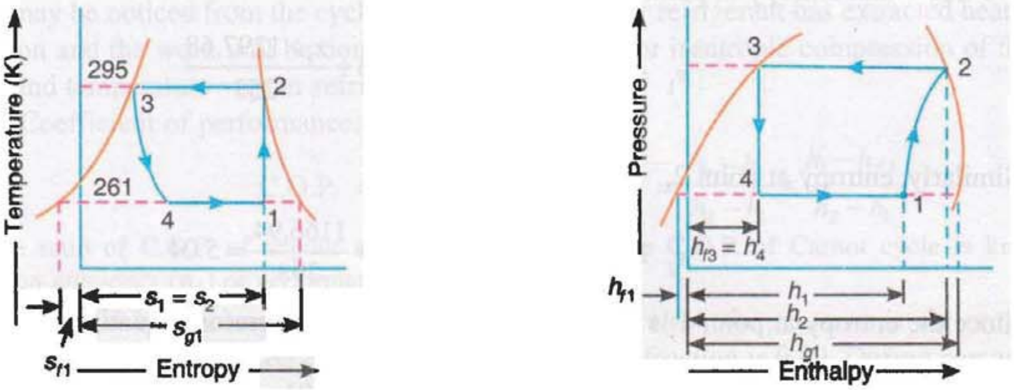
Data :

Pressure (bar)	*Saturation temperature (K)*	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
60	*295*	*151.96*	*293.29*	*0.554*	*1.0332*
25	*261*	*56.32*	*322.58*	*0.226*	*1.2464*

Solution. Given : $p_2 = p_3 = 60$ bar ; $p_1 = p_4 = 25$ bar ; $T_2 = T_3 = 295$ K ; $T_1 = T_4 = 261$ K ; $h_{f3} = h_4 = 151.96$ kJ/kg ; $h_{f1} = 56.32$ kJ/kg ; $h_{g2} = h_2 = 293.29$ kJ/kg ; $h_{g1} = 322.58$ kJ/kg ; *$s_{f2} = 0.554$ kJ/kg K ; $s_{f1} = 0.226$ kJ/kg K ; $s_{g2} = s_2 = 1.0332$ kJ/kg K ; $s_{g1} = 1.2464$ kJ/kg K

1. C.O.P. of the cycle

The *T-s* and *p-h* diagrams are shown in Fig. 4.5 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.5

Let x_1 = Dryness fraction of the vapour refrigerant entering the compressor at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1 s_{fg1} = s_{f1} + x_1(s_{g1} - s_{f1}) \quad \ldots (\because s_{g1} = s_{f1} + s_{fg1})$$

$$= 0.226 + x_1 (1.2464 - 0.226) = 0.226 + 1.0204\, x_1 \quad \ldots (i)$$

and entropy at point 2, $s_2 = s_{g2} = 1.0332$ kJ/kg K ... (Given) ... (*ii*)

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.226 + 1.0204\, x_1 = 1.0332 \quad \text{or} \quad x_1 = 0.791$$

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = h_{f1} + x_1 (h_{g1} - h_{f1}) \ldots (\because h_{g1} = h_{f1} + h_{fg1})$$

$$= 56.32 + 0.791 (322.58 - 56.32) = 266.93 \text{ kJ/kg}$$

∴ C.O.P. of the cycle

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{266.93 - 151.96}{293.29 - 266.93} = 4.36 \textbf{ Ans.}$$

2. Capacity of the refrigerator

We know that the heat extracted or refrigerating effect produced per kg of refrigerant

$$= h_1 - h_{f3} = 266.93 - 151.96 = 114.97 \text{ kJ/kg}$$

Since the fluid flow is at the rate of 5 kg/min, therefore total heat extracted

$$= 5 \times 114.97 = 574.85 \text{ kJ/min}$$

∴ Capacity of the refrigerator

$$= \frac{574.85}{210} = 2.74 \text{ TR } \textbf{Ans.} \quad \ldots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

* Superfluous data

Example 4.4. *28 tonnes of ice from and at 0°C is produced per day in an ammonia refrigerator. The temperature range in the compressor is from 25°C to –15°C. The vapour is dry and saturated at the end of compression and an expansion valve is used. There is no liquid subcooling. Assuming actual C.O.P. of 62% of the theoretical, calculate the power required to drive the compressor. Following properties of ammonia are given:*

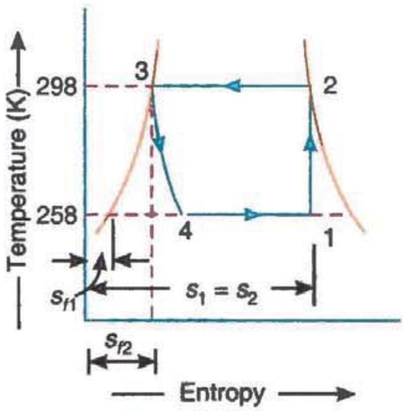
Temperature	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*	
0°C	*Liquid*	*Vapour*	*Liquid*	*Vapour*
25	*298.9*	*1465.84*	*1.1242*	*5.0391*
–15	*112.34*	*1426.54*	*0.4572*	*5.5490*

Take latent heat of ice = 335 kJ/kg.

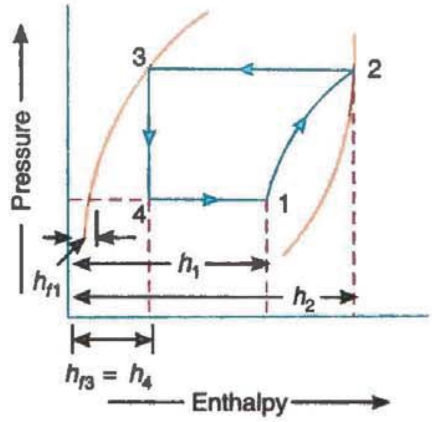
Solution. Given: Ice produced = 28t/day ; $T_2 = T_3 = 25°C = 25 + 273 = 298K$; $T_1 = T_4 = -15°C = -15 + 273 = 258$ K ; $h_{f3} = h_4 = 298.9$ kJ/kg ; $h_{f1} = 112.34$ kJ/kg ; $h_{g2} = h_2 = 1465.84$ kJ/kg ; $h_{g1} = 1426.54$ kJ/kg ; * $s_{f2} = 1.1242$ kJ/kg K ; $s_{f1} = 0.4572$ kJ/kg K ; $s_{g2} = s_2 = 5.0391$ kJ/kg K ; $s_{g1} = 5.5490$ kJ/kg K.

The *T-s* and *p-h* diagrams are shown in Fig. 4.6 (*a*) and (*b*) respectively.

First of all, let us find the dryness fraction (x_1) of the vapour refrigerant entering the compressor at point 1.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.6

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1 s_{fg1} = s_{f1} + x_1(s_{g1} - s_{f1}) \qquad ...(\because s_{g1} = s_{f1} + s_{fg1})$$

$$= 0.4572 + x_1(5.5490 - 0.4572)$$

$$= 0.4572 + 5.0918\, x_1 \qquad ...(i)$$

and entropy at point 2, $\quad s_2 = s_{g2} = 5.0391$ kJ/kg K $\quad$ ... (Given) $\quad$ ...(*ii*)

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.4572 + 5.0918\, x_1 = 5.0391 \quad \text{or} \quad x_1 = 0.9$$

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = h_{f1} + x_1(h_{g1} - h_{f1}) \qquad ...(\because h_{g1} = h_{f1} + h_{fg1})$$

$$= 112.34 + 0.9\,(1426.54 - 112.34) = 1295.12 \text{ kJ/kg}$$

* Superfluous data.

$$\therefore \quad \text{Theoretical C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1295.12 - 298.9}{1465.84 - 1295.12} = \frac{996.22}{170.72} = 5.835$$

Since actual C.O.P. is 62% of theoretical C.O.P., therefore

$$\text{Actual C.O.P} = 0.62 \times 5.835 = 3.618$$

We know that ice produced from and at 0°C

$$= 28 \text{ t/day} = \frac{28 \times 1000}{24 \times 3600} = 0.324 \text{ kg / s}$$

Latent heat of ice = 335 kJ/kg ...(Given)

$\therefore$ Refrigeration effect produced

$$= 0.324 \times 335 = 108.54 \text{ kJ/s}$$

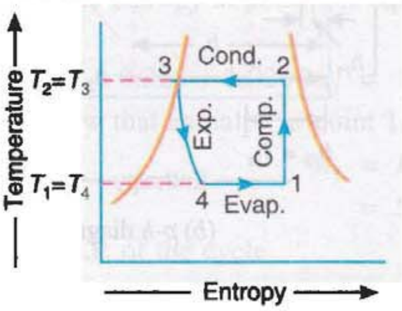
We know that actual C.O.P.,

$$3.618 = \frac{\text{Refrigeration effect}}{\text{workdone}} = \frac{108.54}{\text{workdone}}$$

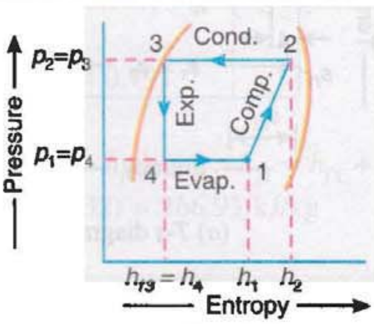
$\therefore$ Workdone or power required to drive the compressor

$$= \frac{108.54}{3.618} = 30 \text{ kJ/s or kW} \quad \textbf{Ans.}$$

4.7 Theoretical Vapour Compression Cycle with Wet Vapour after Compression



(a) T-s diagram.



(b) p-h diagram.

Fig. 4.7. Theoretical vapour compression cycle with wet vapour after compression.

A vapour compression cycle with wet vapour after compression is shown on *T-s* and *p-h* diagrams in Fig. 4.7 (*a*) and (*b*) respectively. In this cycle, the enthalpy at point 2 is found out with the help of dryness fraction at this point. The dryness fraction at points 1 and 2 may be obtained by equating entropies at points 1 and 2.

Now the coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note: The remaining cycle is same as discussed in the last article.

Example 4.5. *Find the theoretical C.O.P. for a CO_2 machine working between the temperature range of 25°C and – 5°C. The dryness fraction of CO_2 gas during the suction stroke is 0.6. Following properties of CO_2 are given :*

Temperature	*Liquid*		*Vapour*		*Latent heat*
°C	*Enthalpy kJ/kg*	*Entropy kJ/kg K*	*Enthalpy kJ/kg*	*Entropy kJ/kg K*	*kJ/kg*
25	164.77	0.5978	282.23	0.9918	117.46
–5	72.57	0.2862	321.33	1.2146	248.76

Solution. Given : $T_2 = T_3 = 25°C = 25 + 273 = 298$ K ; $T_1 = T_4 = -5°C = -5 + 273 = 268$ K ; $x_1 = 0.6$; $h_{f3} = h_{f2} = 164.77$ kJ/kg ; $h_{f1} = h_{f4} = 72.57$ kJ/kg ; $s_{f2} = 0.5978$ kJ/kg K; $s_{f1} = 0.2862$ kJ/kg K ; $h_{2'} = 282.23$ kJ/kg ; $h_{1'} = 321.33$ kJ/kg ; *$s_{2'} = 0.9918$ kJ/kg K ; *$s_{1'} = 1.2146$ kJ/kg K ; $h_{fg2} = 117.46$ kJ/kg ; $h_{fg1} = 248.76$ kJ/kg

The *T-s* and *p-h* diagrams are shown in Fig. 4.8 (*a*) and (*b*) respectively.

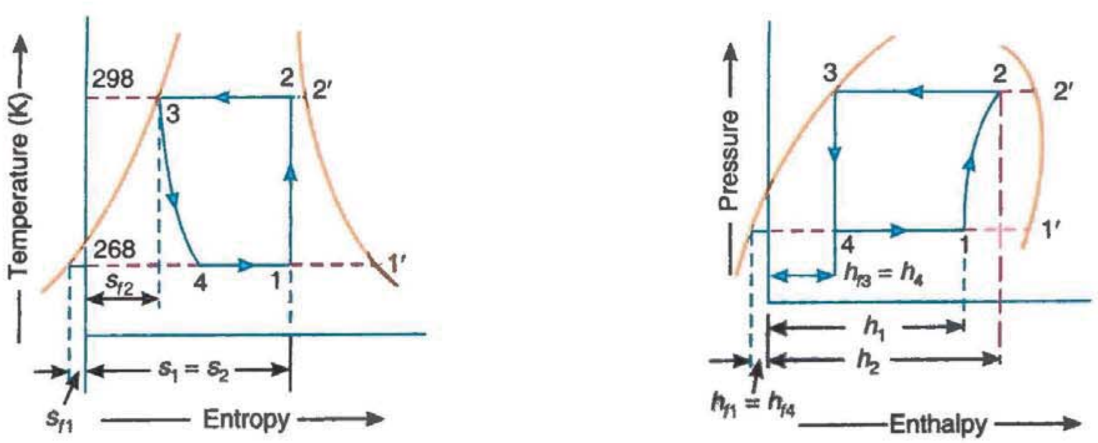
First of all, let us find the dryness fraction at point 2, *i.e.* x_2. We know that the entropy at point 1,

$$s_1 = s_{f1} + \frac{x_1\, h_{fg1}}{T_1} = 0.2862 + \frac{0.6 \times 248.76}{268} = 0.8431 \qquad \dots (i)$$

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{x_2\, h_{fg2}}{T_2} = 0.5978 + \frac{x_2 \times 117.46}{298}$$

$$= 0.5978 + 0.3941\, x_2 \qquad \dots (ii)$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.8

Since the entropy at point 1 (s_1) is equal to entropy at point 2 (s_2), therefore equating equations (*i*) and (*ii*),

$$0.8431 = 0.5978 + 0.3941\, x_2 \quad \text{or} \quad x_2 = 0.622$$

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = 72.57 + 0.6 \times 248.76 = 221.83 \text{ kJ/kg}$$

and enthalpy at point 2,

$$h_2 = h_{f2} + x_2\, h_{fg2} = 164.77 + 0.622 \times 117.46 = 237.83 \text{ kJ/kg}$$

* Superfluous data

$\therefore$ Theoretical C.O.P. $= \dfrac{h_1 - h_{f3}}{h_2 - h_1} = \dfrac{221.83 - 164.77}{237.83 - 221.83} = \dfrac{57.06}{16} = 3.57$ **Ans.**

Example 4.6. *An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of – 10°C and 30°C. The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at 30°C. Assuming actual C.O.P. as 60% of the theoretical, calculate the kilograms of ice produced per kW hour at 0°C from water at 10°C. Latent heat of ice is 335 kJ/kg. Ammonia has the following properties :*

Temperature °C	*Liquid heat (h_f) kJ/kg*	*Latent heat (h_{fg}) kJ/kg*	*Liquid entropy (s_f)*	*Total entropy of dry saturated vapour*
30	*323.08*	*1145.80*	*1.2037*	*4.9842*
–10	*135.37*	*1297.68*	*0.5443*	*5.4770*

Solution. Given : $T_1 = T_4 = -10°C = -10 + 273 = 263$ K ; $T_2 = T_3 = 30°C = 30 + 273 = 303$ K ; $x_2 = 0.95$; $h_{f3} = h_{f2} = 323.08$ kJ/kg ; $h_{f1} = h_{f4} = 135.37$ kJ/kg ; $h_{fg2} = 1145.8$ kJ/kg ; $h_{fg1} = 1297.68$ kJ/kg , $s_{f2} = 1.2037$; $s_{f1} = 0.5443$; * $s_{2'} = 4.9842$; * $s_{1'} = 5.4770$

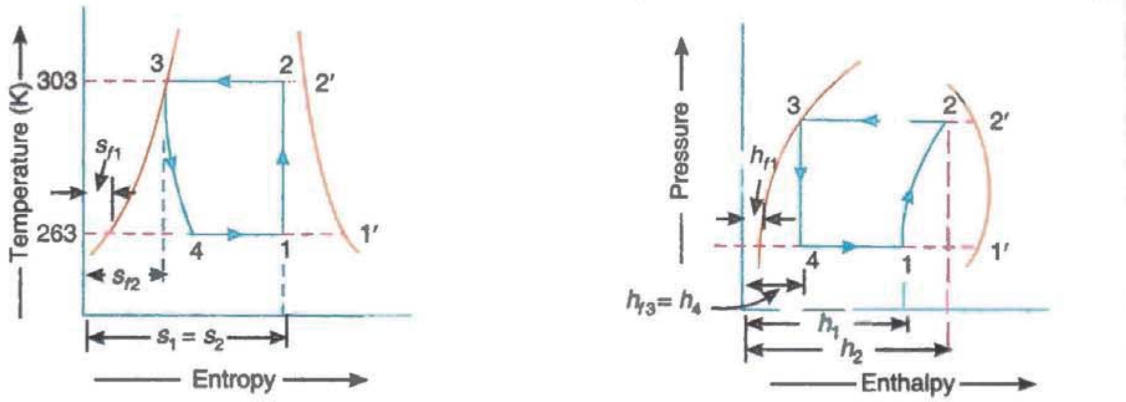
The *T-s* and *p-h* diagrams are shown in Fig. 4.9 (*a*) and (*b*) respectively.

Let x_1 = Dryness fraction at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + \frac{x_1\, h_{fg1}}{T_1} = 0.5443 + \frac{x_1 \times 1297.68}{263}$$

$$= 0.5443 + 4.934\, x_1 \quad \ldots (i)$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.9

Similarly, entropy at point 2,

$$s_2 = s_{f2} + \frac{x_2\, h_{fg2}}{T_2} = 1.2037 + \frac{0.95 \times 1145.8}{303} = 4.796 \quad \ldots (ii)$$

Since the entropy at point 1 (s_1) is equal to entropy at point 2 (s_2), therefore equating equations (*i*) and (*ii*),

$$0.5443 + 4.934\, x_1 = 4.796 \quad \text{or} \quad x_1 = 0.86$$

* Superfluous data.

$\therefore$ Enthalpy at point 1, $h_1 = h_{f1} + x_1\, h_{fg1} = 135.37 + 0.86 \times 1297.68 = 1251.4$ kJ/kg

and enthalpy at point 2, $h_2 = h_{f2} + x_2\, h_{fg2} = 323.08 + 0.95 \times 1145.8 = 1411.6$ kJ/kg

We know that theoretical C.O.P.

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1251.4 - 323.08}{1411.6 - 1251.4} = 5.8$$

$\therefore$ Actual C.O.P. $= 0.6 \times 5.8 = 3.48$

Work to be spent corresponding to 1 kW hour,

$$W = 3600 \text{ kJ}$$

$\therefore$ Actual heat extracted or refrigeration effect produced per kW hour

$$= W \times \text{Actual C.O.P.} = 3600 \times 3.48 = 12\,528 \text{ kJ}$$

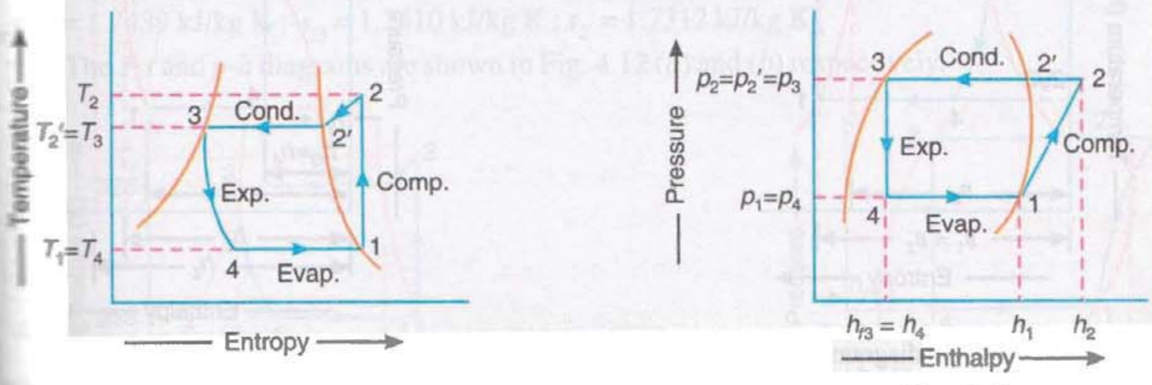
We know that heat extracted from 1 kg of water at 10°C for the formation of 1 kg of ice at 0°C

$$= 1 \times 4.187 \times 10 + 335 = 376.87 \text{ kJ}$$

$\therefore$ Amount of ice produced

$$= \frac{12528}{376.87} = 33.2 \text{ kg / kW hour} \textbf{ Ans.}$$

4.8 Theoretical Vapour Compression Cycle with Superheated Vapour after Compression



(a) T-s diagram. (b) p-h diagram.

Fig. 4.10. Theoretical vapour compression cycle with superheated vapour after compression.

A vapour compression cycle with superheated vapour after compression is shown on *T-s* and *p-h* diagrams in Fig. 4.10 (*a*) and (*b*) respectively. In this cycle, the enthalpy at point 2 is found out with the help of degree of superheat. The degree of superheat may be found out by equating the entropies at points 1 and 2.

Now the coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

A little consideration will show that the superheating increases the refrigerating effect and the amount of work done in the compressor. Since the increase in refrigerating effect is less as compared to the increase in work done, therefore, the net effect of superheating is to have low coefficient of performance.

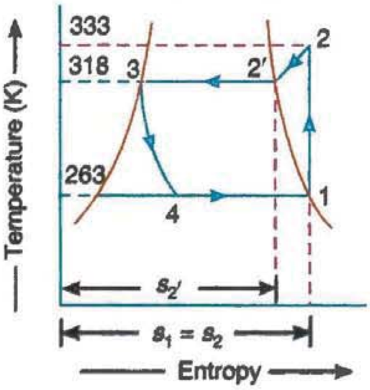
Note : In this cycle, the cooling of superheated vapour will take place in two stages. Firstly, it will be condensed to dry saturated stage at constant pressure (shown by graph 2-2′) and secondly, it will be condensed at constant temperature (shown by graph 2′-3). The remaining cycle is same as discussed in the last article.

Example 4.7. *A vapour compression refrigerator uses methyl chloride (R-40) and operates between temperature limits of – 10°C and 45°C. At entry to the compressor, the refrigerant is dry saturated and after compression it acquires a temperature of 60°C. Find the C.O.P. of the refrigerator. The relevant properties of methyl chloride are as follows :*

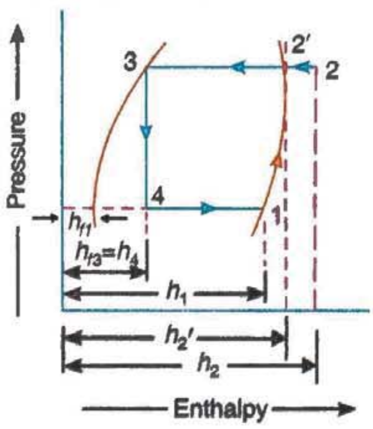
Saturation	*Enthalpy in kJ/kg*		*Entropy in kJ/kg K*	
temperature in °C	*Liquid*	*Vapour*	*Liquid*	*Vapour*
–10	*45.4*	*460.7*	*0.183*	*1.637*
45	*133.0*	*483.6*	*0.485*	*1.587*

Solution. Given : $T_1 = T_4 = -10°C = -10 + 273 = 263$ K ; $T_{2'} = T_3 = 45°C = 45 + 273 = 318$ K ; $T_2 = 60°C = 60 + 273 = 333$ K ; $*h_{f1} = 45.4$ kJ/kg ; $h_{f3} = 133$ kJ/kg ; $h_1 = 460.7$ kJ/kg ; $h_{2'} = 483.6$ kJ/kg ; $*s_{f1} = 0.183$ kJ/kg K ; $*s_{f3} = 0.485$ kJ/kg K ; $s_1 = s_2 = 1.637$ kJ/kg K; $s_{2'} = 1.587$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.11 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.11

Let c_p = Specific heat at constant pressure for superheated vapour.

We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_p \log\left(\frac{T_2}{T_{2'}}\right)$$

$$1.637 = 1.587 + 2.3\, c_p \log\left(\frac{333}{318}\right)$$

$$= 1.587 + 2.3\, c_p \times 0.02 = 1.587 + 0.046\, c_p$$

$$\therefore \quad c_p = 1.09$$

* Superfluous data.

and enthalpy at point 2, $h_2 = h_{2'} + c_p \times \text{Degree of superheat} = h_{2'} + c_p (T_2 - T_2')$

$= 483.6 + 1.09\ (333 - 318) = 500$ kJ/kg

∴ C.O.P. of the refrigerator

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.7 - 133}{500 - 460.7} = 8.34 \textbf{ Ans.}$$

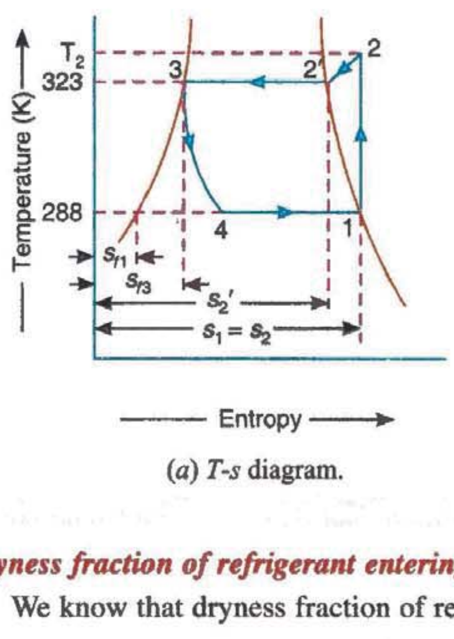
Example 4.8. *A simple refrigerant 134a (tetrafluroethane) heat pump for space heating, operates between temperature limits of 15°C and 50°C. The heat required to be pumped is 100 MJ/h. Determine : 1. The dryness fraction of refrigerant entering the evaporator; 2. The discharge temperature assuming the specific heat of vapour as 0.996 kJ/kg K; 3. The theoretical piston displacement of the compressor; 4. The theoretical power of the compressor; and 5. The C.O.P.*

The specific volume of refrigerant 134a saturated vapour at 15°C is 0.04185 m³/kg. The other relevant properties of R-134a are given below:

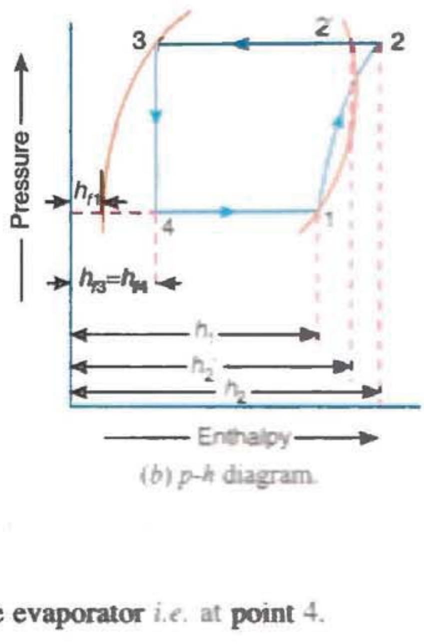
Saturation temperature (°C)	*Pressure (bar)*	*Specific enthalpy (kJ/kg)*		*Specific entropy (kJ/kg K)*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
15	*4.887*	*220.26*	*413.6*	*1.0729*	*1.7439*
50	*13.18*	*271.97*	*430.4*	*1.2410*	*1.7312*

Solution: Given: $T_1 = T_4 = 15°C = 15 + 273 = 288$ K ; $T_{2'} = T_3 = 50°C = 50 + 273 = 323$ K ; $Q = 100$ MJ/h $= 100 \times 10^3$ kJ/h ; $c_p = 0.996$ kJ/kg K ; $v_1 = 0.04185$ m³/kg ; $h_{f1} = 220.26$ kJ/kg ; $h_{f3} = h_4 = 271.97$ kJ/kg ; $h_1 = 413.6$ kJ/kg ; $h_{2'} = 430.4$ kJ/kg ; $s_{f1} = 1.0729$ kJ/kg K ; $s_1 = s_2 = 1.7439$ kJ/kg K ; $s_{f3} = 1.2410$ kJ/kg K ; $s_{2'} = 1.7312$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.12 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.12

1. Dryness fraction of refrigerant entering the evaporator

We know that dryness fraction of refrigerant entering the evaporator *i.e.* at point 4.

$$x_4 = \frac{h_4 - h_{f1}}{h_1 - h_{f1}} = \frac{271.97 - 220.26}{413.6 - 220.26} = \frac{51.71}{193.34} = 0.2675 \textbf{ Ans.}$$

2. Discharge temperature

Let T_2 = Discharge temperature.

We know that entropy at discharge *i.e.* at point 2,

$$s_2 = s_{2'} + 2.3\, c_p \log\left(\frac{T_2}{T_{2'}}\right)$$

$$1.7439 = 1.7312 + 2.3 \times 0.996 \ \log\left(\frac{T_2}{T_{2'}}\right)$$

$$\log\left(\frac{T_2}{T_{2'}}\right) = \frac{1.7439 - 1.7312}{2.3 \times 0.996} = 0.005\,54$$

$$\frac{T_2}{T_{2'}} = 1.0128 \qquad \text{...(Taking antilog of 0.005 54)}$$

$$T_2 = T_{2'} \times 1.0128 = 323 \times 1.0128 = 327.13 \text{ K} = 54.13°\text{C}$$ **Ans.**

3. Theoretical piston displacement of the compressor

We know that enthalpy at discharge *i.e.* at point 2,

$$h_2 = h_{2'} + c_p (T_2 - T_{2'})$$

$$= 430.4 + 0.996\,(327.13 - 323) = 434.5 \text{ kJ/kg}$$

and mass flow rate of the refrigerant,

$$m_R = \frac{Q}{h_2 - h_{f3}} = \frac{100 \times 10^3}{434.5 - 271.97} = 615.3 \text{ kg/h} = 10.254 \text{ kg/min}$$

∴ Theoretical piston displacement of the compressor

$$= m_R \times v_1 = 10.254 \times 0.4185 = 4.29 \text{ m}^3\text{/min}$$ **Ans.**

4. Theoretical power of the compressor

We know that workdone by the compressor

$$= m_R (h_2 - h_1) = 10.254\,(434.5 - 413.6) = 214.3 \text{ kJ/min}$$

∴ Power of the compressor = 214.3/60 = 3.57 kJ/s or kW **Ans.**

5. C.O.P.

We know that $$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{413.6 - 271.97}{434.5 - 413.6} = \frac{141.63}{20.9} = 6.8$$ **Ans.**

Example 4.9. *A refrigeration machine using R-12 as refrigerant operates between the pressures 2.5 bar and 9 bar. The compression is isentropic and there is no undercooling in the condenser.*

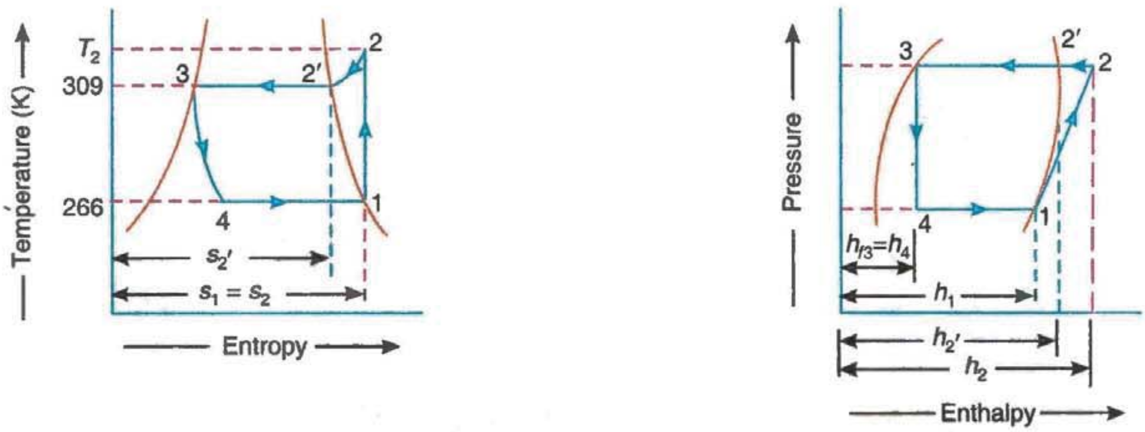
The vapour is in dry saturated condition at the beginning of the compression. Estimate the theoretical coefficient of performance. If the actual coefficient of performance is 0.65 of theoretical value, calculate the net cooling produced per hour. The refrigerant flow is 5 kg per minute. Properties of refrigerant are :

Pressure, bar	*Saturation temperature, °C*	*Enthalpy, kJ/kg*		*Entropy of saturated vapour, kJ/kg K*
		Liquid	*Vapour*	
9.0	*36*	*70.55*	*201.8*	*0.6836*
2.5	*–7*	*29.62*	*184.5*	*0.7001*

Take c_p for superheated vapour at 9 bar as 0.64 kJ/kg K.

Solution. Given : $T_{2'} = T_3 = 36°C = 36 + 273 = 309$ K ; $T_1 = T_4 = -7°C = -7 + 273 = 266$ K ; $(C.O.P.)_{actual} = 0.65\ (C.O.P.)_{th}$; $m = 5$ kg /min ; $h_{f3} = h_4 = 70.55$ kJ/kg ; $^*h_{f1} = h_{f4} = 29.62$ kJ/kg ; $h_2' = 201.8$ kJ / kg ; $h_1 = 184.5$ kJ/kg ; $s_2' = 0.6836$ kJ/kg K ; $s_1 = s_2 = 0.7001$ kJ/kg K ; $c_p = 0.64$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.13 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.13

Theoretical coefficient of performance

First of all, let us find the temperature at point 2 (T_2).

We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\ c_p\ \log\left(\frac{T_2}{T_{2'}}\right)$$

$$0.7001 = 0.6836 + 2.3 \times 0.64\ \log\left(\frac{T_2}{309}\right)$$

$$\log\left(\frac{T_2}{309}\right) = \frac{0.7001 - 0.6836}{2.3 \times 0.64} = 0.0112$$

$$\frac{T_2}{309} = 1.026 \qquad \text{... (Taking antilog of 0.0112)}$$

$$\therefore \qquad T_2 = 1.026 \times 309 = 317 \text{ K}$$

We know that enthalpy of superheated vapour at point 2,

$$h_2 = h_{2'} + c_p\ (T_2 - T_{2'})$$

$$= 201.8 + 0.64\ (317 - 309) = 206.92 \text{ kJ/kg}$$

* Superfluous data.

∴ Theoretical coefficient of performance,

$$(\text{C.O.P.})_{th} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{184.5 - 70.55}{206.92 - 184.5} = 5.1 \quad \textbf{Ans.}$$

Net cooling produced per hour

We also know that actual C.O.P. of the machine,

$$(\text{C.O.P.})_{actual} = 0.65 \times (\text{C.O.P.})_{th} = 0.65 \times 5.1 = 3.315$$

and actual work done, $w_{actual} = h_2 - h_1 = 206.92 - 184.5 = 22.42$ kJ/kg

Refrigeration machine.

We know that net cooling (or refrigerating effect) produced per kg of refrigerant

$$= w_{actual} \times (\text{C.O.P.})_{actual} = 22.42 \times 3.315 = 74.3 \text{ kJ/kg}$$

∴ Net cooling produced per hour

$$= m \times 74.3 = 5 \times 74.3 = 371.5 \text{ kJ/min}$$

$$= \frac{371.5}{210} = 1.77 \text{ TR} \quad \textbf{Ans.} \qquad \ldots (\because 1 \text{ TR} = 210 \text{ kJ/min})$$

Example 4.10. *A simple saturation cycle using R-12 is designed for taking a load of 10 tonnes. The refrigerator and ambient temperature are –0°C and 30°C respectively. A minimum temperature difference of 5°C is required in the evaporator and condenser for heat transfer. Find : 1. Mass flow rate through the system ; 2. Power required in kW ; 3. C.O.P. ; and 4. Cylinder dimensions assuming L / D = 1.2, for a single cylinder, single acting compressor if it runs at 300 r.p.m. with volumetric efficiency of 90%.*

Solution. Given : Q = 10TR = 10 × 210 = 2100 kJ/min

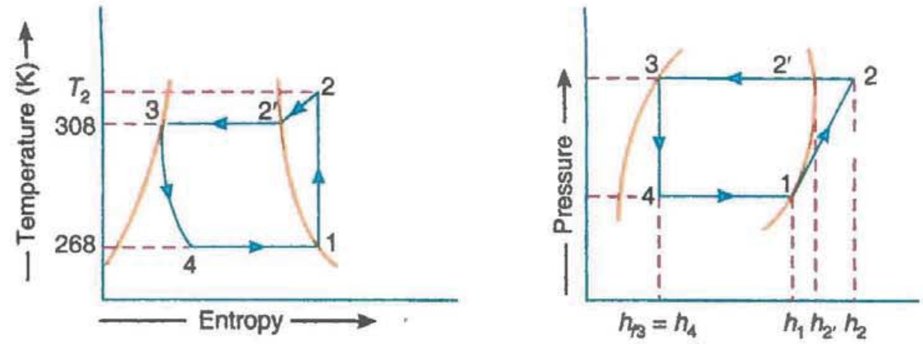
Since a minimum temperature difference of 5°C is required in the evaporator and condenser, therefore evaporator temperature would be

$$T_1 = T_4 = 0 - 5 = -5°C = -5 + 273 = 268 \text{ K}$$

and condenser temperature,

$$T_{2'} = T_3 = 30 + 5 = 35°C = 35 + 273 = 308 \text{ K}$$

The *T-s* and *p-h* diagrams are shown in Fig. 4.14 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.

(*b*) *p-h* diagram.

Fig. 4.14

From *p-h* diagram, we find that enthalpy of dry saturated vapour at –5°C (268 K) *i.e.* at point 1,

$$h_1 = 185 \text{ kJ/kg}$$

Enthalpy of superheated vapour at point 2,

$$h_2 = 206 \text{ kJ/kg}$$

Enthalpy of saturated liquid at 35°C (308 K) *i.e.* at point 3,

$$h_{f3} = h_4 = 70 \text{ kJ/kg}$$

and specific volume of dry saturated vapour at –5°C (268 K) *i.e.* at point 1,

$$v_1 = 0.065 \text{ m}^3\text{/kg}$$

1. Mass flow rate through the system

We know that refrigerating effect per kg of the refrigerant

$$= h_1 - h_{f3} = 185 - 70 = 115 \text{ kJ/kg}$$

$$\therefore \quad \text{Mass flow rate, } m_R = \frac{\text{Refrigerating capacity}}{\text{Refrigerating effect}} = \frac{10 \times 210}{115} = 18.26 \text{ kg / min} \textbf{ Ans.}$$

2. Power required

We know that workdone during compression of the refrigerant

$$= m_R (h_2 - h_1) = 18.26 (206 - 185) = 383.46 \text{ kJ/min}$$

$$\therefore \quad \text{Power required} = 383.46/60 = 6.4 \text{ kJ/s or kW} \textbf{ Ans.}$$

3. C.O.P.

We know that
$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{185 - 70}{206 - 185} = \frac{115}{21} = 5.476 \textbf{ Ans.}$$

4. Cylinder dimensions

Let D = Bore of cylinder,

L = Length of stroke = 1.2 D ...(Given)

N = Speed of compressor = 300 r.p.m. ...(Given)

η_v = Volumetric efficiency = 90% = 0.9 ...(Given)

We know that theoretical suction volume or piston displacement per minute

$$= m_R \times v_1 \times \frac{1}{\eta_v} = 18.26 \times 0.065 \times \frac{1}{0.9} = 1.32 \text{ m}^3/\text{min} \quad ...(i)$$

We also know that suction volume or piston displacement per minute

= Piston area × Stroke × R.P.M.

$$= \frac{\pi}{4} \times D^2 \times L \times N = \frac{\pi}{4} \times D^2 \times 1.2D \times 300 = 282.8D^3 \text{ m}^3/\text{min} \quad ...(ii)$$

Equating equations (*i*) and (*ii*),

$D^3 = 1.32 / 282.8 = 4.667 \times 10^{-3}$ or $D = 0.167\text{m} = 167$ mm **Ans.**

and $L = 1.2\,D = 1.2 \times 167 = 200.4$ mm **Ans.**

Example 4.11. *A water cooler using R-12 works on the condensing and evaporating temperatures of 26°C and 2°C respectively. The vapour leaves the evaporator saturated and dry. The average output of cold water is 100 kg / h cooled from 26°C to 6°C.*

Allowing 20% of useful heat into water cooler and the volumetric efficiency of the compressor as 80% and mechanical efficiency of the compressor and the electric motor as 85% and 95% respectively, find :1. volumetric displacement of the compressor ; and 2. power of the motor. Data for R-12 is given below :

Temperature °C	*Pressure bar*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		*Specific heat kJ/kg K*		*Specific volume of vapour m³/kg*
		Liquid	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*	
26	6.69	60.64	198.10	0.2270	0.6865	0.996	0.674	0.026
2	3.297	37.92	188.39	0.1487	0.6956	1.067	0.620	0.052

Solution. Given : $T_{2'} = T_3 = 26°\text{C} = 26 + 273 = 299$ K ; $T_1 = T_4 = 2°\text{C} = 2 + 273 = 275$ K ; $m_w = 100$ kg/h ; $T_{w1} = 26°\text{C} = 26 + 273 = 299$ K ; $T_{w2} = 6°\text{C} = 6 + 273 = 279$ K ; $\eta_v = 80\% = 0.80$; $\eta_{m1} = 85\% = 0.85$; $\eta_{m2} = 95\% = 0.95$; $h_{f3} = 60.64$ kJ/kg ; $*h_{f1} = 37.92$ kJ/kg ; $h_{2'} = 198.10$ kJ/kg ; $h_1 = 188.39$ kJ/kg ; $*s_{f3} = 0.2270$ kJ/kg K ; $*s_{f1} = 0.1487$ kJ/kg K ; $*s_{2'} = 0.6865$ kJ/kg K ; $s_1 = s_2 = 0.6956$ kJ/kg K ; $*c_{p3} = 0.996$ kJ/kg K ; $*c_{p4} = 1.067$ kJ/kg K; $*c_{p2'} = 0.674$ kJ/kg K ; $c_{p1} = 0.620$ kJ/kg K ; $*v_{2'} = 0.026$ m³ / kg ; $v_1 = 0.052$ m³ / kg

* Superfluous data.

1. Volumetric displacement of the compressor

The *T-s* and *p-h* diagrams are shown in Fig. 4.15 (*a*) and (*b*) respectively. Since 20% of the useful heat is lost into water cooler, therefore actual heat extracted from the water cooler,

$$h_E = 1.2\, m_w \times c_w\,(T_{w1} - T_{w2})$$
$$= 1.2 \times 100 \times 4.187\,(299 - 279) = 10\,050 \text{ kJ/h} = 167.5 \text{ kJ/min}$$

... ($\because$ Sp. heat of water, $c_w = 4.187$ kJ/kg K)

We know that heat extracted or the net refrigerating effect per kg of the refrigerant

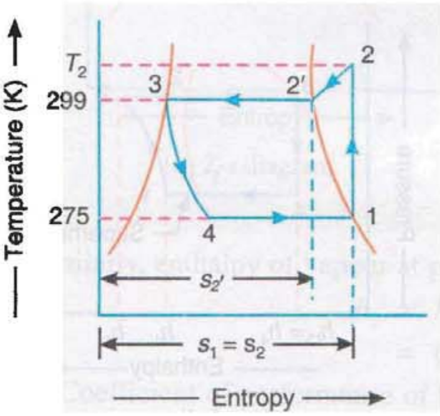
$$= h_1 - h_{f3} = 188.39 - 60.64 = 127.75 \text{ kJ/kg}$$

$\therefore$ Mass flow of the refrigerant,

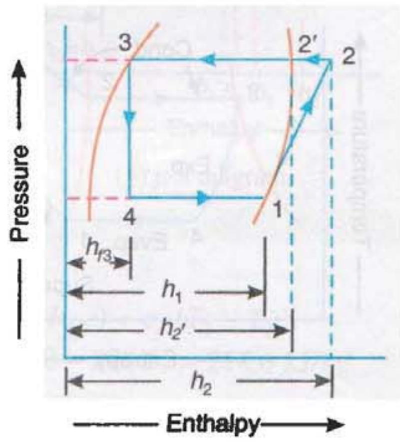
$$m_R = \frac{167.5}{127.75} = 1.3 \text{ kg/min}$$

and volumetric displacement of the compressor

$$= \frac{m_R \times v_1}{\eta_v} = \frac{1.3 \times 0.052}{0.80} = 0.085 \text{ m}^3\text{/min } \textbf{Ans.}$$



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.15

2. Power of the motor

First of all, let us find the temperature at point 2 (T_2). We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{p2'} \log\left(\frac{T_2}{T_{2'}}\right)$$

$$0.6956 = 0.6865 + 2.3 \times 0.674 \log\left(\frac{T_2}{299}\right)$$

$$\log\left(\frac{T_2}{299}\right) = \frac{0.6956 - 0.6865}{2.3 \times 0.674} = 0.005\,87$$

or $$\frac{T_2}{299} = 1.0136$$ ... (Taking anti-log of 0.005 87)

$\therefore$ $$T_2 = 299 \times 1.0136 = 303 \text{ K}$$

We know that enthalpy at point 2,

$$h_2 = h_{2'} + c_{p2'}(T_2 - T_{2'})$$
$$= 198.10 + 0.674\,(303 - 299) = 200.8 \text{ kJ/kg}$$

We also know that work done by the compressor per kg of the refrigerant

$$= h_2 - h_1 = 200.8 - 188.39 = 12.41 \text{ kJ/kg}$$

and work done per minute $= m_R \times 12.41 = 1.3 \times 12.41 = 16.133$ kJ/min

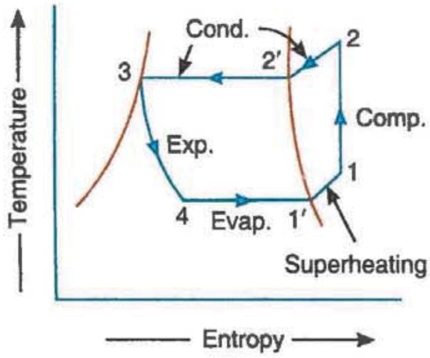
$$= 0.27 \text{ kJ/s} = 0.27 \text{ kW}$$

∴ Power required for the compressor

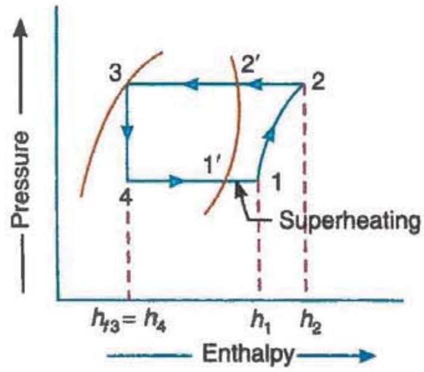
$$= \frac{0.27}{\eta_{m1}} = \frac{0.27}{0.85} = 0.317 \text{ kW}$$

and power of the motor $= \dfrac{0.317}{\eta_{m2}} = \dfrac{0.317}{0.95} = 0.334$ kW **Ans.**

4.9 Theoretical Vapour Compression Cycle with Superheated Vapour before Compression



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.16. Theoretical vapour compression cycle with superheated vapour before compression.

A vapour compression cycle with superheated vapour before compression is shown on *T-s* and *p-h* diagrams in Fig. 4.16 (*a*) and (*b*) respectively. In this cycle, the evaporation starts at point 4 and continues upto point 1′, when it is dry saturated. The vapour is now superheated before entering the compressor upto the point 1.

The coefficient of performance may be found out as usual from the relation,

$$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note: In this cycle, the heat is absorbed (or extracted) in two stages. Firstly from point 4 to point 1′ and secondly from point 1′ to point 1. The remaining cycle is same as discussed in the previous article.

Example 4.12. *A vapour compression refrigeration plant works between pressure limits of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression, its temperature being 37°C. The vapour is superheated by 5°C before entering the compressor.*

If the specific heat of superheated vapour is 0.63 kJ/kg K, find the coefficient of performance of the plant. Use the data given below :

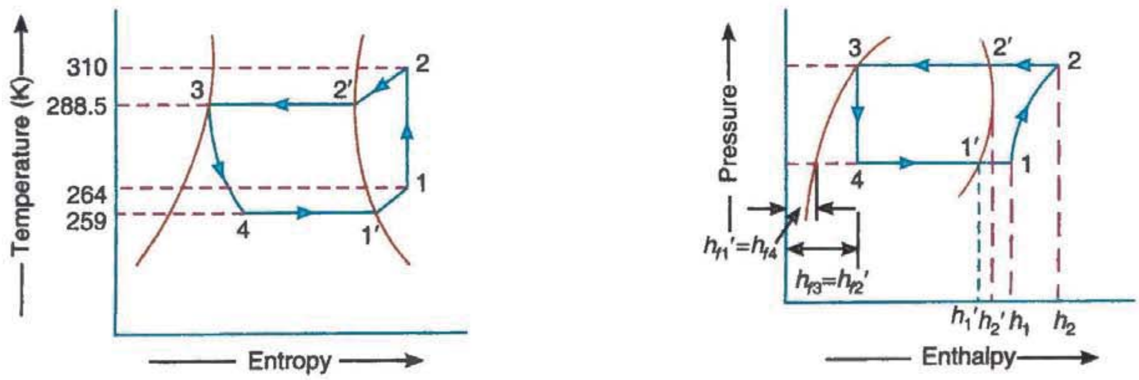
Pressure, bar	Saturation temperature, °C	Liquid heat, kJ/kg	Latent heat, kJ/kg
5.3	15.5	56.15	144.9
2.1	−14.0	25.12	158.7

Solution. Given : $p_2 = 5.3$ bar ; $p_1 = 2.1$ bar ; $T_2 = 37°C = 37 + 273 = 310$ K ; $T_1 - T_{1'} = 5°C$; $c_p = 0.63$ kJ/kg K ; $T_{2'} = 15.5°C = 15.5 + 273 = 288.5$ K ; $T_{1'} = -14°C = -14 + 273 = 259$ K ; $h_{f3} = h_{f2'} = 56.15$ kJ/kg ; $h_{f1'} = 25.12$ kJ/kg ; $h_{fg2'} = 144.9$ kJ/kg ; $h_{fg1'} = 158.7$ kJ/kg

The *T-s* and *p-h* diagrams are shown in Fig. 4.17 (*a*) and (*b*) respectively.

We know that enthalpy of vapour at point 1,

$$h_1 = h_{1'} + c_p (T_1 - T_{1'}) = (h_{f1'} + h_{fg1'}) + c_p (T_1 - T_{1'})$$
$$= (25.12 + 158.7) + 0.63 \times 5 = 186.97 \text{ kJ/kg}$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.17

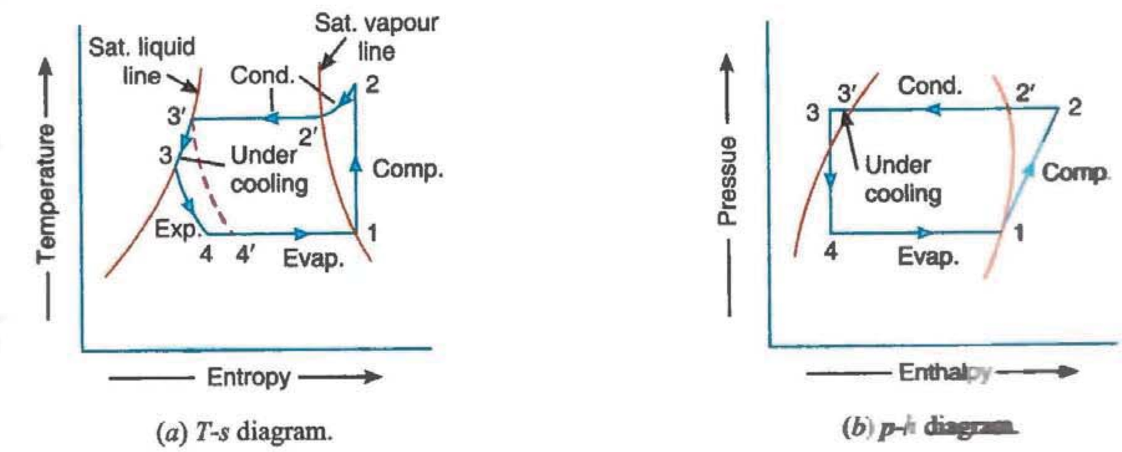
Similarly, enthalpy of vapour at point 2,

$$h_2 = h_{2'} + c_p (T_2 - T_{2'}) = (h_{f2'} + h_{fg2'}) + c_p (T_2 - T_{2'})$$
$$= (56.15 + 144.9) + 0.63\,(310 - 288.5) = 214.6 \text{ kJ/kg}$$

∴ Coefficient of performance of the plant,

$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{186.97 - 56.15}{214.6 - 186.97} = \frac{130.82}{27.63} = 4.735 \textbf{ Ans.}$$

4.10 Theoretical Vapour Compression Cycle with Under-cooling or Subcooling of Refrigerant



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.18. Theoretical vapour compression cycle with undercooling or subcooling of the refrigerant.

Sometimes, the refrigerant, after condensation process 2′-3′, is cooled below the saturation temperature (T_3') before expansion by throttling. Such a process is called *undercooling* or *subcooling* of the refrigerant and is generally done along the liquid line as shown in Fig. 4.18 (*a*) and (*b*). The ultimate effect of the undercooling is to increase the value of coefficient of performance under the same set of conditions.

The process of undercooling is generallly brought about by circulating more quantity of cooling water through the condenser or by using water colder than the main circulating water. Sometimes, this process is also brought about by employing a heat exchanger. In actual practice, the refrigerant is superheated after compression and undercooled before throttling, as shown in Fig. 4.18 (*a*) and (*b*). A little consideration will show, that the refrigerating effect is increased by adopting both the superheating and undercooling process as compared to a cycle without them, which is shown by dotted lines in Fig. 4.18 (*a*).

In this case, the refrigerating effect or heat absorbed or extracted,

$$R_E = h_1 - h_4 = h_1 - h_{f3} \qquad \ldots (\because h_4 = h_{f3})$$

and work done,

$$w = h_2 - h_1$$

$$\therefore \qquad \text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Work done}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Note: The value of h_{f3} may be found out from the relation,

$$h_{f3} = h_{f3}' - c_p \times \text{Degree of undercooling}$$

Example 4.13. *A vapour compression refrigerator uses R-12 as refrigerant and the liquid evaporates in the evaporator at – 15°C. The temperature of this refrigerant at the delivery from the compressor is 15°C when the vapour is condensed at 10°C. Find the coefficient of performance if (i) there is no undercooling, and (ii) the liquid is cooled by 5°C before expansion by throttling.*

Take specific heat at constant pressure for the superheated vapour as 0.64 kJ/kg K and that for liquid as 0.94 kJ/kg K. The other properties of refrigerant are as follows :

Temperature in °C	*Enthalpy in kJ/ kg*		*Specific entropy in kJ / kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–15	*22.3*	*180.88*	*0.0904*	*0.7051*
+10	*45.4*	*191.76*	*0.1750*	*0.6921*

Solution. Given : $T_1 = T_4 = -15°C = -15 + 273 = 258$ K ; $T_2 = 15°C = 15 + 273 = 288$ K ; $T_3 = 10°C = 10 + 273 = 283$ K ; $c_{pv} = 0.64$ kJ/kg K ; $c_{pl} = 0.94$ kJ/kg K ; $h_{f1} = 22.3$ kJ/kg ; $h_{f3}' = 45.4$ kJ/kg ; $h_1' = 180.88$ kJ/kg ; $h_2' = 191.76$ kJ/kg ; $s_{f1} = 0.0904$ kJ/kg K ; *$s_{f3} = 0.1750$ kJ/kg K ; $s_{g1} = 0.7051$ kJ/kg K ; $s_2' = 0.6921$ kJ/kg K

(i) Coefficient of performance if there is no undercooling

The *T-s* and *p-h* diagrams, when there is no undercooling, are shown in Fig. 4.19 (*a*) and (*b*) respectively.

Let $\qquad x_1$ = Dryness fraction of the refrigerant at point 1.

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1 s_{fg1} = s_{f1} + x_1 (s_{g1} - s_{f1}) \qquad \ldots (\because s_{g1} = s_{f1} + s_{fg1})$$

$$= 0.0904 + x_1 (0.7051 - 0.0904) = 0.0904 + 0.6147\, x_1 \qquad \ldots (i)$$

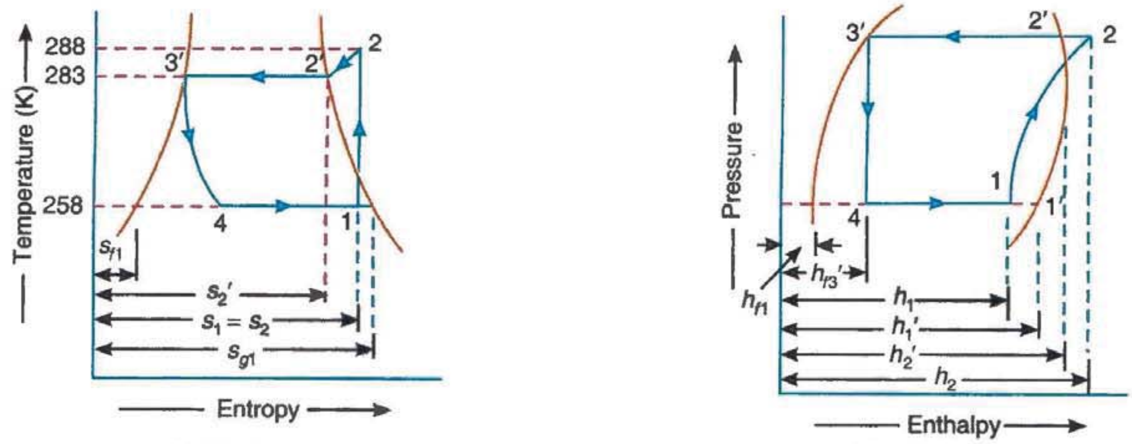
* Superfluous data.

and entropy at point 2,

$$s_2 = s_{2'} + 2.3\ c_{pv}\ \log\left(\frac{T_2}{T_{2'}}\right)$$
$$= 0.6921 + 2.3 \times 0.64\ \log\left(\frac{288}{283}\right)$$
$$= 0.6921 + 2.3 \times 0.64 \times 0.0077 = 0.7034 \qquad \text{... }(ii)$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.0904 + 0.6147\ x_1 = 0.7034 \qquad \text{or} \qquad x_1 = 0.997$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.19

We know that the enthalpy at point 1,

$$h_1 = h_{f1} + x_1\ h_{fg1} = h_{f1} + x_1\ (h_{g1} - h_{f1})$$
$$= 22.3 + 0.997\ (180.88 - 22.3) = 180.4 \text{ kJ/kg}$$
$$\text{... } (\because h_{g1} = h_1')$$

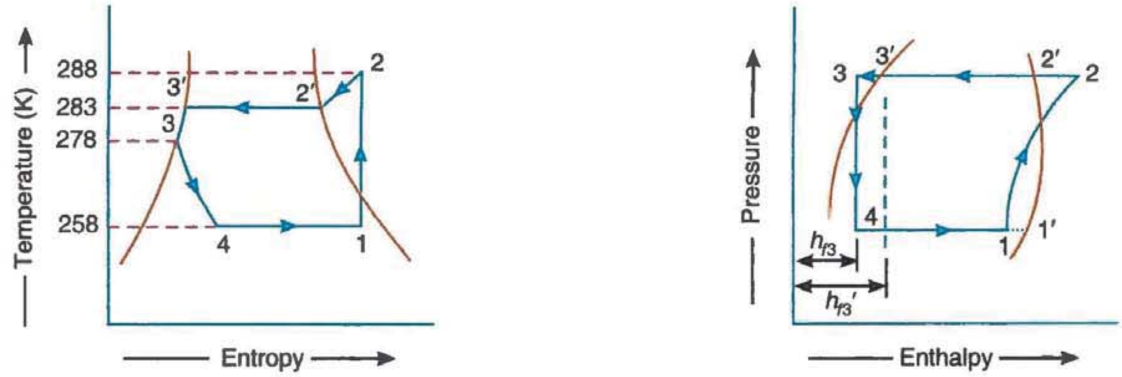
and enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv}\ (T_2 - T_{2'})$$
$$= 191.76 + 0.64\ (288 - 283) = 194.96 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3'}}{h_2 - h_1} = \frac{180.4 - 45.4}{194.96 - 180.4} = 9.27 \textbf{ Ans.}$$

(ii) Coefficient of performance when there is an undercooling of 5°C

The *T-s* and *p-h* diagrams, when there is an undercooling of 5°C, are shown in Fig. 4.20 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *T-s* diagram.

Fig. 4.20

We know that enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl} \times \text{Degree of undercooling}$$
$$= 45.4 - 0.94 \times 5 = 40.7 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{180.4 - 40.7}{194.96 - 180.4} = 9.59 \textbf{ Ans.}$$

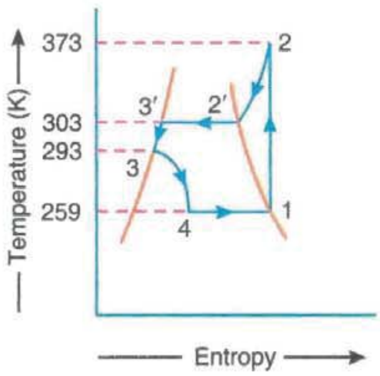
Example 4.14. *A simple NH_3 vapour compression system has compressor with piston displacement of 2 m³/min, a condenser pressure of 12 bar and evaporator pressure of 2.5 bar. The liquid is sub-cooled to 20°C by soldering the liquid line to suction line. The temperature of vapour leaving the compressor is 100°C, heat rejected to compressor cooling water is 5000 kJ/hour, and volumetric efficiency of compressor is 0.8.*

Compute : Capacity ; Indicated power ; and C.O.P. of the system.

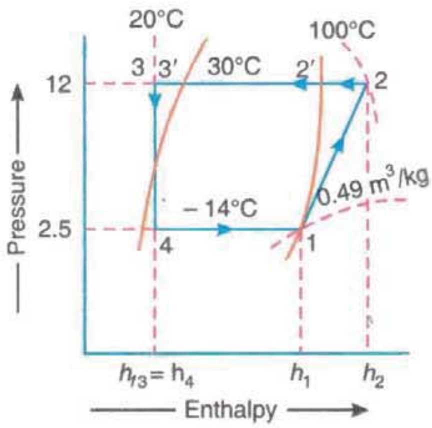
Solution. Given : $v_p = 2$ m³/min ; $p_2 = p_{2'} = p_{3'} = p_3 = 12$ bar ; $p_1 = p_4 = 2.5$ bar ; $T_3 = 20°\text{C} = 20 + 273 = 293$ K ; $T_2 = 100°\text{C} = 100 + 273 = 373$ K ; $\eta_v = 0.8$

Capacity of the system

The *T-s* and *p-h* diagrams are shown in Fig. 4.21 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.21

From *p-h* diagram, we find that the evaporating temperature corresponding to 2.5 bar is

$$T_1 = T_4 = -14°\text{C} = -14 + 273 = 259 \text{ K}$$

Condensing temperature corresponding to 12 bar is

$$T_{2'} = T_{3'} = 30°\text{C} = 30 + 273 = 303 \text{ K}$$

Specific volume of dry saturated vapour at 2.5 bar (*i.e.* at point 1),

$$v_1 = 0.49 \text{ m}^3\text{/kg}$$

Enthalpy of dry saturated vapour at point 1,

$$h_1 = 1428 \text{ kJ/kg}$$

Enthalpy of superheated vapour at point 2,

$$h_2 = 1630 \text{ kJ/kg}$$

and enthalpy of sub-cooled liquid at 20°C at point 3,

$$h_{f3} = h_4 = 270 \text{ kJ/kg}$$

Let $\qquad m_R$ = Mass flow of the refrigerant in kg/min.

We know that piston displacement,

$$v_p = \frac{m_R \times v_1}{\eta_v} \quad \text{or} \quad m_R = \frac{v_p \times \eta_v}{v_1} = \frac{2 \times 0.8}{0.49} = 3.265 \text{ kg / min}$$

We know that refrigerating effect per kg of refrigerant

$$= h_1 - h_{f3} = 1428 - 270 = 1158 \text{ kJ/kg}$$

and total refrigerating effect $= m_R (h_1 - h_{f3}) = 3.265 (1428 - 270) = 3781$ kJ/min

$\therefore$ Capacity of the system $= 3781 / 210 = 18$ TR **Ans.**

Indicated power of the system

We know that work done during compression of the refrigerant

$$= m_R (h_2 - h_1) = 3.265 (1630 - 1428) = 659.53 \text{ kJ/min}$$

Heat rejected to compressor cooling water

$$= 5000 \text{ kJ/h} = 5000/60 = 83.33 \text{ kJ/min} \qquad \text{... (Given)}$$

$\therefore$ Total work done by the system

$$= 659.53 + 83.33 = 742.86 \text{ kJ/min}$$

and indicated power of the system

$$= 742.86/60 = 12.38 \text{ kW}$$ **Ans.**

C.O.P. of the system

We know that C.O.P. of the system

$$= \frac{\text{Total refrigerating effect}}{\text{Total work done}} = \frac{3781}{742.86} = 5.1$$ **Ans.**

Example 4.15. *Saturated ammonia at 2.5 bar enters a 160mm × 150mm (bore × stroke) twin cylinder, single acting compressor whose volumetric efficiency is 79% and speed is 250 r.p.m. The head pressure is 12 bar. The subcooled liquid ammonia at 22°C enters the expansion valve. For a standard refrigeration cycle, find: 1. The ammonia circulated in kg / min.; 2. The refrigeration in TR ; and 3. The C.O.P. of the refrigeration cycle. Refer to the following table for the properties of ammonia:*

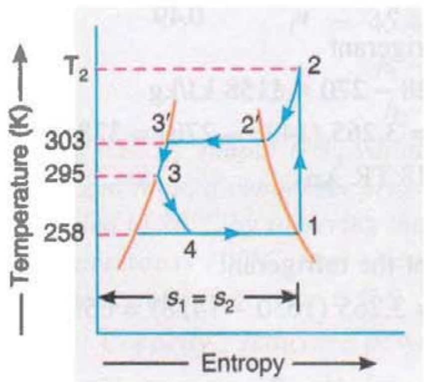
Pressure (bar)	*Saturation temperature (°C)*	*Specific volume of vapour (m³/kg)*	*Specific enthalpy (kJ/kg)*		*Specific entropy (kJ/kg K)*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
2.5	*–15*	*0.5098*	*112.4*	*1426.58*	*0.4572*	*5.5497*
12	*30*	*0.1107*	*323.08*	*1468.87*	*1.2037*	*4.9842*

Assume specific heat at constant pressure for liquid ammonia as 4.606 kJ/kg K and for superheated ammonia vapour as 2.763 kJ/kg K.

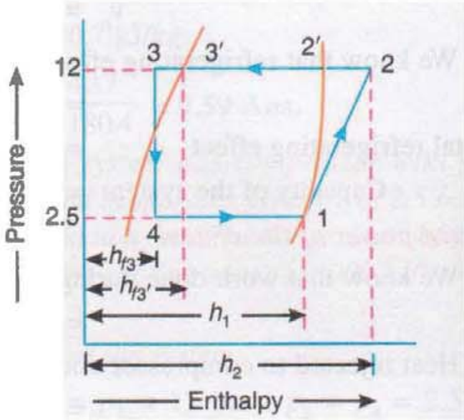
Solution. Given : $p_1 = p_4 = 2.5$ bar ; $D = 160$ mm $= 0.16$ m ; $L = 150$ mm $= 0.15$ m ; No. of cylinders = 2 ; $\eta_v = 79\% = 0.79$; $N = 250$ r.p.m. ; $p_2 = p_3 = 12$ bar ; $T_3 = 22°C = 22 + 273 = 295$ K ; $T_1 = T_4 = -15°C = -15 + 273 = 258$ K ; $T_{2'} = T_{3'} = 30°C = 30 + 273 = 303$ K ; $v_1 = 0.5098$ m³/kg; $*v_{2'} = 0.1107$ m³/kg $*h_{f1} = 112.4$ kJ/kg ; $h_{f3'} = 323.08$ kJ/kg ; $h_1 = 1426.58$ kJ/kg ; $h_{2'} = 1468.87$ kJ/kg ; $s_{f1} = 0.4572$ kJ/kg K ; $s_{f3} = 1.2037$ kJ/kg K ; $s_1 = s_2 = 5.5497$ kJ/kg K ; $s_{2'} = 4.9842$ kJ/kg K ; $c_{pl} = 4.606$ kJ/kg K ; $c_{pv} = 2.763$ kJ/kg K

* Superfluous data.

The T-s and p-h diagrams are shown in Fig. 4.22 (*a*) and (*b*) respectively.



(*a*) T-s diagram.



(*b*) p-h diagram.

Fig. 4.22

1. Ammonia circulated in kg/min

Let m_R = Mass flow rate of ammonia in kg/min.

We know that suction volume or piston displacement per minute

$$= \text{Piston area} \times \text{Stroke} \times \text{R.P.M} \times \text{No. of cylinders}$$

$$= \frac{\pi}{4} \times D^2 \times L \times N \times 2$$

$$= \frac{\pi}{4}(0.16)^2 0.15 \times 250 \times 2 = 1.508 \text{ m}^3/\text{min} \quad \text{...}(i)$$

We also know that piston displacement per minute

$$= m_R \times v_1 \times \frac{1}{\eta_v} = m_R \times 0.5098 \times \frac{1}{0.79} = 0.6453\, m_R \quad \text{...}(ii)$$

Equating equations (*i*) and (*ii*),

$$m_R = 1.508 / 0.6453 = 2.34 \text{ kg / min} \textbf{ Ans.}$$

2. Refrigeration in TR

We know that enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl}(T_{3'} - T_3)$$

$$= 323.08 - 4.606(303 - 295) = 286.23 \text{ kJ/kg}$$

and total refrigeration effect $= m_R(h_1 - h_{f3})$

$$= 2.34\,(1426.58 - 286.23) = 2668.4 \text{ kJ/min}$$

∴ Refrigeration or capacity of the system

$$= 2668.4 / 210 = 12.7 \text{ TR } \textbf{Ans.} \quad \text{... } (\because 1\text{TR} = 210 \text{ kJ/min})$$

3. C.O.P. of the refrigeration cycle

First of all, let us find the temperature of the superheated vapour at point 2 (T_2). We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right)$$

$$5.5497 = 4.9842 + 2.3 \times 2.763 \log\left(\frac{T_2}{T_{2'}}\right)$$

$$\log\left(\frac{T_2}{T_{2'}}\right) = \frac{5.5497 - 4.9842}{2.3 \times 2.763} = 0.089$$

or $$\frac{T_2}{T_{2'}} = 1.227 \qquad \text{...(Taking antilog of 0.089)}$$

$\therefore$ $$T_2 = T_{2'} \times 1.227 = 303 \times 1.227 = 371.78 \text{ K}$$

$$= 371.78 - 273 = 98.78°\text{C}$$

We know that enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv}(T_2 - T_{2'})$$

$$= 1468.87 + 2.763\,(371.78 - 303) = 1658.9 \text{ kJ/kg}$$

$\therefore$ C.O.P. of the refrigeration cycle

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1426.58 - 286.23}{1658.9 - 1426.58} = \frac{1140.35}{232.32} = 4.91 \textbf{ Ans.}$$

Example 4.16. *A vapour compression refrigerator uses methyl chloride (R-40) and operates between pressure limits of 177.4 kPa and 967.5 kPa. At entry to the compressor, the methyl chloride is dry saturated and after compression has a temperature of 102°C. The compressor has a bore and stroke of 75 mm and runs at 8 rev/s with a volumetric efficiency of 80%. The temperature of the liquid refrigerant as it leaves the condenser is 35°C and its specific heat capacity is 1.624 kJ/kg K. The specific heat capacity of the superheated vapour may be assumed to be constant. Determine : 1. refrigerator C.O.P.; 2. mass flow rate of refrigerant; and 3. cooling water required by the condenser if its temperature rise is limited to 12°C. Specific heat capacity of water = 4.187 kJ/kg K.*

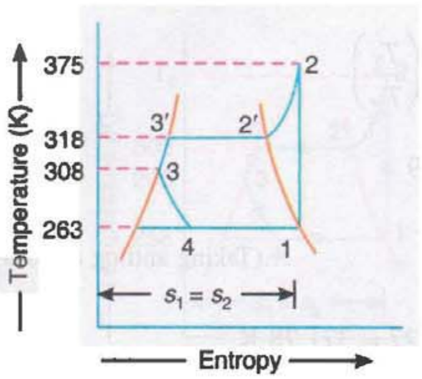
The relevant properties of methyl chloride are as follows :

Sat. temp. °C	*Pressure kPa*	*Specific volume m³/kg*		*Specific enthalpy kJ/kg*		*Specific entropy kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*
– 10	*177.4*	*0.00102*	*0.233*	*45.38*	*460.76*	*0.183*	*1.762*
45	*967.5*	*0.00115*	*0.046*	*132.98*	*483.6*	*0.485*	*1.587*

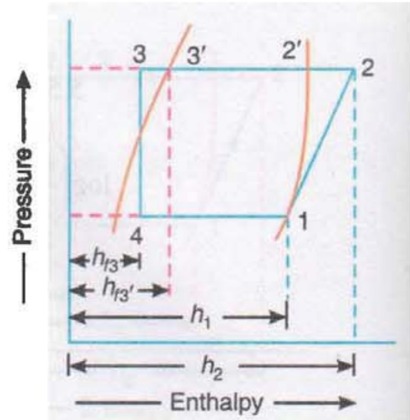
Solution. Given : $p_1 = p_4 = 177.4$ kPa ; $p_2 = p_3 = 967.5$ kPa ; $T_2 = 102°\text{C} = 102 + 273 = 375$ K ; $D = L = 75$ mm $= 0.075$ m ; $N = 8$ r.p.s. $= 480$ r.p.m. ; $\eta_v = 80\% = 0.8$; $T_3 = 35°\text{C} = 35 + 273 = 308$ K ; $c_{pl} = c_{pv} = 1.624$ kJ/kg K ; $c_{pw} = 4.187$ kJ/kg K ; $T_1 = T_4 = -10°\text{C} = -10 + 273 = 263$ K ; $T_{2}' = T_{3}' = 45°\text{C} = 45 + 273 = 318$ K ; $v_1 = 0.233$ m³/kg ; $v_{2}' = 0.046$ m³/kg ; *$h_{f1} = 45.38$ kJ/kg ; $h_{f3}' = 132.98$ kJ/kg ; $h_1 = 460.76$ kJ/kg ; $h_{2}' = 483.6$ kJ/kg ; $s_{f1} = 0.183$ kJ/kg K ; *$s_{f3} = 0.485$ kJ/kg K ; *$s_1 = s_2 = 1.762$ kJ/kg K ; $s_{2}' = 1.587$ kJ/kg K

* Superfluous data.

The *T-s* and *p-h* diagrams are shown in Fig. 4.23 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.23

1. Refrigerator C.O.P.

We know that enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv}(T_2 - T_{2'})$$
$$= 483.6 + 1.624\,(375 - 318) = 576.2 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl}(T_{3'} - T_3)$$
$$= 132.98 - 1.624\,(318 - 308) = 116.74 \text{ kJ/kg}$$

We know that refrigerator C.O.P.

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.76 - 116.74}{576.2 - 460.76} = 2.98 \textbf{ Ans.}$$

2. Mass flow rate of refrigerant

Let $\quad m_R$ = Mass flow rate of refrigerant in kg / min.

We know that suction volume or piston displacement per minute,

= Piston area × Stroke × R.P.M.

$$= \frac{\pi}{4}(0.075)^2\, 0.075 \times 480 = 0.16 \text{ m}^3\text{/min} \qquad \dots (i)$$

We also know that piston displacement per minute

$$= m_R \times v_1 \times \frac{1}{\eta_v} = m_R \times 0.233 \times \frac{1}{0.8} = 0.29\, m_R \qquad \dots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_R = 0.16 / 0.29 = 0.55 \text{ kg / min} \textbf{ Ans.}$$

3. Cooling water required by the condenser

Let $\quad m_w$ = Cooling water required by the condenser in kg / min.

We know that heat given out by the refrigerant in the condenser

$$= m_R(h_2 - h_{f3}) = 0.55\,(576.2 - 116.74) \text{ kg / min}$$
$$= 252.7 \text{ kJ/min} \qquad \dots (iii)$$

and heat taken by water in the condenser

$$= m_w \times c_{pw} \times \text{Rise in temperature}$$
$$= m_w \times 4.187 \times 12 = 50.244\, m_w \quad \ldots (iv)$$

Equating equations (*iii*) and (*iv*),

$$m_w = 252.7/50.244 = 5.03 \text{ kg / min} \textbf{ Ans.}$$

Example 4.17. *The following data refer to a single stage vapour compression system:*

Refrigerant used (Ozone friendly) R-134a; Condensing temperature = 35°C; Evaporator temperature = –10°C; Compressor R.P.M. = 2800; Clearance volume/Swept volume = 0.03; Swept volume = 269.4 × 10^{-6} m³; Expansion index = 1.12; Compression efficiency = 0.8; Condensate subcooling = 5°C.

Find : 1. Capacity of the system in TR ; 2. Power required ; 3. C.O.P.; 4. Heat rejection to condenser ; and 5. Refrigeration efficiency.

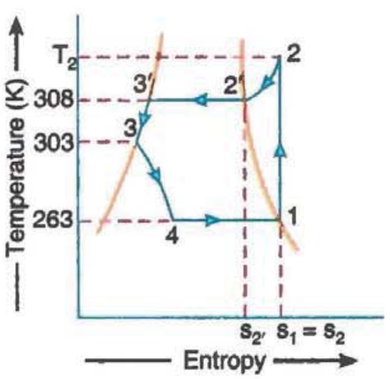
The properties of R-134a are given below:

Sat.temp. °C	*Pressure bar*	*Specific volume of vapour, m³/kg*	*Specific enthalpy kJ/kg*		*Specific entropy kJ/kg K*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
–10	*2.014*	*0.0994*	*186.7*	*392.4*	*0.9512*	*1.733*
35	*8.870*	*—*	*249.1*	*417.6*	*1.1680*	*1.715*

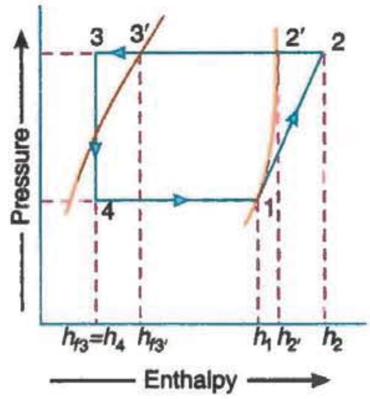
Assume isentropic compression and suction vapour as dry saturated. The specific heat of vapour refrigerant may be taken as 1.1 kJ/kg K and for liquid refrigerant as 1.458 kJ/kg K.

Solution. Given : $T_{2'} = T_{3'} = 35°\text{C} = 35 + 273 = 308$ K ; $T_1 = T_4 = -10°\text{C} = -10 + 273 = 263$ K ; $N = 2800$ r.p.m.; $v_c/v_s = C = 0.03$; $v_s = 269.4 \times 10^{-6}$ m³ ; $n = 1.12$; $\eta_c = 0.8$; $T_3 = 35 - 5 = 30°\text{C} = 30 + 273 = 303$ K ; $p_1 = p_4 = 2.014$ bar ; $p_{2'} = p_{3'} = 8.870$ bar ; $v_1 = 0.0994$ m³/kg ; $^*h_{f1} = 186.7$ kJ/kg ; $h_{f3'} = 249.1$ kJ/kg ; $h_1 = 392.4$ kJ/kg ; $h_{2'} = 417.6$ kJ/kg ; $s_{f1} = 0.9512$ kJ/kg K ; $^* s_{f3} = 1.1680$ kJ/kg K ; $s_1 = s_2 = 1.733$ kJ/kg K ; $s_{2'} = 1.715$ kJ/kg K ; $c_{pv} = 1.1$ kJ/kg K ; $c_{pl} = 1.458$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.24 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.24

* Superfluous data.

First of all, let us find the temperature of superheated vapour at point 2 (T_2). We know that entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right)$$

$$1.733 = 1.715 + 2.3 \times 1.1 \log\left(\frac{T_2}{308}\right)$$

$$\log\left(\frac{T_2}{308}\right) = \frac{1.733 - 1.715}{2.3 \times 1.1} = 0.007\,114$$

$$\frac{T_2}{308} = 1.0165 \qquad \text{... (Taking antilog of 0.007 114)}$$

$\therefore$ $T_2 = 1.0165 \times 308 = 313.08$ K

We know that enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv}(T_2 - T_{2'})$$

$$= 417.6 + 1.1(313.08 - 308) = 423.2 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl}(T_{3'} - T_3)$$

$$= 249.1 - 1.458\,(308 - 303) = 241.81 \text{ kJ/kg}$$

We know that volumetric efficiency of the compressor,

$$\eta_v = 1 + C - C\left(\frac{p_2}{p_1}\right)^{1/n} = 1 + 0.03 - 0.03\left[\frac{8.87}{2.014}\right]^{1/1.12}$$

$$= 1.03 - 0.113 = 0.917$$

Let m_R = Mass flow rate of the refrigerant in kg / min.

We know that piston displacement per minute

$$= \text{Swept volume} \times \text{R.P.M.}$$

$$= 269.4 \times 10^{-6} \times 2800 = 0.754\,32 \text{ m}^3\text{/min} \qquad \text{...(i)}$$

We also know that piston displacement per minute

$$= m_R \times v_1 \times \frac{1}{\eta_v} = m_R \times 0.0994 \times \frac{1}{0.917} = 0.1084\, m_R \qquad \text{...(ii)}$$

Equating equations (*i*) and (*ii*),

$$m_R = 0.75432 / 0.1084 = 6.96 \text{ kg/min}$$

1. Capacity of the system

We know that refrigerating effect per minute

$$= m_R(h_1 - h_{f3}) = 6.96\,(392.4 - 241.81) = 1048 \text{ kJ/min}$$

∴ Capacity of the system

$= 1048 / 210 = 4.991$ TR **Ans.** ... ($\because$ 1 TR = 210 kJ/min)

2. Power required

We know that workdone during compression of the refrigerant

$= m_R (h_2 - h_1) = 6.96\ (423.2 - 392.4) = 214.47$ kJ/min

∴ Power required $= 214.47/60 = 3.57$ kJ/s or kW **Ans.**

3. C.O.P.

We know that $$\text{C.O.P.} = \frac{\text{Refrigerating effect}}{\text{Workdone}} = \frac{1048}{214.4} = 4.89 \text{ Ans.}$$

4. Heat rejected to condenser

We know that heat rejected to condenser

$= m_R (h_2 - h_{f3})$

$= 6.96\ (423.2 - 241.81) = 1262.47$ kJ/min **Ans.**

Note: The heat rejected to condenser in also equal to refrigerating effect + workdone (1048 + 214.47 = 1262.47 kJ/min).

5. Refrigeration efficiency

We know that C.O.P of the Carnot cycle,

$$(\text{C.O.P.})_{Carnot} = \frac{T_1}{T_{2'} - T_1} = \frac{263}{308 - 263} = 5.844$$

$$\therefore \text{ Refrigeration efficiency} = \frac{(\text{C.O.P.})_{Cycle}}{(\text{C.O.P.})_{Carnot}} = \frac{4.89}{5.844} = 0.8367 \text{ or } 83.67\% \text{ Ans.}$$

Example 4.18. *A commercial refrigerator operates with R-12 between 1.2368 bar and 13.672 bar. The vapour is dry and saturated at the compressor inlet. Assuming isentropic compression, determine the theoretical C.O.P. of the plant. The isentropic discharge temperature is 64.86°C. If the actual C.O.P. of the plant is 80% of the theoretical, calculate the power required to run the compressor to obtain a refrigerating capacity of 1 TR. If the liquid is sub-cooled through 10°C after condensation, calculate the power required. The properties of R-12 are given below :*

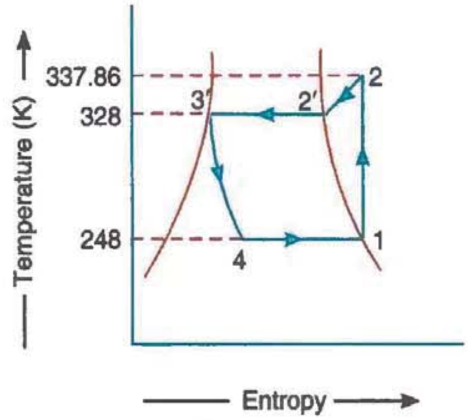
Saturation temp. (°C)	*Saturation pressure (bar)*	*Enthalpy (kJ/kg)*		*Entropy (kJ/kg K)*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
–25	*1.2368*	*13.33*	*176.48*	*0.0552*	*0.7126*
55	*13.672*	*90.28*	*207.95*	*0.3197*	*0.6774*

Properties of superheated R-12

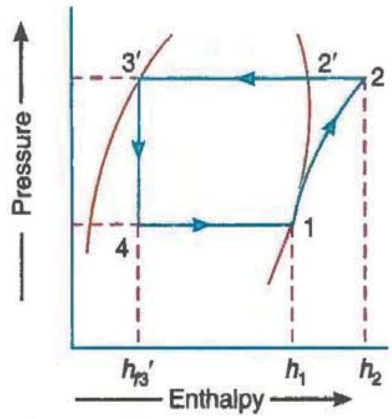
Temperature (°C)	*Pressure (bar)*	*Enthalpy (kJ/kg)*	*Entropy (kJ/kg K)*
64.86	*13.672*	*220.6*	*0.7126*

Assume specific heat of liquid to be 1.055 kJ/kg K.

Solution. Given : $p_1 = p_4 = 1.2368$ bar ; $p_2 = p_3 = 13.672$ bar ; $T_2 = 64.86°C = 64.86 + 273 = 337.86$ K ; $(C.O.P.)_{actual}$ = 80% $(C.O.P)_{th}$; Q = 1TR ; $T_1 = -25°C = -25 + 273 = 248$ K ; $T_2' = 55°C = 55 + 273 = 328$ K ; $^*h_{f1} = 13.33$ kJ/kg ; $h_1 = 176.48$ kJ/kg ; $^*s_{f1} = 0.0552$ kJ/kg K ; $^*s_1 = s_2 = 0.7126$ kJ/kg K ; $h_{f3}' = 90.28$ kJ/kg ; $^*h_2' = 207.95$ kJ/kg K ; $^*s_{f3} = 0.3197$ kJ/kg K ; $^*s_2' = 0.6774$ kJ/kg K ; $h_2 = 220.6$ kJ/kg ; $c_{pl} = 1.055$ kJ/kg K



(a) T-s diagram.



(b) p-h diagram.

The T-s and p-h diagrams are shown in Fig. 4.25 (a) and (b) respectively.

Theoretical C.O.P. of the plant

We know that theoretical C.O.P. of the plant

$$= \frac{h_1 - h_{f3}'}{h_2 - h_1} = \frac{176.48 - 90.28}{220.6 - 176.48} = 1.95 \textbf{ Ans.}$$

Power required to run the compressor

Since the actual C.O.P. of the plant is 80% of the theoretical, therefore,

$$(C.O.P.)_{actual} = 0.8 \times 1.95 = 1.56$$

and actual work done by the compressor,

$$w_{actual} = h_2 - h_1 = 220.6 - 176.48 = 44.12 \text{ kJ/kg}$$

∴ Net refrigerating effect produced per kg of refrigerant

$$= w_{actual} \times (C.O.P.)_{actual}$$
$$= 44.12 \times 1.56 = 68.83 \text{ kJ/kg}$$

We know that refrigerating capacity

$$= 1 \text{ TR} = 210 \text{ kJ/min}$$

∴ Mass flow of refrigerant,

$$m_R = 210/68.83 = 3.05 \text{ kg/min}$$

and work done during compression of the refrigerant

$$= m_R (h_2 - h_1) = 3.05 (220.6 - 176.48) = 134.57 \text{ kJ/min}$$

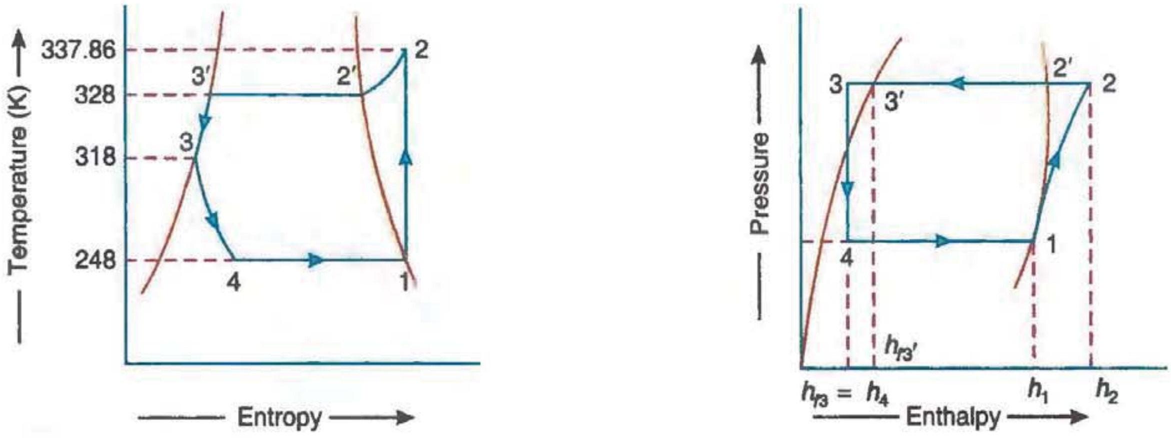
∴ Power required to run the compressor

$$= 134.57/60 = 2.24 \text{ kW } \textbf{Ans.}$$

* Superfluous data.

Power required if the liquid is subcooled through 10°C

The T-s and p-h diagrams with subcooling of liquid are shown in Fig. 4.26 (a) and (b) respectively.



(a) T-s diagram. (b) p-h diagram.

We know that enthalpy of liquid refrigerant at point 3,

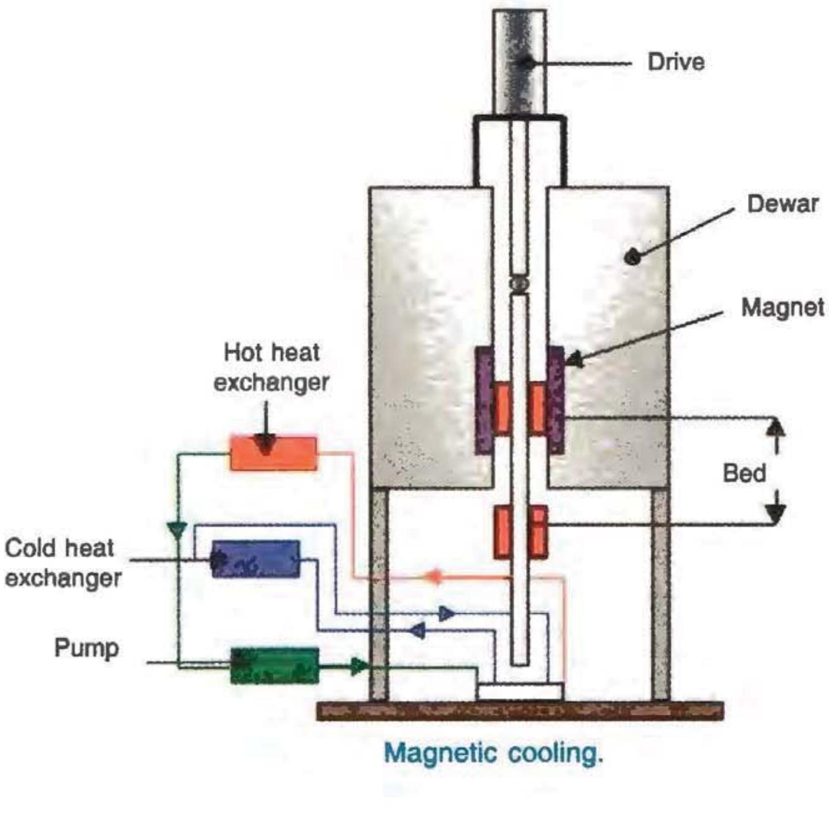
$$h_{f3} = h_{f3}' - c_{pl} \times \text{Degree of subcooling}$$
$$= 90.28 - 1.055 \times 10 = 79.73 \text{ kJ/kg}$$

$$\therefore \quad \text{Theoretical C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{176.48 - 79.73}{220.6 - 176.48} = 2.2$$

and $$(\text{C.O.P.})_{actual} = 0.8 \times 2.2 = 1.76$$

Actual work done by the compressor,

$$w_{actual} = h_2 - h_1 = 220.6 - 176.48 = 44.12 \text{ kJ/kg}$$



Magnetic cooling.

∴ Net refrigeration effect produced per kg of refrigerant

$$= w_{actual} \times (\text{C.O.P.})_{actual} = 44.12 \times 1.76 = 77.65 \text{ kJ/kg}$$

We know that refrigerating capacity

$$= 1 \text{ TR} = 210 \text{ kJ/min}$$

∴ Mass flow of the refrigerant,

$$m_R = 210 / 77.65 = 2.7 \text{ kg/min}$$

and work done during compression of the refrigerant

$$= m_R (h_2 - h_1) = 2.7 (220.6 - 176.48) = 119.12 \text{ kJ/min}$$

∴ Power required $= 119.12 / 60 = 1.985$ kW **Ans.**

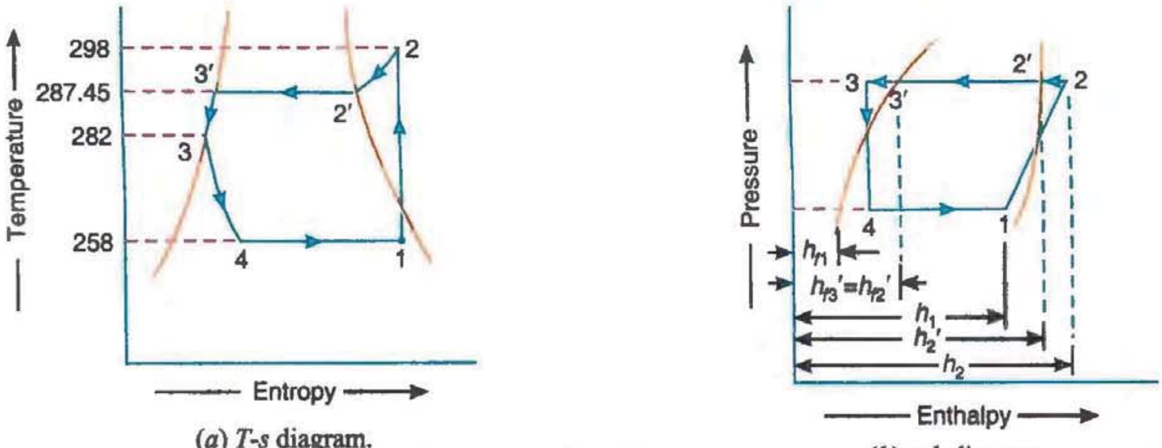
Example 4.19. *A vapour compression refrigerator works between the pressures 4.93 bar and 1.86 bar. The vapour is superheated at the end of compression, its temperature being 25°C. The liquid is cooled to 9°C before throttling. The vapour is 95% dry before compression. Using the data given below, calculate the coefficient of performance and refrigerating effect per kg of the working substance circulated :*

Pressure ,bar	*Saturation temp., °C*	*Total heat (liquid), kJ/kg*	*Latent heat, kJ/kg*
1.86	*–15*	*21.67*	*161.41*
4.93	*14.45*	*49.07*	*147.80*

The specific heat at constant pressure for the superheated vapour is 0.645 kJ/kg K and for the liquid is 0.963 kJ/kg K.

Solution. Given : $p_2 = p_3 = 4.93$ bar ; $p_1 = p_4 = 1.86$ bar ; $T_2 = 25°\text{C} = 25 + 273 = 298$ K ; $T_3 = 9°\text{C} = 9 + 273 = 282$ K ; $x_1 = 95\% = 0.95$; $T_{3'} = T_{2'} = 14.45°\text{C} = 14.45 + 273 = 287.45$ K; $T_1 = T_4 = -15°\text{C} = -15 + 273 = 258$ K ; $h_{f1} = 21.67$ kJ/kg ; $h_{f3'} = h_{f2'} = 49.07$ kJ/kg ; $h_{fg1} = 161.41$ kJ/kg ; $h_{fg2'} = 147.8$ kJ/kg ; $c_{pv} = 0.645$ kJ/kg K ; $c_{pl} = 0.963$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.27 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram.

(*b*) *p-h* diagram.

Fig. 4.27

Coefficient of performance

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1 h_{fg1} = 21.67 + 0.95 \times 161.41 = 175 \text{ kJ/kg}$$

Similarly, enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv} \times \text{Degree of superheat}$$

$$= (h_{f2'} + h_{fg2'}) + c_{pv} (T_2 - T_{2'})$$

$$= (49.07 + 147.8) + 0.645 (298 - 287.45) = 203.67 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3}' - c_{pl} \times \text{Degree of undercooling}$$
$$= h_{f3}' - c_{pl}(T_3' - T_3)$$
$$= 49.07 - 0.963\,(287.45 - 282) = 43.82 \text{ kJ/kg}$$

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{175 - 43.82}{203.67 - 175} = 4.57 \textbf{ Ans.}$$

Refrigerating effect per kg of the working substance

We know that the refrigerating effect

$$= h_1 - h_{f3} = 175 - 43.82 = 131.18 \text{ kJ/kg} \textbf{ Ans.}$$

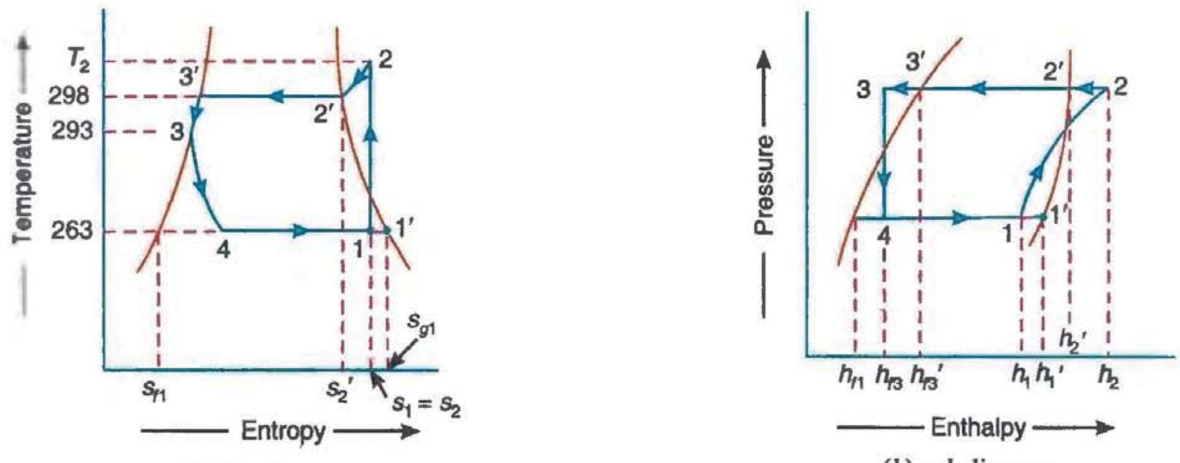
Example 4.20. *In a 15 TR ammonia refrigeration plant, the condensing temperature is 25°C and evaporating temperature – 10°C. The refrigerant ammonia is sub-cooled by 5°C before passing through the throttle valve. The vapour leaving the evaporator is 0.97 dry. Find : 1. coefficient of performance ; and 2. power required.*

Use the following properties of ammonia : —

Saturation temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		*Specific heat, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*
25	*298.9*	*1465.84*	*1.1242*	*5.0391*	*4.6*	*2.8*
–10	*135.37*	*1433.05*	*0.5443*	*5.4770*	—	—

Solution. Given : $Q = 15$ TR ; $T_2' = T_3' = 25°\text{C} = 25 + 273 = 298$ K ; $T_1 = T_4 = -10°\text{C} = -10 + 273 = 263$ K ; $T_3 = 25 - 5 = 20°\text{C} = 20 + 273 = 293$ K ; $x_1 = 0.97$; $h_{f3}' = 298.9$ kJ/kg ; $h_2' = 1465.84$ kJ/kg ; *$s_{f3}' = 1.1242$ kJ/kg K ; $s_{g2}' = s_2' = 5.0391$ kJ/kg K ; $c_{pl} = 4.6$ kJ/kg K ; $c_{pv} = 2.8$ kJ/kg K ; $h_{f1} = 135.37$ kJ/kg ; $h_1' = 1433.05$ kJ/kg ; $s_{f1} = 0.5443$ kJ/kg ; $s_{g1} = 5.4770$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.28 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.28

First of all, let us find the temperature of refrigerant at point 2 (T_2).

* Superfluous data.

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1\, s_{fg1} = s_{f1} + x_1\,(s_{g1} - s_{f1}) \qquad \ldots (\because s_{g1} = s_{f1} + s_{fg1})$$

$$= 0.5443 + 0.97\,(5.4770 - 0.5443) = 5.329 \text{ kJ/kg K} \quad \ldots (i)$$

and entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right) = 5.0391 + 2.3 \times 2.8 \log\left(\frac{T_2}{298}\right)$$

$$= 5.0391 + 6.44 \log\left(\frac{T_2}{298}\right) \qquad \ldots (ii)$$

Since entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$5.329 = 5.0391 + 6.44 \log\left(\frac{T_2}{298}\right)$$

$$\log\left(\frac{T_2}{298}\right) = \frac{5.329 - 5.0391}{6.44} = 0.045$$

or

$$\frac{T_2}{298} = 1.109 \qquad \ldots \text{(Taking antilog of 0.045)}$$

$$\therefore \quad T_2 = 298 \times 1.109 = 330 \text{ K}$$

1. *Coefficient of performance*

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1\, h_{fg1} = h_{f1} + x_1\,(h_{g1} - h_{f1}) \quad \ldots (\because h_{g1} = h_{f1} + h_{fg1})$$

$$= 135.37 + 0.97\,(1433.05 - 135.37) = 1394.12 \text{ kJ/kg}$$

$$\ldots (\because h_{g1} = h_{1'})$$

Enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv} \times \text{Degree of superheat} = h_{2'} + c_{pv}\,(T_2 - T_{2'})$$

$$= 1465.84 + 2.8\,(330 - 298) = 1555.44 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl} \times \text{Degree of undercooling}$$

$$= 298.9 - 4.6 \times 5 = 275.9 \text{ kJ/kg}$$

Ammonia refrigeration plant.

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1394.12 - 275.9}{1555.44 - 1394.12} = 6.93 \text{ Ans.}$$

2. Power required

We know that the heat extracted or refrigerating effect produced per kg of refrigerant

$$R_E = h_1 - h_{f3} = 1394.12 - 275.9 = 1118.22 \text{ kJ/kg}$$

and refrigerating capacity of the system,

$$Q = 15 \text{ TR} = 15 \times 210 = 3150 \text{ kJ/min} \quad \text{... (Given)}$$

∴ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{3150}{1118.22} = 2.81 \text{ kg/min}$$

Work done during compression of the refrigerant

$$= m_R (h_2 - h_1) = 2.81 (1555.44 - 1394.12) = 453.3 \text{ kJ/min}$$

∴ Power required $= 453.3 / 60 = 7.55$ kW **Ans.**

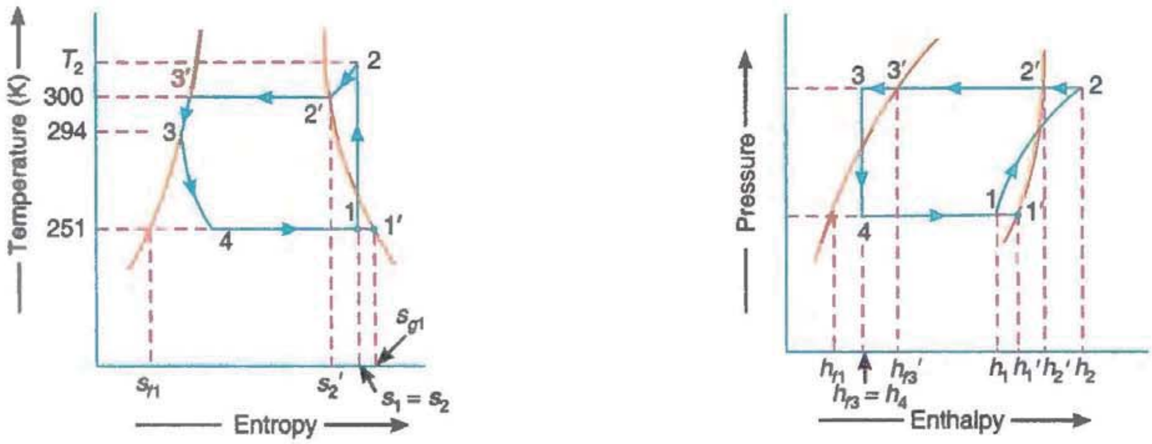
Example 4.21. *A refrigeration system of 10.5 tonnes capacity at an evaporator temperature of –12°C and condenser temperature of 27°C is needed in a food storage locker. The refrigerant ammonia is subcooled by 6°C before entering the expansion valve. The vapour is 0.95 dry as it leaves the evaporator coil. The compression in the compressor is isentropic. Find:*

1. Condition of vapour at outlet of the compressor ; 2. Condition of vapour at entrance to the evaporator ; 3. C.O.P. ; and 4. Power required in kW.

Neglect valve throttling and clearance effect.

Solution. Given : $Q = 10.5$ TR ; $T_1 = T_4 = -12°C = -12 + 273 = 251K$; $T_{2'} = T_{3'} = 27°C = 27 + 273 = 300K$; $T_3 = 27 - 6 = 21°C = 21 + 273 = 294K$; $x_1 = 0.95$

The *T-s* and *p-h* diagrams are shown in Fig. 4.29 (*a*) and (*b*) respectively.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.29

1. Condition of vapour at outlet of the Compressor

Let T_2 = Temperature of the vapour refrigerant at the outlet of the compressor.

From the Refrigeration tables for ammonia, we find that corresponding to 27°C, entropy at point 2′,

$$s_{2'} = s_{g2'} = 5.0170 \text{ kJ/kg K}$$

and corresponding to –12°C, $s_{f1} = 0.5096$ kJ/kg K, and $s_{g1} = 5.5055$ kJ/kg K

We know that entropy at point 1,

$$s_1 = s_{f1} + x_1\, s_{fg1} = s_{f1} + x_1(s_{g1} - s_{f1})$$

$$= 0.5096 + 0.95(5.5055 - 0.5096) = 5.2557 \text{ kJ/kg K} \quad ...(i)$$

Since s_1 (at –12°C) is greater than $s_{2'}$ (at 27°C), therefore condition of vapour at point 2 is superheated. Taking the specific heat at constant pressure for the superheated vapour (c_{pv}) as 2.8 kJ/kg K, we have entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right) = 5.0170 + 2.3 \times 2.8 \log\left(\frac{T_2}{300}\right)$$

$$= 5.0170 + 6.44 \log\left(\frac{T_2}{300}\right) \quad ...(ii)$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$5.2557 = 5.0170 + 6.44 \log\left(\frac{T_2}{300}\right)$$

$$\log\left(\frac{T_2}{300}\right) = \frac{5.2557 - 5.0170}{6.44} = 0.037\,06$$

or $$\frac{T_2}{300} = 1.089 \quad \text{(Taking antilog of 0.037 06)}$$

$\therefore$ $$T_2 = 300 \times 1.089 = 326.7 \text{ K}$$

$$= 326.7 - 273 = 53.7°\text{C} \quad \textbf{Ans.}$$

In other words, the vapour refrigerant is superheated by (53.7 – 27) = 26.7°C.

2. Condition of vapour at entrance to evaporator

Let x_4 = Dryness fraction of vapour refrigerant at entrance to evaporator at point 4.

From the Refrigeration tables for ammonia, we find that at 27°C, $h_{f3'}$ = 308.63 kJ/kg ; and corresponding to –12°C, $h_{f4} = 126.16$ kJ/kg and $h_{g4} = 1430.54$ kJ/kg.

We know that enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl} \times \text{Degree of undercooling} = h_{f3'} - c_{pl}(T_{3'} - T_3)$$

$$= 308.63 - 4.6(300 - 294) = 281.06 \text{ kJ/kg} \quad ...(iii)$$

...(Taking specific heat of liquid ammonia as 4.6 kJ/kg K) and enthalpy of vapour refrigerant at point 4,

$$h_4 = h_{f4} + x_4 h_{fg4} = h_{f4} + x_4(h_{g4} - h_{f4}) \qquad ...(\because h_{g4} = h_{f4} + h_{...}$$

$$= 126.16 + x_4\,(1430.54 - 126.16)$$

$$= 126.16 + 1304.38\,x_4 \qquad ...(iv)$$

Since enthalpy of liquid refrigerant at point 3 (h_{f3}) is equal to enthalpy of vapour refrigerant at point 4 (h_4), therefore equating equations (*iii*) and (*iv*),

$$281.06 = 126.16 + 1304.38x_4 \quad \text{or} \quad x_4 = 0.1187 \quad \textbf{Ans.}$$

3. C.O.P.

From Refrigeration tables for ammonia, we find that corresponding to –12°C (*i.e.* at point 1),

$$h_{f1} = 126.16 \text{ kJ/kg} \text{ ; and } h_{g1} = 1430.54 \text{ kJ/kg}$$

and corresponding to 27°C (*i.e.* at point 2′),

$$h_{2'} = 1467.22 \text{ kJ/kg}$$

We know that enthalpy at point 1,

$$h_1 = h_{f1} + x_1\,h_{fg1} = h_{f1} + x_1(h_{g1} - h_{f1})$$

$$= 126.16 + 0.95\,(1430.54 - 126.16) = 1365.32 \text{ kJ/kg}$$

and enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv} \times \text{Degree of superheat} = h_{2'} + c_{pv}(T_2 - T_{2'})$$

$$= 1467.22 + 2.8\,(326.7 - 300) = 1541.98 \text{ kJ/kg}$$

We know that

$$\text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{1365.32 - 281.06}{1541.98 - 1365.32} = \frac{1084.26}{176.66} = 6.137 \quad \textbf{Ans.}$$

4. Power required

We know that refrigerating effect produced per kg of refrigerant,

$$R_E = h_1 - h_{f3} = 1365.32 - 281.06 = 1084.26 \text{ kJ/kg}$$

and refrigerating capacity of the system,

$$Q = 10.5 \text{ TR} = 10.5 \times 210 = 2205 \text{ kJ/min}$$

∴ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2205}{1084.26} = 2.0336 \text{ kg / min}$$

Workdone during compression of the refrigerant

$$= m_R(h_2 - h_1) = 2.0336\,(1541.98 - 1365.32) = 359.26 \text{ kJ/min}$$

∴ Power required = 359.26 / 60 = 5.987 kJ/s or kW **Ans.**

Example 4.22. *The following data refers to a 20 TR ice plant using ammonia as refrigerant :*

The temperature of water entering and leaving the condenser are 20°C and 27°C and temperature of brine in the evaporator is – 15°C.

Before entering the expansion valve, ammonia is cooled to 20°C and the ammonia enters the compressor dry saturated.

Calculate for one tonne of refrigeration the power expended, the amount of cooling water in the condenser and the coefficient of performance of the plant.

Use the properties given in the table below :

Saturation temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		*Specific heat, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*	*Liquid*	*Vapour*
–15	*112.34*	*1426.54*	*0.4572*	*5.5490*	*4.396*	*2.303*
25	*298.90*	*1465.84*	*1.1242*	*5.0391*	*4.606*	*2.805*

Solution. Given : Q = 20 TR ; T_{w2} = 20°C = 20 + 273 = 293 K ; T_{w1} = 27°C = 27 + 273 = 300 K ; $T_{2'} = T_{3'}$ = 25°C = 25 + 273 = 298 K ; $T_1 = T_4$ = – 15°C = – 15 + 273 = 258 K ; T_3 = 20°C = 20 + 273 = 293 K ; $*h_{f1}$ = 112.34 kJ/kg ; $h_{f3'}$ = 298.90 kJ/kg ; h_1 = 1426.54 kJ/kg ; $h_{2'}$ = 1465.84 kJ/kg ; $*s_{f1}$ = 0.4572 kJ/kg K ; $*s_{f3}$ = 1.1242 kJ/kg K ; $s_1 = s_2$ = 5.5490 kJ/kg K ; $s_{2'}$ = 5.0391 kJ/kg K ; $*c_{pl4}$ = 4.396 kJ/kg K ; c_{pl3} = 4.606 kJ/kg K ; $*c_{pv1}$ = 2.303 kJ/kg K ; $c_{pv2'}$ = 2.805 kJ/kg K

Power expended per TR

The *T-s* and *p-h* diagrams are shown in Fig. 4.30 (*a*) and (*b*) respectively. First of all, let us find the temperature of refrigerant at point 2 (T_2). We know that entropy at point 2,

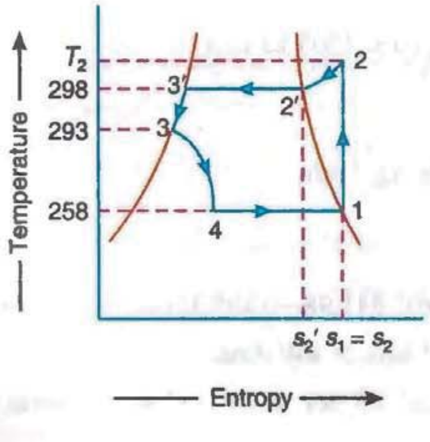
$$s_2 = s_{2'} + 2.3\, c_{pv2'} \log\left(\frac{T_2}{T_{2'}}\right)$$

$$5.5490 = 5.0391 + 2.3 \times 2.805 \log\left(\frac{T_2}{298}\right)$$

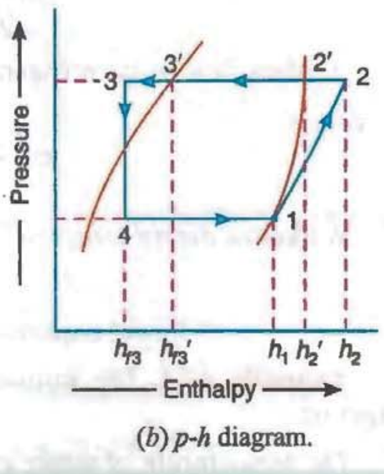
$$\log\left(\frac{T_2}{298}\right) = \frac{5.5490 - 5.0391}{2.3 \times 2.805} = 0.079$$

$$\frac{T_2}{298} = 1.2 \qquad \text{... (Taking antilog of 0.079)}$$

or $$T_2 = 1.2 \times 298 = 357.6 \text{ K or } 84.6° \text{ C}$$



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.30

* Superfluous data.

∴ Enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv2'} (T_2 - T_{2'})$$
$$= 1465.84 + 2.805 (357.6 - 298) = 1633.02 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl3} \times \text{Degree of undercooling}$$
$$= h_{f3'} - c_{pl3} (T_{3'} - T_3)$$
$$= 298.9 - 4.606 (298 - 293) = 275.87 \text{ kJ/kg}$$

We know that heat extracted or refrigerating effect produced per kg of the refrigerant,

$$R_E = h_1 - h_{f3} = 1426.54 - 275.87 = 1150.67 \text{ kJ/kg}$$

and capacity of the ice plant,

$$Q = 20 \text{ TR} = 20 \times 210 = 4200 \text{ kJ/min}$$

∴ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{4200}{1150.67} = 3.65 \text{ kg/min}$$

Work done by the compressor per minute

$$= m_R (h_2 - h_1) = 3.65 (1633.02 - 1426.54) = 753.65 \text{ kJ/min}$$

∴ Power expended per TR

$$= \frac{753.65}{60 \times 20} = 0.628 \text{ kW/TR} \quad \textbf{Ans.}$$

Amount of cooling water in the condenser

Let m_w = Amount of cooling water in the condenser.

We know that heat given out by the refrigerant in the condenser

$$= m_R (h_2 - h_{f3})$$
$$= 3.65 (1633.02 - 275.87) = 4953.6 \text{ kJ/min} \quad \ldots (i)$$

Since the specific heat of water, $c_w = 4.187$ kJ/kg K, therefore heat taken by water in the condenser

$$= m_w \times c_w (T_{w1} - T_{w2})$$
$$= m_w \times 4.187 (300 - 293) = 29.3 \, m_w \text{ kJ/min} \quad \ldots (ii)$$

Since the heat given by the refrigerant in the condenser is equal to the heat taken by water in the condenser, therefore equating equations (*i*) and (*ii*),

$$29.3 \, m_w = 4953.6 \quad \text{or} \quad m_w = 169 \text{ kg/min} \quad \textbf{Ans.}$$

Example 4.23. *A vapour compression refrigeration machine, with Freon-12 as refrigerant, has a capacity of 12 tonne of refrigeration operating between – 28°C and 26°C. The refrigerant is subcooled by 4°C before entering the expansion valve and the vapour is superheated by 5°C before leaving the evaporator. The machine has a six-cylinder single-acting compressor with stroke equal to 1.25 times the bore. It has a clearance of 3% of the stroke volume. Determine : 1. Theoretical power required ; 2. C.O.P., 3. Volumetric efficiency ; and 4. Bore and stroke of cylinder. The speed of compressor is 1000 r.p.m.*

The following properties of Freon-12 may be used :

Sat. temp., °C	*Pressure, bar*	*Sp. volume of vapour, m³/kg*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
–28	1.093	0.1475	10.64	175.11	0.0444	0.7153
26	6.697	0.0262	60.67	198.11	0.2271	0.6865

Specific heat of liquid refrigerant = 0.963 kJ/kg K and specific heat of superheated vapour = 0.615 kJ/kg K.

Solution. Given : $Q = 12$ TR ; $T_1' = -28°C = -28 + 273 = 245$ K ; $T_2' = T_3' = 26°C = 26 + 273 = 299$ K ; $T_3' - T_3 = 4°C$ or $T_3 = 22°C = 22 + 273 = 295$ K ; $T_1 - T_1' = 5°C$ or $T_1 = -23°C = -23 + 273 = 250$ K ; $L = 1.25D$; Clearance volume = 3% Stroke volume : $N = 1000$ r.p.m. ; $v_1' = 0.1475$ m³/kg ; $v_2' = 0.0262$ m³/kg ; $h_{f1} = 10.64$ kJ/kg ; $h_{f3}' = 60.67$ kJ/kg ; $h_1' = 175.11$ kJ/kg ; $h_2' = 198.11$ kJ/kg ; $s_{f1} = 0.0444$ kJ/kg K ; $s_{f3} = 0.2271$ kJ/kg K : $s_1' = 0.7153$ kJ/kg K ; $s_2' = 0.6865$ kJ/kg K ; $c_{pl} = 0.963$ kJ/kg K ; $c_{pv} = 0.615$ kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.31 (*a*) and (*b*) respectively.

1. Theoretical power required

First of all, let us find the temperature of superheated vapour at point 2 (T_2).

We know that entropy at point 1,

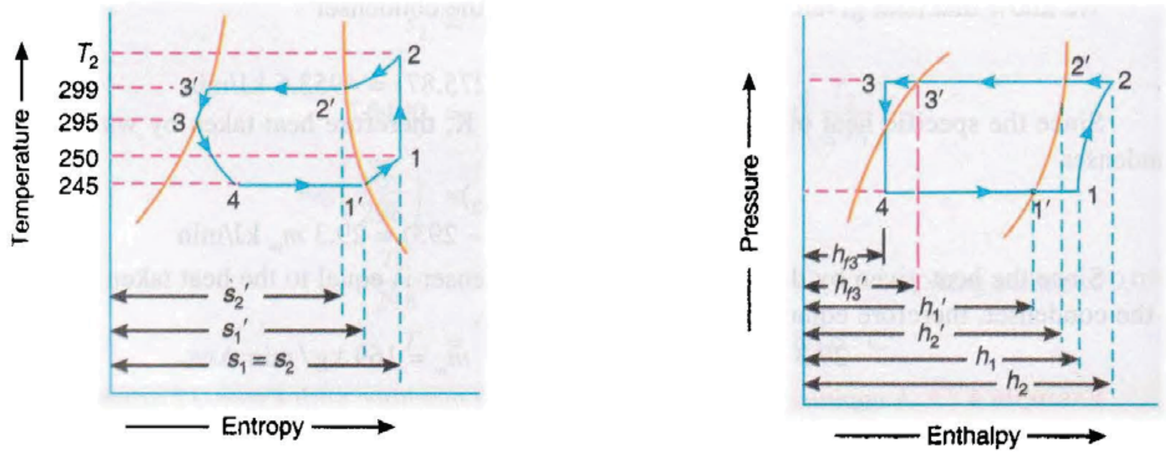
$$s_1 = s_1' + 2.3\, c_{pv} \log\left(\frac{T_1}{T_1'}\right)$$

$$= 0.7153 + 2.3 \times 0.615 \log\left(\frac{250}{245}\right) = 0.7277 \qquad \ldots (i)$$

and entropy at point 2,

$$s_2 = s_2' + 2.3\, c_{pv} \log\left(\frac{T_2}{T_2'}\right) = 0.6865 + 2.3 \times 0.615 \log\left(\frac{T_2}{299}\right)$$

$$= 0.6865 + 1.4145 \log\left(\frac{T_2}{299}\right) \qquad \ldots (ii)$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.31

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.7277 = 0.6865 + 1.4145 \log\left(\frac{T_2}{299}\right)$$

$$\log\left(\frac{T_2}{299}\right) = \frac{0.7277 - 0.6865}{1.4145} = 0.0291$$

$$\frac{T_2}{299} = 1.0693 \qquad \ldots \text{(Taking antilog of 0.0291)}$$

$$\therefore \quad T_2 = 299 \times 1.0693 = 319.7 \text{ K}$$

We know that enthalpy at point 1,

$$h_1 = h_{1'} + c_{pv}(T_1 - T_{1'})$$
$$= 175.11 + 0.615\,(250 - 245) = 178.18 \text{ kJ/kg}$$

Enthalpy at point 2.
$$h_2 = h_{2'} + c_{pv}(T_2 - T_{2'})$$
$$= 198.11 + 0.615\,(319.7 - 299) = 210.84 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl}(T_{3'} - T_3)$$
$$= 60.67 - 0.963\,(299 - 295) = 64.52 \text{ kJ/kg}$$

We know that heat extracted or refrigerating effect per kg of the refrigerant,

$$R_E = h_1 - h_{f3} = 178.18 - 64.52 = 113.66 \text{ kJ/kg}$$

and refrigerating capacity of the system,

$$Q = 12 \text{ TR} = 12 \times 210 = 2520 \text{ kJ/min} \qquad \text{...(Given)}$$

$\therefore$ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2520}{113.66} = 22.17 \text{ kg/min}$$

Work done during compression of the refrigerant

$$= m_R\,(h_2 - h_1) = 22.17\,(210.84 - 178.18) = 724 \text{ kJ/min}$$

$\therefore$ Theoretical power required

$$= 724/60 = 12.07 \text{ kW} \quad \textbf{Ans.}$$

2. *C.O.P.*

We know that C.O.P.

$$= \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{178.18 - 64.52}{210.84 - 178.18} = 3.48 \quad \textbf{Ans.}$$

3. *Volumetric efficiency*

Let v_2 = Specific volume at point 2, and

C = Clearance = 3% = 0.03. ... (Given)

First of all, let us find the specific volume at suction to the compressor, *i.e.* at point 1. Applying Charles' law to points 1 and 1′,

$$\frac{v_1}{T_1} = \frac{v_{1'}}{T_{1'}} \quad \text{or} \quad v_1 = v_{1'} \times \frac{T_1}{T_{1'}} = 0.1475 \times \frac{250}{245} = 0.1505 \text{ m}^3\text{/kg}$$

Again applying Charles' law to points 2 and 2′,

$$\frac{v_2}{T_2} = \frac{v_{2'}}{T_{2'}} \quad \text{or} \quad v_2 = v_{2'} \times \frac{T_2}{T_{2'}} = 0.0262 \times \frac{319.7}{299} = 0.028 \text{ m}^3\text{/kg}$$

We know that volumetric efficiency,

$$\eta_v = 1 + C - C\left(\frac{v_1}{v_2}\right) = 1 + 0.03 - 0.03\left(\frac{0.1505}{0.028}\right)$$
$$= 0.87 \text{ or } 87\% \quad \textbf{Ans.}$$

4. *Bore and stroke of cylinder*

Let D = Bore of cylinder,

L = Length of cylinder = 1.25 D, and ... (Given)

N = Speed of compressor = 1000 r.p.m. ... (Given)

We know that theoretical suction volume or piston displacement per minute

$$= m_R \times v_1 \times \frac{1}{\eta_v} = 22.17 \times 0.1505 \times \frac{1}{0.87} = 3.84 \text{ m}^3/\text{min}$$

Since the machine has six cylinder, single acting compressor, therefore, theoretical suction volume or piston displacement per cylinder per minute

$$= \frac{3.84}{6} = 0.64 \text{ m}^3/\text{min} \quad \text{... (iii)}$$

We also know that suction volume or piston displacement per minute

$$= \text{Piston area} \times \text{Stroke} \times \text{R.P.M.}$$

$$= \frac{\pi}{4} \times D^2 \times L \times N = \frac{\pi}{4} \times D^2 \times 1.25\, D \times 1000$$

$$= 982\, D^3 \text{ m}^3/\text{min} \quad \text{... (iv)}$$

Equating equations (*iii*) and (*iv*),

$$D^3 = 0.64/982 = 0.000\,652$$

$\therefore$ $D = 0.0867$ m = 86.7 mm **Ans.**

and $L = 1.25 \times 86.7 = 108.4$ mm **Ans.**

Example 4.24. *A food storage locker requires a refrigeration capacity of 12 TR and works between the evaporating temperature of – 8°C and condensing temperature of 30°C. The refrigerant R-12 is subcooled by 5°C before entry to expansion valve and the vapour is superheated to – 2°C before leaving the evaporator coils. Assuming a two cylinder, single acting compressor operating at 1000 r.p.m. with stroke equal to 1.5 times the bore, determine : 1. coefficient of performance; 2. theoretical power per tonne of refrigeration; and 3. bore and stroke of compressor when (a) there is no clearance; and (b) there is a clearance of 2%.*

Use the following data for R-12 :

Saturation temperature, °C	*Pressure, bar*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		*Specific volume of vapour, m³/kg*
		Liquid	*Vapour*	*Liquid*	*Vapour*	
–8	*2.354*	*28.72*	*184.07*	*0.1149*	*0.7007*	*0.0790*
30	*7.451*	*64.59*	*199.62*	*0.2400*	*0.6853*	*0.0235*

The specific heat of liquid R-12 is 1.235 kJ/kg K, and of vapour R-12 is 0.733 kJ/kg K.

Solution. Given : Q = 12 TR ; $T_{1'}$ = – 8°C = – 8 + 273 = 265 K ; $T_{2'}$ = 30°C = 30 + 273 = 303 K ; $T_{3'} - T_3$ = 5°C ; T_1 = – 2°C = – 2 + 273 = 271 K ; h_{f1} = 28.72 kJ/kg ; $h_{f3'}$ = 64.59 kJ/kg ; $h_{1'}$ = 184.07 kJ/kg ; $h_{2'}$ = 199.62 kJ/kg ; *s_{f1} = 0.1149 kJ/kg K ; *s_{f3} = 0.2400 kJ/kg K ; $s_{1'}$ = 0.7007 kJ/kg K ; $s_{2'}$ = 0.6853 kJ/kg K ; $v_{1'}$ = 0.079 m³/kg ; $v_{2'}$ = 0.0235 m³/kg ; c_{pl} = 1.235 kJ/kg K ; c_{pv} = 0.733 kJ/kg K

The *T-s* and *p-h* diagrams are shown in Fig. 4.32 (*a*) and (*b*) respectively.

1. ***Coefficient of performance***

First of all, let us find the temperature of superheated vapour at point 2 (T_2).

* Superfluous data.

We know that entropy at point 1,

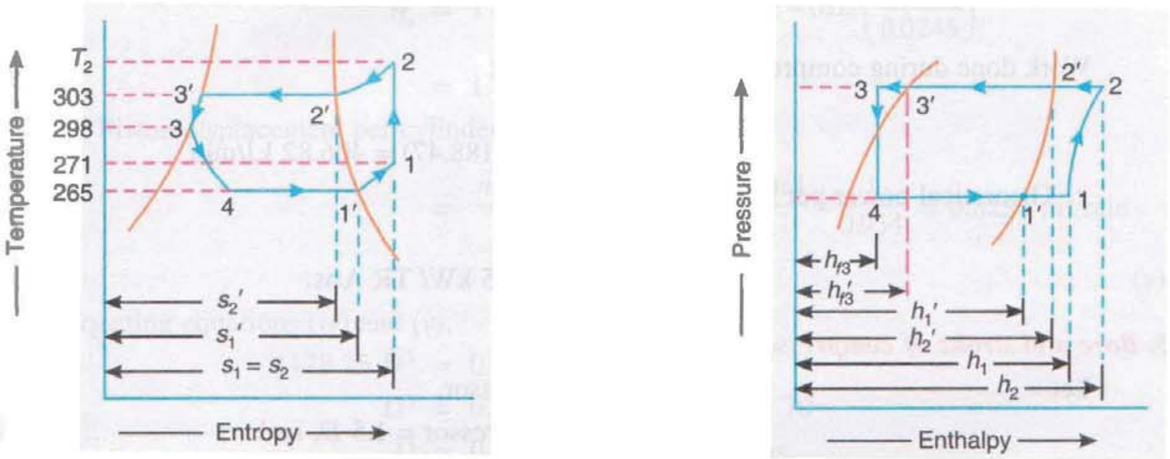
$$s_1 = s_{1'} + 2.3\, c_{pv} \log\left(\frac{T_1}{T_{1'}}\right)$$

$$= 0.7007 + 2.3 \times 0.733 \log\left(\frac{271}{265}\right) = 0.7171 \qquad \ldots (i)$$

and entropy at point 2,

$$s_2 = s_{2'} + 2.3\, c_{pv} \log\left(\frac{T_2}{T_{2'}}\right) = 0.6853 + 2.3 \times 0.733 \log\left(\frac{T_2}{303}\right)$$

$$= 0.6853 + 1.686 \log\left(\frac{T_2}{303}\right) \qquad \ldots (ii)$$



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.32

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (*i*) and (*ii*),

$$0.7171 = 0.6853 + 1.686 \log\left(\frac{T_2}{303}\right)$$

or

$$\log\left(\frac{T_2}{303}\right) = \frac{0.7171 - 0.6853}{1.686} = 0.0188$$

$$\left(\frac{T_2}{303}\right) = 1.0444 \qquad \ldots \text{(Taking antilog of 0.0188)}$$

$$\therefore \qquad T_2 = 1.0444 \times 303 = 316.4 \text{ K}$$

We know that enthalpy at point 1,

$$h_1 = h_{1'} + c_{pv} (T_1 - T_{1'})$$

$$= 184.07 + 0.733 (271 - 265) = 188.47 \text{ kJ/kg}$$

Enthalpy at point 2,

$$h_2 = h_{2'} + c_{pv} (T_2 - T_{2'})$$

$$= 199.62 + 0.733 (316.4 - 303) = 209.44 \text{ kJ/kg}$$

and enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3'} - c_{pl} (T_{3'} - T_3)$$

$$= 64.59 - 1.235 \times 5 = 58.42 \text{ kJ/kg}$$

$$\therefore \qquad \text{C.O.P.} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{188.47 - 58.42}{209.44 - 188.47} = \frac{130.05}{20.97} = 6.2 \text{ Ans.}$$

2. Theoretical power per tonne of refrigeration

We know that the heat extracted or refrigerating effect per kg of the refrigerant,

$$R_E = h_1 - h_{f3} = 188.47 - 58.42 = 130.05 \text{ kJ/kg}$$

and the refrigerating capacity of the system,

$$Q = 12 \text{ TR} = 12 \times 210 = 2520 \text{ kJ/min} \qquad \text{... (Given)}$$

∴ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2520}{130.05} = 19.4 \text{ kg/min}$$

Work done during compression of the refrigerant

$$= m_R (h_2 - h_1)$$
$$= 19.4 (209.44 - 188.47) = 406.82 \text{ kJ/min}$$

∴ Theoretical power per tonne of refrigeration

$$= \frac{406.82}{60 \times 12} = 0.565 \text{ kW/ TR Ans.}$$

3. Bore and stroke of compressor

Let D = Bore of compressor,

L = Stroke of compressor = 1.5 D, and ... (Given

N = Speed of compressor = 1000 r.p.m. ... (Given

First of all, let us find the specific volume at suction to the compressor, *i.e.* at point 1 Applying Charles' law,

$$\frac{v_1}{T_1} = \frac{v_1'}{T_1'}$$

or

$$v_1 = v_1' \times \frac{T_1}{T_1'} = 0.0790 \times \frac{271}{265} = 0.081 \text{ m}^3\text{/kg}$$

(a) When there is no clearance

We know that theoretical suction volume or piston displacement per minute

$$= m_R \times v_1 = 19.4 \times 0.081 = 1.57 \text{ m}^3\text{/min}$$

and theoretical suction volume or piston displacement per cylinder per minute

$$= 1.57/2 = 0.785 \text{ m}^3\text{/min} \qquad \text{... (}iii$$

Also theoretical suction volume or piston displacement per minute

$$= \text{Piston area} \times \text{Stroke} \times \text{R.P.M.}$$

$$= \frac{\pi}{4} \times D^2 \times L \times N = \frac{\pi}{4} \times D^2 \times 1.5 D \times 1000$$

$$= 1178.25\, D^3 \text{ m}^3\text{/min} \qquad \text{...(}iv$$

Equating equations (*iii*) and (*iv*),

$$1178.25\, D^3 = 0.785$$

or

$$D^3 = 0.785/1178.25 = 0.000\,666$$

$\therefore \quad D = 0.087 \text{ m} = 87 \text{ mm}$ **Ans.**

and $\quad L = 1.5\,D = 1.5 \times 87 = 130.5 \text{ mm}$ **Ans.**

(b) When there is a clearance of 2%

Let $\quad v_2$ = Specific volume at point 2, and

C = Clearance = 2% = 0.02 ... (Given)

Applying Charles' law to points 2 and 2′,

$$\frac{v_2}{T_2} = \frac{v_2'}{T_2'} \quad \text{or} \quad v_2 = v_2' \times \frac{T_2}{T_2'} = 0.0235 \times \frac{361.4}{303} = 0.0245 \text{ m}^3/\text{kg}$$

We know that volumetric efficiency of the compressor,

$$\eta_v = 1 + C - C\left(\frac{v_1}{v_2}\right) = 1 + 0.02 - 0.02\left(\frac{0.081}{0.0245}\right)$$

$$= 1.02 - 0.066 = 0.954$$

$\therefore$ Piston displacement per cylinder per min

$$= \frac{m_R \times v_1}{2} \times \frac{1}{\eta_v} = \frac{19.4 \times 0.081}{2} \times \frac{1}{0.954} = 0.8236 \text{ m}^3/\text{min} \quad \ldots (v)$$

Equating equations (*iv*) and (*v*),

$$1178.25\,D^3 = 0.8236$$

or $\quad D^3 = 0.8236 / 1178.25 = 0.000\,70$

$\therefore \quad D = 0.0887 \text{ m} = 88.7 \text{ mm}$ **Ans.**

and $\quad L = 1.5\,D = 1.5 \times 88.7 = 133 \text{ mm}$ **Ans.**

4.11 Actual Vapour Compression Cycle

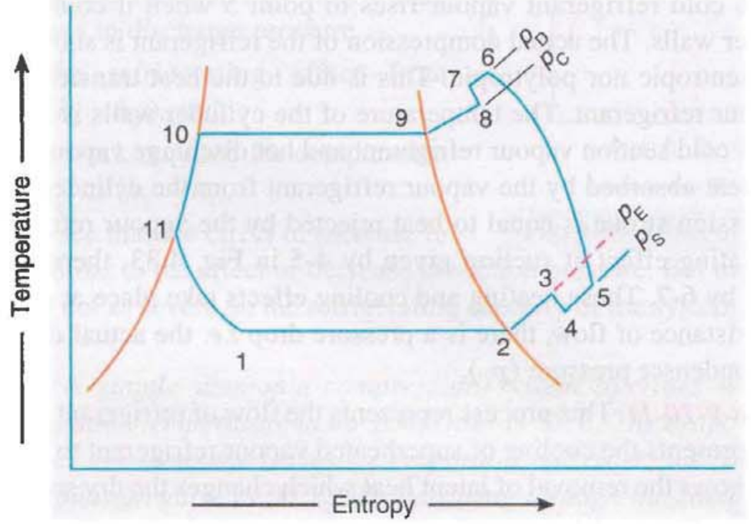


Fig. 4.33. Actual vapour compression cycle.

The actual vapour compression cycle differs from the theoretical vapour compression in many ways, some of which are unavoidable and cause losses. The main deviations betw theoretical cycle and actual cycle are as follows :

1. The vapour refrigerant leaving the evaporator is in superheated state.
2. The compression of refrigerant is neither isentropic nor polytropic.
3. The liquid refrigerant before entering the expansion valve is sub-cooled in the condenser.
4. The pressure drops in the evaporator and condenser.

The actual vapour compression cycle on *T-s* diagram is shown in Fig. 4.33. The various processes are discussed below :

(a) Process 1-2-3. This process shows the flow of refrigerant in the evaporator. The point 1 represents the entry of refrigerant into the evaporator and the point 3 represents the exit of refrigerant from evaporator in a superheated state. The point 3 also represents the entry of refrigerant into the compressor in a superheated condition. The superheating of vapour refrigerant from point 2 to point 3 may be due to :

(*i*) automatic control of expansion valve so that the refrigerant leaves the evaporator as the superheated vapour.

(*ii*) picking up of larger amount of heat from the evaporator through pipes located within the cooled space.

(*iii*) picking up of heat from the suction pipe, *i.e.* the pipe connecting the evaporator delivery and the compressor suction valve.

In the first and second case of superheating the vapour refrigerant, the refrigerating effect as well as the compressor work is increased. The coefficient of performance, as compared to saturation cycle at the same suction pressure may be greater, less or unchanged.

The superheating also causes increase in the required displacement of compressor and load on the compressor and condenser. This is indicated by 2-3 on *T-s* diagram as shown in Fig. 4.33.

(b) Process 3-4-5-6-7-8. This process represents the flow of refrigerant through the compressor. When the refrigerant enters the compressor through the suction valve at point 3, the pressure falls to point 4 due to frictional resistance to flow. Thus the actual suction pressure (p_S) is lower than the evaporator pressure (p_E). During suction and prior to compression, the temperature of the cold refrigerant vapour rises to point 5 when it comes in contact with the compressor cylinder walls. The actual compression of the refrigerant is shown by 5-6 in Fig. 4.33, which is neither isentropic nor polytropic. This is due to the heat transfer between the cylinder walls and the vapour refrigerant. The temperature of the cylinder walls is some-what in between the temperatures of cold suction vapour refrigerant and hot discharge vapour refrigerant. It may be assumed that the heat absorbed by the vapour refrigerant from the cylinder walls during the first part of the compression stroke is equal to heat rejected by the vapour refrigerant to the cylinder walls. Like the heating effect at suction given by 4-5 in Fig. 4.33, there is a cooling effect at discharge as given by 6-7. These heating and cooling effects take place at constant pressure. Due to the frictional resistance of flow, there is a pressure drop *i.e.* the actual discharge pressure (p_D) is more than the condenser pressure (p_C).

(c) Process 8-9-10-11. This process represents the flow of refrigerant through the condenser. The process 8-9 represents the cooling of superheated vapour refrigerant to the dry saturated state. The process 9-10 shows the removal of latent heat which changes the dry saturated refrigerant into liquid refrigerant. The process 10-11 represents the sub-cooling of liquid refrigerant in the condenser before passing through the expansion valve. This is desirable as it increases the refrigerating effect per kg of the refrigerant flow. It also reduces the volume of the refrigerant partially evaporated from the liquid refrigerant while passing through the expansion valve. The increase in refrigerating effect can be obtained by large quantities of circulating cooling water which should be at a temperature much lower than the condensing temperatures.

(d) Process 11-1. This process represents the expansion of subcooled liquid refrigerant by throttling from the condenser pressure to the evaporator pressure.

4.12 Effect of Suction Pressure

We have discussed in the previous article that in actual practice, the suction pressure (or evaporator pressure) decreases due to the frictional resistance of flow of the refrigerant. Let us consider a theoretical vapour compression cycle 1′-2′-3-4′ when the suction pressure decreases from p_S to p_S' as shown on *p-h* diagram in Fig. 4.34.

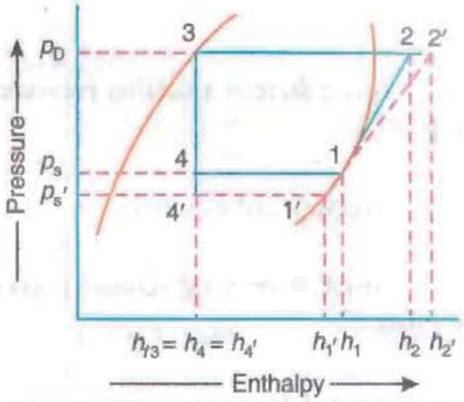


Fig. 4.34. Effect of suction pressure.

It may be noted that the decrease in suction pressure

1. decreases the refrigerating effect from $(h_1 - h_4)$ to $(h_{1'} - h_{4'})$, and
2. increases the work required for compression from $(h_2 - h_1)$ to $(h_{2'} - h_{1'})$.

Since the C.O.P. of the system is the ratio of refrigeraing effect to the work done, therefore with the decrease in suction pressure, the net effect is to decrease the C.O.P. of the refrigerating system for the same amount of refrigerant flow. Hence with the decrease in suction pressure, the refrigerating capacity of the system decreases and the refrigeration cost increases.

4.13 Effect of Discharge Pressure

We have already discussed that in actual practice, the discharge pressure (or condenser pressure) increases due to frictional resistance of flow of the refrigerant. Let us consider a theoretical vapour compression cycle 1-2′-3′-4′ when the discharge pressure increases from p_D to p_D' as shown on *p-h* diagram in Fig. 4.35. It may be noted that the increase in discharge pressure

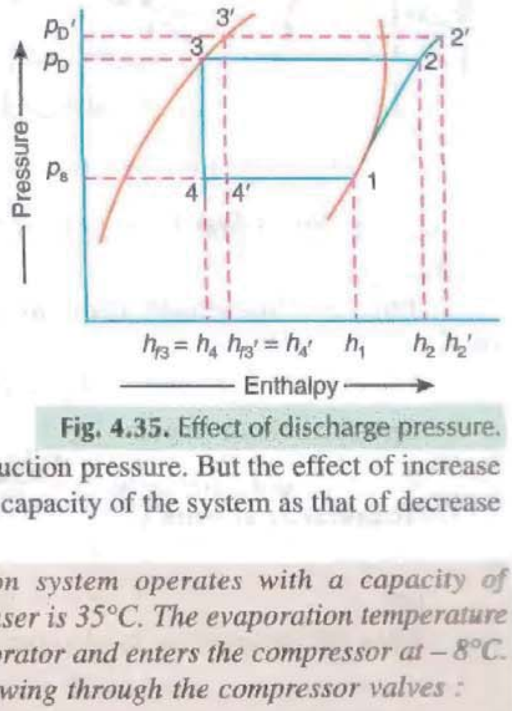


Fig. 4.35. Effect of discharge pressure.

1. decreases the refrigerating effect from $(h_1 - h_4)$ to $(h_1 - h_{4'})$, and
2. increases the work required for compression from $(h_2 - h_1)$ to $(h_{2'} - h_1)$.

From above, we see that the effect of increase in discharge pressure is similar to the effect of decrease in suction pressure. But the effect of increase in discharge pressure is not as severe on the refrigerating capacity of the system as that of decrease in suction pressure.

Example 4.25. *A simple ammonia-compression system operates with a capacity of 150 tonnes. The condensation temperature in the condenser is 35°C. The evaporation temperature in brine cooler is – 25°C. The ammonia leaves the evaporator and enters the compressor at – 8°C. Ammonia enters the expansion valve at 30°C. Wire drawing through the compressor valves :*

Suction = 0.118 bar ; Discharge = 0.23 bar ; Compression index = 1.22 ; Volumetric efficiency = 0.75.

Calculate : 1. Power ; 2. Heat transferred to cylinder water jacket ; 3. Piston displacement ; 4. Heat transfer in condenser ; and 5. Coefficient of performance.

Solution. Given : $Q = 150$ TR ; $T_2 = T_2'' = T_3' = 35°C = 35 + 273 = 308$ K ; $T_1'' = T_4 = -25°C = -25 + 273 = 248$ K ; $T_1 = -8°C = -8 + 273 = 265$ K ; $n = 1.22$; $\eta_v = 0.75$

The *T-s* and *p-h* diagrams are shown in Fig. 4.36 (*a*) and (*b*) respectively.

From *p-h* diagram, we find that the pressure corresponding to evaporation temperature of − 25°C,

$$p_1 = p_1'' = p_4 = 1.518 \text{ bar}$$

Since there is a suction pressure drop of 0.118 bar due to *wire drawing, therefore pressure at point 1′,

$$p_1' = 1.518 - 0.118 = 1.4 \text{ bar} = 1.4 \times 10^5 \text{ N/m}^2$$

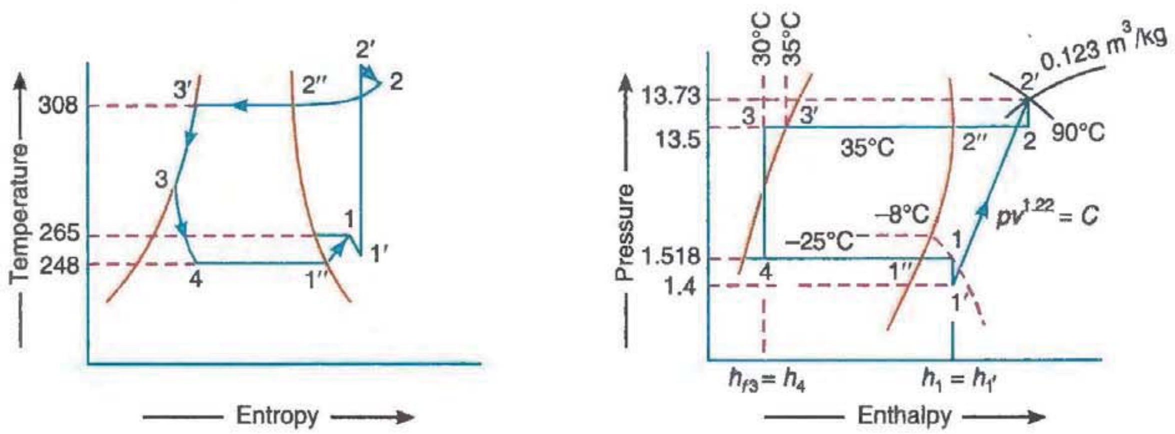
Pressure corresponding to condensation temperature of 35°C

$$= 13.5 \text{ bar}$$

Since there is a discharge pressure drop of 0.23 bar due to wire drawing, therefore pressure at point 2′,

$$p_2' = 13.5 + 0.23 = 13.73 \text{ bar} = 13.73 \times 10^5 \text{ N/m}^2$$

Note: In Fig. 4.36, point 1 represents the inlet of the suction valve and point 1′ is the outlet of the suction valve. The point 2′ represents the inlet of discharge valve and point 2 is the outlet of the discharge valve.



(*a*) *T-s* diagram. (*b*) *p-h* diagram.

Fig. 4.36

From *p-h* diagram, we also find that enthalpy of superheated ammonia vapours at point 1 or 1′,

$$h_1 = h_1' = 1440 \text{ kJ/kg}$$

Specific volume at point 1′,

$$v_1' = 0.8 \text{ m}^3\text{/kg}$$

Temperature at point 1′,

$$T_1' = -9°C = -9 + 273 = 264 \text{ K}$$

Let v_2' = Specific volume at point 2′.

Since the compression is according to $pv^{1.22} = C$, therefore

$$p_1' (v_1')^n = p_2' (v_2')^n$$

or

$$v_2' = v_1' \left(\frac{p_1'}{p_2'}\right)^{\frac{1}{n}} = 0.8 \left(\frac{1.4}{13.73}\right)^{\frac{1}{1.22}} = 0.123 \text{ m}^3\text{/kg}$$

* Wire drawing is a constant enthalpy process.

Now plot a point 2′ on the *p-h* diagram corresponding to $p_{2'}$ = 13.73 bar and $v_{2'}$ = 0.123 m³/kg. From the *p-h* diagram, we find that

Enthalpy of superheated ammonia vapours at point 2 or 2′,

$$h_2 = h_{2'} = 1620 \text{ kJ/kg}$$

Temperature at point 2′.

$$T_{2'} = 90°\text{C}$$

and enthalpy of liquid ammonia at point 3,

$$h_{f3} = h_4 = 320 \text{ kJ/kg}$$

1. Power

We know that refrigerating effect per kg,

$$R_E = h_1 - h_{f3} = 1440 - 320 = 1120 \text{ kJ/kg}$$

and refrigerating capacity $= 150 \text{ TR} = 150 \times 210 = 31\,500 \text{ kJ/min}$ ... (Given)

∴ Mass flow of the refrigerant,

$$m_R = \frac{31\,500}{1120} = 28.12 \text{ kg / min}$$

We know that work done by the compressor per minute

$$= m_R \times \frac{n}{n-1}(p_{2'}v_{2'} - p_{1'}v_{1'})$$

$$= 28.12 \times \frac{1.22}{1.22-1}(13.73 \times 10^5 \times 0.123 - 1.4 \times 10^5 \times 0.8)$$

$$= 89 \times 10^5 \text{ J/min} = 8900 \text{ kJ/min}$$

∴ Power $= 8900/60 = 148.3$ kW **Ans.**

2. Heat transferred to cylinder water jacket

We know that actual work done by the compressor

$$= m_R(h_{2'} - h_{1'}) = 28.12(1620 - 1440) = 5062 \text{ kJ/min}$$

∴ Heat transferred to cylinder water jacket

$$= 8900 - 5062 = 3838 \text{ kJ/min}$$ **Ans.**

3. Piston displacement

We know that piston displacement

$$= \frac{m_R \times v_{1'}}{\eta_v} = \frac{28.12 \times 0.8}{0.75} = 30 \text{ m}^3\text{/min}$$ **Ans.**

4. Heat transfer in condenser

We know that heat transfer in condenser

$$= m_R(h_2 - h_{f3}) = 28.12(1620 - 320) = 36\,556 \text{ kJ/min}$$ **Ans.**

5. Coefficient of performance

We know that coefficient of performance,

$$\text{C.O.P} = \frac{\text{Refrigerating capacity}}{\text{Work done}} = \frac{31\,500}{8900} = 3.54$$ **Ans.**

Geared turbo - compressor for the compression of vapours.

4.14 Improvements in Simple Saturation Cycle

The simple saturation cycle may be improved by the following methods :

1. By introducing the flash chamber between the expansion valve and the evaporator.
2. By using the accumulator or pre-cooler.
3. By subcooling the liquid refrigerant by the vapour refrigerant.
4. By subcooling the liquid refrigerant leaving the condenser by liquid refrigerant from the expansion valve.

The effect of the above mentioned methods on the simple saturation cycle are discussed, in detail, in the following pages :

4.15 Simple Saturation Cycle with Flash Chamber

We have already discussed that when the high pressure liquid refrigerant from the condenser passes through the expansion valve, some of it evaporates. This partial evaporation of the liquid refrigerant is known as *flash*. It may be noted that the vapour formed during expansion is of no use to the evaporator in producing refrigerating effect as compared to the liquid refrigerant which carries the heat in the form of latent heat. This formed vapour, which is incapable of producing any refrigeration effect, can be by - passed around the evaporator and supplied directly to the suction of the compressor. This is done

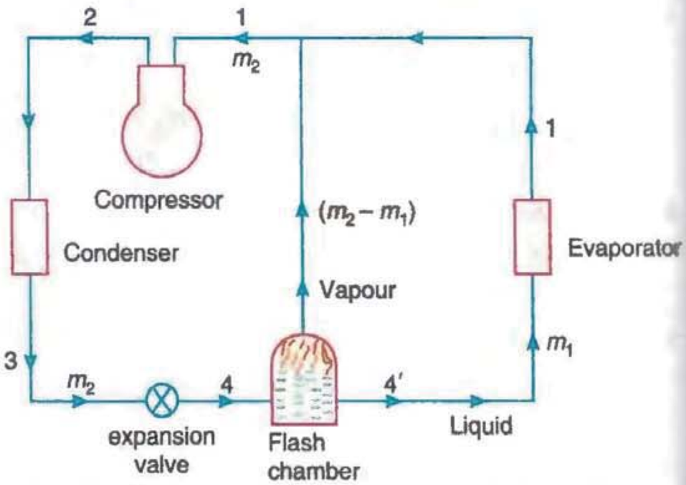


Fig. 4.37. Simple saturation cycle with flash chamber.

by introducing a flash chamber between the expansion valve and the evaporator as shown in Fig. 4.37. The flash chamber is an insulated container and it separates the liquid and vapour due to centrifugal effect. Thus the mass of the refrigerant passing through the evaporator reduces.

Let us consider that a certain amount of refrigerant is circulating through the condenser. This refrigerant after passing through the expansion valve, is supplied to the flash chamber which separates the liquid and vapour refrigerant. The liquid refrigerant from the flash chamber is supplied to the evaporator and the vapour refrigerant flows directly from the flash chamber to the suction of the compressor. The *p-h* diagram of the cycle is shown in Fig. 4.38.

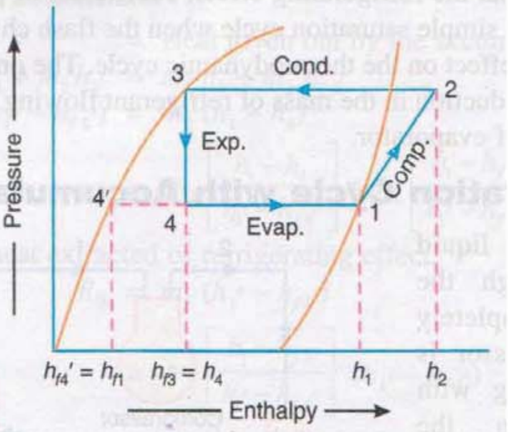


Fig. 4.38. *p-h* diagram of simple saturation cycle with flash chamber.

Let m_1 = Mass of liquid refrigerant supplied to the evaporator, and

m_2 = Mass of refrigerant (liquid and vapour) circulating through the condenser, or leaving the expansion valve.

∴ Mass of vapour refrigerant flowing directly from the flash chamber to the suction of the compressor

$$= m_2 - m_1$$

Now considering the thermal equilibrium of the flash chamber. Since the flash chamber is an insulated vessel, therefore, there is no heat exchange between the flash chamber and the atmosphere. In other words, the heat taken and given out by the flash chamber are same. Mathematically,

Heat taken by the flash chamber

= Heat given out by the flash chamber

or
$$m_2 h_4 = m_1 h_{f4'} + (m_2 - m_1)\, h_1$$

$$m_2 (h_1 - h_4) = m_1 (h_1 - h_{f4'})$$

$$\therefore \quad m_1 = m_2 \left[\frac{h_1 - h_4}{h_1 - h_{f4'}}\right] = m_2 \left[\frac{h_1 - h_{f3}}{h_1 - h_{f4'}}\right] \quad \ldots (\because h_4 = h_{f3}) \ldots (i)$$

We know that the heat extracted or refrigerating effect,

$$R_E = m_1 (h_1 - h_{f4'})$$

$$= m_2 \left(\frac{h_1 - h_{f3}}{h_1 - h_{f4'}}\right) (h_1 - h_{f4'}) \quad \ldots \text{[From equation (i)]}$$

$$= m_2 (h_1 - h_{f3})$$

and workdone in compressor,

$$W = m_2 (h_2 - h_1)$$

$$\therefore \qquad \text{C.O.P.} = \frac{R_E}{W} = \frac{m_2(h_1 - h_{f3})}{m_2(h_2 - h_1)} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

and power required to drive the compressor,

$$P = \frac{m_2(h_2 - h_1)}{60} \text{ kW}$$

From above, we see that the refrigerating effect, coefficient of performance and the power required are same as that of a simple saturation cycle when the flash chamber is not used. Thus the use of flash chamber has no effect on the thermodynamic cycle. The only effect resulting from the use of flash chamber is the reduction in the mass of refrigerant flowing through the evaporator and hence the reduction in size of evaporator.

4.16 Simple Saturation Cycle with Accumulator or Pre-cooler

Sometimes, the liquid refrigerant passing through the evaporator is not completely evaporated. If the compressor is supplied with liquid along with vapour refrigerant, then the compressor has to do an additional work of evaporating and raising the temperature of liquid refrigerant. It will also upset the normal working of the compressor which is meant only for compressing the pure vapour refrigerant.

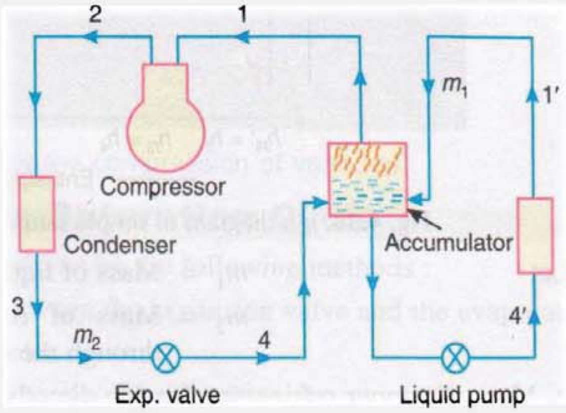


Fig. 4.39. Simple saturation cycle with accumulator or pre-cooler

In order to avoid this difficulty, an insulated vessel, known as accumulator or pre-cooler, is used in the system, as shown in Fig. 4.39. The accumulator receives the discharge (a mixture of liquid and vapour refrigerant) from the expansion valve and supplies the liquid refrigerant only to the evaporator, as in the case of flash chamber.

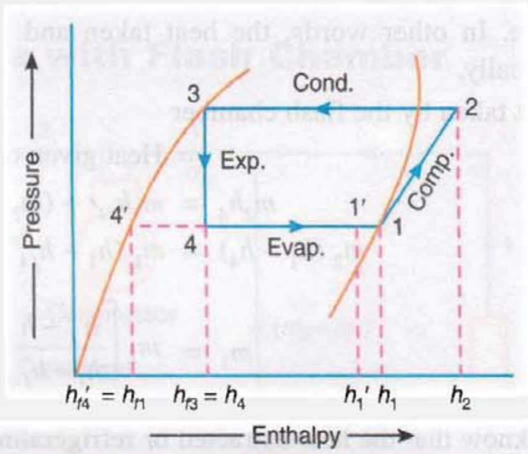


Fig. 4.40. *p-h* diagram of simple saturation cycle with accumulator

The discharge from the evaporator is sent again to the accumulator which helps to keep off the liquid from entering the compressor. Thus the accumulator supplies dry and saturated vapour to the compressor. A liquid pump is provided in the system in order to maintain circulation of the refrigerant in the evaporator.

Let $\quad m_1$ = Mass of liquid refrigerant circulating through the evaporator, and

m_2 = Mass of refrigerant flowing in the condenser.

When all the liquid refrigerant does not evaporate in the evaporator, it is represented by point 1′ on *p-h* diagram as shown in Fig. 4.40. Let the mass of refrigerant that leaves the evaporator at point 1′ is same *i.e.* m_1.

Consider the thermal equilibrium of the accumulator. Since the accumulator is an insulated vessel, therefore there is no heat exchange between the accumulator and the atmosphere. In other words, the heat taken in and given out by the accumulator is equal. Mathematically,

Heat taken in by the accumulator

= Heat given out by the accumulator

$$m_2 h_4 + m_1 h_{1'} = m_2 h_1 + m_1 h_{f4'}$$

$$m_1 (h_{1'} - h_{f4'}) = m_2 (h_1 - h_4)$$

$$m_1 = m_2 \left[\frac{h_1 - h_4}{h_{1'} - h_{f4'}}\right] = m_2 \left[\frac{h_1 - h_{f3}}{h_{1'} - h_{f4'}}\right] \quad \ldots (\because h_4 = h_{f3}) \ldots (i)$$

We know that the heat extracted or refrigerating effect,

$$R_E = m_1 (h_{1'} - h_{f4'})$$

$$= m_2 \left[\frac{h_1 - h_{f3}}{h_{1'} - h_{f4'}}\right] (h_{1'} - h_{f4'}) \quad \ldots \text{[From equation } (i)]$$

$$= m_2 (h_1 - h_{f3})$$

and workdone in compressor, $W = m_2 (h_2 - h_1)$

$$\therefore \quad \text{C.O.P.} = \frac{R_E}{W} = \frac{m_2(h_1 - h_{f3})}{m_2(h_2 - h_1)} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

and power required to drive the compressor,

$$P = \frac{m_2(h_2 - h_1)}{60} \text{ kW}$$

From above, we see that when the accumulator is used in the system, the refrigerating effect, coefficient of performance, and power required is same as the simple saturation cycle. The accumulator is used only to protect the liquid refrigerant to flow into the compressor and thus dry compression is always ensured.

4.17 Simple Saturation Cycle with Sub-cooling of Liquid Refrigerant by Vapour Refrigerant

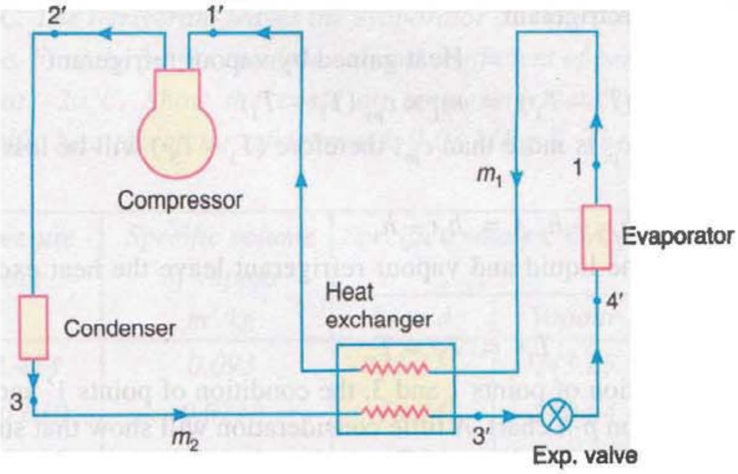


Fig. 4.41. Simple saturation cycle with sub-cooling of liquid refrigerant by vapour refrigerant.

We know that the liquid refrigerant leaving the condenser is at a higher temperature than the vapour refrigerant leaving the evaporator. The liquid refrigerant leaving the condenser can be subcooled by passing it through a heat exchanger which is supplied with saturated vapour from the evaporator as shown in Fig. 4.41. In the heat exchanger, the liquid refrigerant gives heat to the vapour refrigerant. The *p-h* diagram of the cycle is shown in Fig. 4.42.

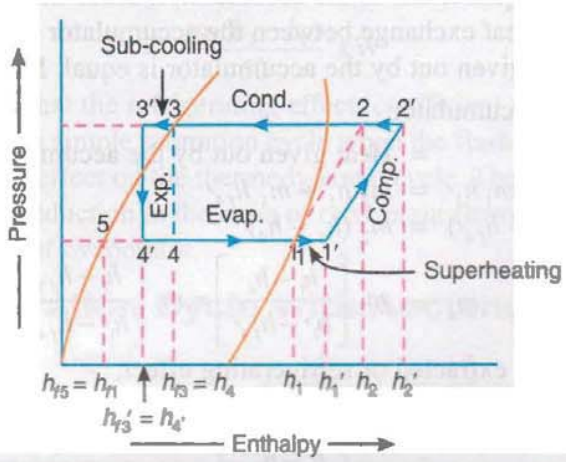


Fig. 4.42. *p-h* diagram of simple saturation cycle with sub-cooling of liquid refrigerant by vapour refrigerant.

Let
m_1 = Mass of the vapour refrigerant,
m_2 = Mass of the liquid refrigerant,
T_3 = Temperature of liquid refrigerant entering the heat exchanger,
$T_{3'}$ = Temperature of liquid refrigerant leaving the heat exchanger,
T_1 = Temperature of vapour refrigerant entering the heat exchanger,
$T_{1'}$ = Temperature of vapour refrigerant leaving the heat exchanger,
c_{pv} = Specific heat of vapour refrigerant, and
c_{pl} = Specific heat of liquid refrigerant.

Considering the thermal equilibrium of heat exchanger,

Heat lost by liquid refrigerant

= Heat gained by vapour refrigerant

i.e.
$$m_2 \times c_{pl} (T_3 - T_{3'}) = m_1 \times c_{pv} (T_{1'} - T_1)$$

Since $m_2 = m_1$ and c_{pl} is more than c_{pv}, therefore $(T_3 - T_{3'})$ will be less than $(T_{1'} - T_1)$. For energy balance,

$$h_{f3} - h_{f3}' = h_{1'} - h_1$$

In the ideal case, the liquid and vapour refrigerant leave the heat exchanger at the same temperature, say (T_m).

$$\therefore \quad T_{1'} = T_{3'} = T_m$$

Knowing the condition of points 1 and 3, the condition of points 1′ and 3′ can be obtained by trial and error method on *p-h* chart. A little consideration will show that sub-cooling increases the refrigerating effect from $(h_1 - h_{f3})$ to $(h_1 - h_{f3}')$ per kg of refrigerant as compared with the simple saturation cycle.

If Q tonnes of refrigeration is the load on the evaporator, then the mass of refrigerant (m_R) required to be circulated through the evaporator for sub-cooled cycle is given by

$$m_R = \frac{210\,Q}{h_1 - h_{f3'}} \text{ kg / min}$$

∴ Power required to drive the compressor,

$$P_1 = \frac{m_R\,(h_{2'} - h_{1'})}{60} \text{ kW}$$

$$= \frac{210\,Q}{60}\left[\frac{h_{2'} - h_{1'}}{h_1 - h_{f3'}}\right] \text{ kW}$$

and power required to drive the compressor without heat exchanger (*i.e.* for simple saturation cycle),

$$P_2 = \frac{m_R\,(h_2 - h_1)}{60} \text{ kW}$$

$$= \frac{210\,Q}{60}\left[\frac{h_2 - h_1}{h_1 - h_{f3}}\right] \text{ kW}$$

∴ Excess power required to drive the compressor as compared to simple saturation cycle,

$$P_{excess} = P_1 - P_2$$

$$= \frac{210\,Q}{60}\left[\frac{h_{2'} - h_{1'}}{h_1 - h_{f3'}} - \frac{h_2 - h_1}{h_1 - h_{f3}}\right] \text{ kW}$$

From above, we see that sub-cooling the liquid refrigerant by vapour refrigerant, the coefficient of performance of the cycle is reduced. Even with theoretical loss resulting from the above type of sub-cooling, there are many actual installations which adopt this process.

Example 4.26. *For a vapour compression refrigeration system using R-22 as refrigerant, condenser outlet temperature is 40°C and the evaporator inlet temperature is –20°C. In order to avoid flashing of the refrigerant, a liquid-suction vapour heat exchanger is provided where liquid is subcooled to 26°C. The refrigerant leaves the evaporator as saturated vapour. The compression process is isentropic. Find the power requirement and coefficient of performance if capacity of the system is 10 kW at –20°C. Show the cycle on temperature-entropy and pressure-enthalpy diagrams. The specific heat of vapour refrigerant is 1.03 kJ/kg K. The thermodynamic properties are given below:*

Saturation temperature °C	*Pressure bar*	*Specific volume of vapour m³/kg*	*Specific enthalpy, kJ/kg*		*Specific entropy, kJ/kg K*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
–20	2.458	0.093	22.21	243.25	0.0908	0.9638
26	10.819	0.0218	79.74	260.64	0.2935	0.9014
40	15.489	0.0148	97.94	263.21	0.3563	0.8822

Solution. Given: $T_2 = T_3 = 40°C = 40 + 273 = 313\,K$; $T_1 = T_{4'} = -20°C = -20 + 273 = 253\,K$; $T_{3'} = 26°C = 26 + 273 = 299K$; $Q = 10$ kW $= 10$ kJ/s ; $c_{pv} = 1.03$ kJ/kg K ; $*v_1 = 0.093$ m³/kg ; $*h_{f1} = 22.21$ kJ/kg ; $h_1 = 243.25$ kJ/kg ; $*s_{f1} = 0.0908$ kJ/kg K ; $s_1 = 0.9638$ kJ/kg K ; $h_{f3'} = h_{4'} = 79.74$ kJ/kg; $*v_2 = 0.0148$ m³/kg ; $h_{f3} = 97.94$ kJ/kg ; $h_2 = 263.21$ kJ/kg ; $*s_{f3} = 0.3563$ kJ/kg K ; $s_2 = 0.8822$ kJ/kg K

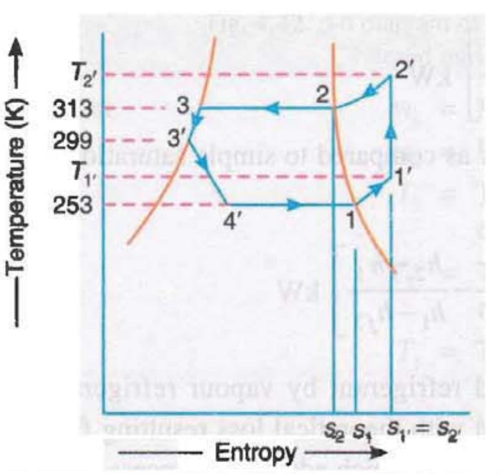
The *T-s* and *p-h* diagrams are shown in Fig. 4.43 (*a*) and (*b*) respectively. Considering the thermal equilibrium of the heat exchanger,

Heat lost by liquid refrigerant = Heat gained by vapour refrigerant

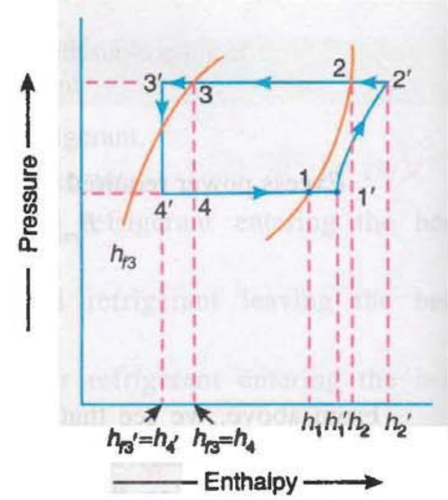
i.e.
$$h_{f3} - h_{f3'} = h_{1'} - h_1$$

$$97.94 - 79.74 = h_{1'} - 243.25$$

$$\therefore \quad h_{1'} = 97.94 - 79.74 + 243.25 = 261.45 \text{ kJ/kg}$$



(*a*) *T-s* diagram.



(*b*) *p-h* diagram.

Fig. 4.43

Let $T_{1'}$ = Temperature of vapour refrigerant leaving the heat exchanger.

We know that $c_{pv}(T_{1'} - T_1) = h_{1'} - h_1$

$$1.03(T_{1'} - 253) = 261.45 - 243.25 = 18.2$$

$$\therefore \quad T_{1'} = \frac{18.2}{1.03} + 253 = 270.67\text{K}$$

Now let as find the temperature of superheated vapour at point $2'(T_{2'})$. We know that

* Superfluous data.

Entropy at point 1′,

$$s_{1'} = s_1 + 2.3\, c_{pv} \log\left(\frac{T_{1'}}{T_1}\right)$$

$$= 0.9638 + 2.3 \times 1.03 \log\left(\frac{270.67}{253}\right)$$

$$= 0.9638 + 2.3 \times 1.03 \times 0.0293 = 1.0332 \qquad \ldots(i)$$

and entropy at point 2′, $s_{2'} = s_2 + 2.3 c_{pv} \log\left(\frac{T_{2'}}{T_2}\right) = 0.8822 + 2.3 \times 1.03 \log\left(\frac{T_{2'}}{313}\right)$

$$= 0.8822 + 2.369 \log\left(\frac{T_{2'}}{313}\right) \qquad \ldots(ii)$$

Since the entropy at point 1′ is equal to entropy at point 2′, therefore equating equations (*i*) and (*ii*),

$$1.0332 = 0.8822 + 2.369 \log\left(\frac{T_{2'}}{313}\right)$$

or

$$\log\left(\frac{T_{2'}}{313}\right) = \frac{1.0332 - 0.8822}{2.369} = 0.063\ 74$$

$$\frac{T_{2'}}{313} = 1.158 \qquad \ldots\text{(Taking antilog of 0.063 74)}$$

$$\therefore \qquad T_{2'} = 1.158 \times 313 = 362.45 \text{ K}$$

and enthalpy at point 2′, $h_{2'} = h_2 + c_{pv}(T_{2'} - T_2)$

$$= 263.21 + 1.03\,(362.45 - 313) = 314.14 \text{ kJ/kg}$$

Power requirement

We know that refrigerating effect per kg of the refrigerant,

$$R_E = h_1 - h_{f3'} = 243.25 - 79.74 = 163.51 \text{ kJ/kg}$$

∴ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{10}{163.51} = 0.0611 \text{ kg/s}$$

and workdone during compression of the refrigerant

$$= m_R\,(h_{2'} - h_{1'}) = 0.0611(314.14 - 261.45) = 3.22 \text{ kJ/s}$$

∴ Power required = 3.22 kJ/s or kW **Ans.**

2. Coefficient of performance

We know that coefficient of performance,

$$\text{C.O.P.} = \frac{h_1 - h_{f3'}}{h_{2'} - h_{1'}} = \frac{243.25 - 79.74}{314.14 - 261.45} = \frac{163.51}{52.69} = 3.1 \textbf{ Ans.}$$

4.18 Simple Saturation Cycle with Sub-cooling of Liquid Refrigerant by Liquid Refrigerant

We know that the liquid refrigerant leaving the condenser is at a higher temperature than the liquid refrigerant leaving the expansion valve. The liquid refrigerant leaving the condenser

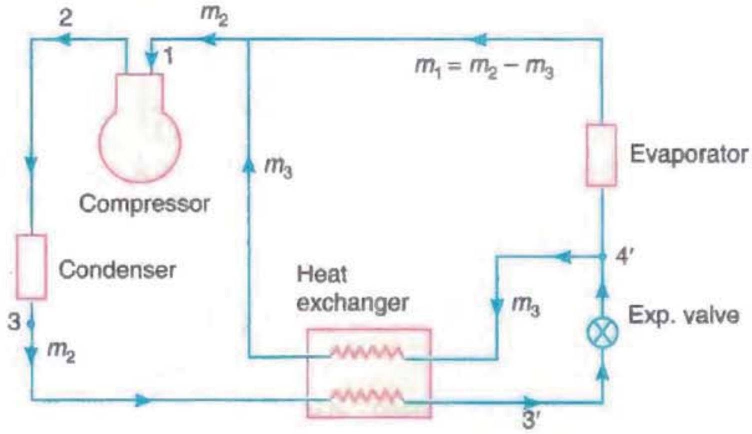


Fig. 4.44. Simple saturation cycle with sub-cooling of liquid refrigerant by liquid refrigerant.

can be sub-cooled by passing it through a heat exchanger which is supplied with liquid refrigerant from the expansion valve, as shown in Fig. 4.44. In the heat exchanger, the liquid refrigerant from the condenser gives heat to the liquid refrigerant from the expansion valve. The *p-h* diagram of the cycle is shown in Fig. 4.45.

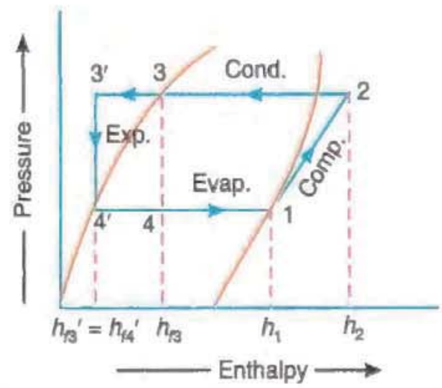


Fig. 4.45. *p-h* diagram of simple saturation cycle with sub-cooling of liquid refrigerant by liquid refrigerant.

Let m_1 = Mass of refrigerant leaving the evaporator,

m_2 = Mass of liquid refrigerant passing through the condenser, and

m_3 = Mass of liquid refrigerant supplied to the heat exchanger, from the expansion valve.

Considering the thermal equilibrium of the heat exchanger,

Heat lost by liquid refrigerant from condenser

= Heat gained by liquid refrigerant from expansion valve

i.e.
$$m_2 (h_{f3} - h_{f3'}) = m_3 (h_1 - h_{f4'})$$

$$\therefore \quad m_3 = m_2 \left[\frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}}\right] = m_2 \left[\frac{h_{f3} - h_{f3'}}{h_1 - h_{f3'}}\right]$$

$$\dots (\because h_{f4'} = h_{f3'}) \quad \dots (i)$$

We know that refrigerating effect,

$$R_E = m_1(h_1 - h_{f4'}) = (m_2 - m_3)(h_1 - h_{f4'}) \quad \ldots (\because m_1 = m_2 - m_3)$$

$$= \left[m_2 - m_2\left(\frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}}\right)\right](h_1 - h_{f4'})$$

... [From equation (*i*)]

$$= m_2(h_1 - h_{f4'}) - m_2(h_{f3} - h_{f3'})$$

$$= m_2 h_1 - m_2 h_{f4'} - m_2 h_{f3} + m_2 h_{f3'}$$

$$= m_2 h_1 - m_2 h_{f3} = m_2(h_1 - h_{f3}) \quad \ldots (\because m_2 h_{f4'} = m_2 h_{f3'})$$

and work done in compressor,

$$W = m_2(h_2 - h_1)$$

$$\therefore \quad \text{C.O.P.} = \frac{R_E}{W} = \frac{m_2(h_1 - h_{f3})}{m_2(h_2 - h_1)} = \frac{h_1 - h_{f3}}{h_2 - h_1} \quad \ldots (ii)$$

If Q tonnes of refrigeration is the load on the evaporator, then the mass of refrigerant (m_1) required to be circulated through the evaporator is given by

$$m_1 = \frac{210\,Q}{h_1 - h_{f4'}}$$

or

$$m_2 - m_3 = \frac{210\,Q}{h_1 - h_{f4'}}$$

Substituting the value of m_3 from equation (*i*),

$$m_2 - m_2\left[\frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}}\right] = \frac{210\,Q}{h_1 - h_{f4'}}$$

$$m_2\left[1 - \frac{h_{f3} - h_{f3'}}{h_1 - h_{f4'}}\right] = \frac{210\,Q}{h_1 - h_{f4'}}$$

$$m_2\left[\frac{h_1 - h_{f4'} - h_{f3} + h_{f3'}}{h_1 - h_{f4'}}\right] = \frac{210\,Q}{h_1 - h_{f4'}}$$

$$\therefore \quad m_2 = \frac{210\,Q}{h_1 - h_{f3}} \quad \ldots (\because h_{f4'} = h_{f3'})$$

We know that power required to drive the compressor,

$$P = \frac{m_2(h_2 - h_1)}{60}\ \text{kW}$$

Substituting the value of m_2 in the above equation,

$$P = \frac{210\,Q}{60}\left[\frac{h_2 - h_1}{h_1 - h_{f3}}\right]\ \text{kW}$$

Notes: 1. From equation (*i*), we see that the mass of refrigerant required in the heat exchanger is exactly equal to the mass of flash that would form in a simple saturation cycle.

2. Since the C.O.P. of the cycle and power required to drive the compressor is same as that of simple saturation cycle, therefore this arrangement of sub-cooling the liquid refrigerant is of no advantage. In other words, the above method of subcooling is thermodynamically same as the simple saturation cycle.

EXERCISES

1. An ammonia refrigerator works between – 6.7°C and 26.7° C, the vapour being dry at the end of isentropic compression. There is no under-cooling of liquid ammonia and the liquid is expanded through a throttle valve after leaving the condenser. Sketch the cycle on the *T-s* and *p-h* diagram and calculate the refrigeration effect per kg of ammonia and the theoretical coefficient of performance of the unit with the help of the properties given below :

Temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–6.7	*152.18*	*1437.03*	*0.6016*	*5.4308*
26.7	*307.18*	*1467.03*	*1.1515*	*5.0203*

[Ans. 1028.3 kJ/kg ; 7.2]

2. An ammonia refrigerator produces 30 tonnes of ice from and at 0°C in 24 hours. The temperature range of the compressor is from 25°C to – 15°C. The vapour is dry saturated at the end of compression and an expansion valve is used. Assume a coefficient of performance to be 60% of the theoretical value. Calculate the power required to drive the compressor. Latent heat of ice = 335 kJ/ kg. Properties of ammonia are :

Temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
25	*298.9*	*1465.84*	*1.1242*	*5.0391*
–15	*112.34*	*1426.54*	*0.4572*	*5.5490*

[Ans. 33.24 kW]

3. An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of – 10°C and 30°C. The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at 30°C. If the actual coefficient of performance is 60% of the theoretical, find the ice produced per kW hour at 0°C from water at 10°C. The latent heat of ice is 335 kJ/kg. The ammonia has the following properties :

Temperature, °C	*Liquid heat, kJ/kg*	*Latent heat, kJ/kg*	*Entropy, kJ/kg K*	
			Liquid	*Vapour*
30	*323.08*	*1145.79*	*1.2037*	*4.9842*
–10	*135.37*	*1297.68*	*0.5443*	*5.4770*

[Ans. 33.24 kg/kWh]

4. A vapour compression works on a simple saturation cycle with R-12 as the refrigerant which operates between the condenser temperature of 40°C and an evaporator temperature of –5°C. For the modified cycle, the evaporator temperature is changed to –10°C and other operating conditions are the same as the original cycle. Compare the power requirement for both cycles. Both system develops 15 tonnes of refrigeration.

[Ans. 10.7 kW, 12.47 kW]

5. A R-12 refrigerating machine works on vapour-compression cycle. The temperature of refrigerant in the evaporator is – 20°C. The vapour is dry saturated when it enters the compressor and leaves it in a superheated condition. The condenser temperature is 30°C. Assuming specific heat at constant pressure for R-12 in the superheated condition as 1.884 kJ/kg K, determine :

   1. condition of vapour at the entrance to the condenser ;
   2. condition of vapour at the entrance to the evaporator ; and
   3. theoretical C.O.P. of the machine.

The properties of R-12 are :

Temperature, °C	Enthalpy, kJ/kg		Entropy, kJ/kg K	
	Liquid	Vapour	Liquid	Vapour
–20	17.82	178.73	0.0731	0.7087
30	64.59	199.62	0.2400	0.6843

[Ans. 33.8°C ; 29% dry ; 4.07]

6. It is proposed to replace R-12 by ozone friendly R-134a in a refrigeration plant of 10TR capacity with evaporator and condenser temperatures of 0°C and 40°C respectively. Considering standard saturation cycle (evaporator exit and condenser exit as saturated states), compare the mass flow rate, compressor work (in kW), condenser heat rejection (in kW) and C.O.P for the two refrigerants. The saturation properties and vapour specific heats are as follows:

Refrigerant	Saturation temperature (°C)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Specific heat (kJ/kg K)
		Liquid	Vapour	Liquid	Vapour	
R-134a	0	200.81	398.78	1.0025	1.7261	—
	40	255.73	419.63	1.1884	1.7128	1.068
R-12	0	36.1	187.5	0.0142	0.6966	—
	40	74.6	203.2	0.2718	0.6825	0.776

[Ans. For R-134a : 0.245 kg/s ; 6.13 kW ; 41.13 kW ; 5.7
For R-12 : 0.31 kg/s ; 6.25 kW ; 41.25 kW ; 5.6]

7. A CO_2 refrigerating plant fitted with an expansion valve, works between the pressure limits of 54.81 bar and 20.93 bar. The vapour is compressed isentropically and leaves the compressor cylinder at 32°C. The condensation takes place at 18°C in the condenser and there is no undercooling of the liquid. Determine the theoretical coefficient of performance of the plant. The properties of CO_2 are :—

Pressure, bar	Saturation temperature, °C	Enthalpy, kJ/kg		Entropy, kJ/kg K	
		Liquid	Vapour	Liquid	Vapour
54.81	+18	137.48	302.55	0.5065	1.0738
20.93	–18	43.27	323.06	0.1733	1.2692

[Ans. 4.92]

8. A vapour compression plant using R-12 operates between 35°C condensing temperature and –5°C evaporation temperature with saturated vapour leaving the evaporator. The plant consists of twin cylinder, single acting compressor with 100 mm diameter and 120 mm stroke running at 300 r.p.m. The volumetric efficiency is 85% and the mechanical efficiency is 90%. Assuming isentropic compression, determine : 1. C.O.P. ; 2. Power required ; and 3. Tonnage capacity of the plant.

[Ans. 5.476 ; 2.87 kW ; 4.1 TR]

9. A single stage NH_3 refrigeration system has cooling capacity of 500 kW. The evaporator and condenser temperatures are – 10°C and 30°C respectively. Assuming saturation cycle, determine : 1. mass flow rate of refrigerant ; 2. adiabatic discharge temperature ; 3. compressor work in kW ; 4. condenser heat rejection ; 5. C.O.P. ; and 6. compressor swept volume in m^3/min, if volumetric efficiency is 70%.

The following values may be taken :

h_g (– 10°C) = 1431.6 kJ/kg ; h_f (30°C) = 322.8 kJ/kg ; v_g (–10°C) = 0.4185 m³/kg ;
s_g (– 10°C) = 5.4717 kJ/kg K.

The properties of superheated NH_3 at condenser pressure of 11.66 bar (30°C) are as follows :

At 85°C, h = 1621.8 kJ/kg ; s = 5.5484 kJ/kg K. At 90°C, h = 1634.5 kJ/kg ; s = 5.4838 kJ/kg K.

[Ans. 0.45 kg/s ; 88.3°C ; 89.5 kW ; 590 kW ; 5.585 ; 16.2 m³/min]

10. A refrigeration system of 10TR capacity at an evaporator temperature of –12°C, needs a condenser temperature of +28°C. The refrigerant NH_3 is subcooled by 5°C before entering the expansion valve. The vapour is 0.95 dry when it leaves the evaporator. Using p-h chart for NH_3, find : 1. Conditions of vapour at the outlet of compressor ; 2. Condition of vapour at entrance of evaporator ; 3. C.O.P. ; and 4. Power required. **[Ans. 60°C ; 0.12 ; 5.7 ; 6.14 kW]**

11. The evaporator and condenser temperatures in an NH_3 refrigeration system are – 10°C and 40°C respectively. Determine per TR basis : 1. mass flow rate ; 2. compressor work ; 3. condenser heat rejection; 4. C.O.P. ; and 5. refrigerating efficiency. Use only the properties of NH_3 given below :

Saturation temperature (t) °C	*Pressure (p) bar*	*Specific volume of vapour (v_g) m³/kg*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
			Liquid	*Vapour*	*Liquid*	*Vapour*
–10	2.908	0.418	134.95	1431.41	0.5435	5.4712
40	15.55	0.0833	371.47	1472.02	1.3574	4.8728

For superheated NH_3 at 15.55 bar, the following values may be taken :

Superheat K	*Specific volume (v) m³/kg*	*Enthalpy (h) kJ/kg*	*Entropy (s) kJ/kg K*
60	0.108	1647.9	5.3883
80	0.116	1700.3	5.5253

[Ans. 0.198 kg/min ; 0.82 kW ; 4.32 kW ; 4.27 ; 81.2%]

12. In a vapour compression refrigeration system using R-12, the evaporator pressure is 1.4 bar and the condenser pressure is 8 bar. The refrigerant leaves the condenser sub-cooled to 30°C. The vapour leaving the evaporator is dry and saturated. The compression process is isentropic. The amount of heat rejected in the condenser is 13.42 MJ/min. Determine : 1. refrigerating effect in kJ/kg; 2. refrigerating load in TR; 3. compressor input in kW ; and 4. C.O.P.
Show the cycle on a p-h diagram. **[Ans. 114 kJ/kg ; 49 TR ; 51.4 kW ; 3.35]**

13. A vapour compression refrigerator works between the temperature limits of – 20°C and 25°C. The refrigerant leaves the compressor in dry saturated condition. If the liquid refrigerant is undercooled to 20°C before entering the throttle valve, determine :
    1. work required to drive the compressor ;
    2. refrigerating effect produced per kg of the refrigerant; and
    3. theoretical C.O.P.

    Assume specific heat of the refrigerant as 4.8. The properties of the refrigerant are :

Temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–20	89.78	1420.02	0.3684	5.6244
25	298.90	1465.84	1.1242	5.0391

[Ans. 189.7 kJ/kg ; 990.2 kJ/kg ; 5.01]

14. A food storage chamber requires a refrigeration system of 12 TR capacity with an evaporator temperature of –8°C and condenser temperature of 30°C. The refrigerant R-12 is subcooled by 5°C before entering the throttle valve, and the vapour is superheated by 6°C before entering the compressor. If the liquid and vapour specific heats are 1.235 and 0.733 kJ/kg K respectively, find : 1. refrigerating effect per kg; 2. mass of refrigerant circulated per minute; and 3. coefficient of performance.

The relevant properties of the refrigerant R-12 are given below :

Saturation temperature, °C	*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*	
	Liquid	*Vapour*	*Liquid*	*Vapour*
–8	*28.70*	*184.06*	*0.1148*	*0.7007*
30	*64.59*	*199.62*	*0.2400*	*0.6853*

[Ans. 130.05 kJ/kg ; 19.4 kg ; 6.2 **]**

15. The following data refer to a single cylinder, single acting compressor of an ammonia refrigeration system :

Bore	= 100 mm
Stroke	= 150 mm
Speed	= 200 r.p.m.
Indicated mean effective pressure	= 3.2 bar
Condenser pressure	= 10 bar
Evaporator pressure	= 3 bar
Temperature of water at entry to condenser	= 55°C
Temperature of water at exit from condenser	= 20°C
Rate of cooling water flowing in the condenser	= 12.5 kg/min
Inlet water temperature	= 12.5°C
Outlet water temperature	= 20.5°C

If the mass of ice produced per hour from water at 15°C is 50 kg and the latent heat of ice is 335 kJ/kg, find ; (*a*) coefficient of performance ; (*b*) mass flow of ammonia per minute ; and (*c*) condition of ammonia entering the compressor.

The relevant properties of ammonia are given below :

Pressure, bar	*Saturation temperature, °C*	*Enthalpy, kJ/kg*		*Specific heat, kJ/kg K*	
		Liquid	*Vapour*	*Liquid*	*Vapour*
12	*31*	*327.9*	*1469.5*	*4.6*	*2.8*
2.9	*–10*	*135.4*	*1433*	*—*	*—*

[Ans. 4.46 ; 0.326 kg ; 0.86 **]**

16. A freezer of 20 TR capacity has evaporator and condenser temperatures of – 30°C and 25°C respectively. The refrigerant R-12 is sub-cooled by 4°C before it enters the expansion valve and is superheated by 5°C before leaving the evaporator. The compression is isentropic and the valve throttling and clearance are to be neglected. If a six cylinder, single acting compressor with stroke equal to bore running at 1000 r.p.m. is used, determine (*a*) C.O.P. of the refrigerating system, (*b*) mass of refrigerant to be circulated per min, (*c*) theoretical piston displacement per minute, and (*d*) theoretical bore and stroke of the compressor. The specific heat of liquid R-12 is 1.235 kJ/kg K and of vapour R-12 is 0.733 kJ/kg K.

The properties of R-12 are given below :

Saturation temp., °C	Pressure, bar	Enthalpy, kJ/kg		Entropy, kJ/kg K		Specific volume, m^3/kg	
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
–30	1.0044	8.86	174.20	0.0371	0.7171	0.006 73	0.1596
25	6.5184	59.7	197.73	0.2239	0.6868	0.007 64	0.0269

[Ans. 3.64 ; 34.12 kg/min ; 5.56 m^3/min ; 0.106 m]

17. A vapour compression refrigeration system of 2400 kJ/min capacity works at an evaporator temperature of 263 K and a condenser temperature of 303 K. The refrigerant used is R-12 and is subcooled by 6°C before entering the expansion valve and vapour is superheated by 7°C before leaving the evaporator coil. The compression of refrigerant is reversible adiabatic. The refrigeration compressor is two cylinder, single acting with stroke equal to 1.25 times the bore and runs at 1000 r.p.m. Take liquid specific heat = 1.235 kJ/kg K and vapour specific heat = 0.733 kJ/kgK. Determine : 1. Refrigerating effect per kg ; 2. Mass of refrigerant circulated per minute ; 3. Theoretical piston displacement per minute ; 4. Power required to run the compressor ; 5. Heat removed in through condenser ; and 6. Bore and stroke of the compressor.

The properties of R-12 are given below:

Saturation temp., K	Pressure bar	Specific volume of vapour, m^3/kg	Enthalpy, kJ/kg		Entropy, kJ/kg K	
			Liquid	Vapour	Liquid	Vapour
263	2.19	0.0767	26.9	183.2	0.1080	0.7020
303	7.45	0.0235	64.6	199.6	0.2399	0.6854

[Ans. 131.14 kJ/kg; 18.3 kg/min; 1.44 m^3/min; 7 kW; 2820 kJ/min; 90mm, 112.5 mm]

18. A refrigeration plant of 8 TR capacity has its evaporation temperature of – 8°C and condenser temperature of 30°C. The refrigerant is sub-cooled by 5°C before entering into the expansion valve and vapour is superheated by 6°C before leaving the refrigerator. The suction pressure drop is 0.2 bar in the suction valve and discharge pressure drop is 0.1 bar in the discharge valve.

If the refrigerant used is R-12, find out the C.O.P. of the plant and theoretical power required for the compressor. Assume compression is isentropic. Use *p-h* chart for calculation.

QUESTIONS

1. Mention the advantages of vapour compression refrigeration system over air refrigeration system.
2. Describe the mechanism of a simple vapour compression refrigeration system.
3. Explain with reference to *T-s* diagram, the stages involved in vapour compression process of refrigeration. Establish an expression for the coefficient of performance.
4. Sketch the *T-s* and *p-h* diagrams for the vapour compression cycles when the vapour after compression is (*i*) dry saturated, and (*ii*) wet.
5. Why in practice a throttle valve is used in vapour compression refrigerator rather than an expansion cylinder to reduce pressure between the condenser and the evaporator ?
6. Explain the effect of subcooling of condensate with the help of *T-s* and *p-h* diagrams in vapour compression systems.
7. Describe a simple vapour compression refrigeration system without liquid subcooling and with superheated vapour after compression. Show the entire system on *T-s* and *p-h* planes. Why is superheating considered to be good in certain cases ?
8. How does an actual vapour compression cycle differ from that of a theoretical cycle ?

OBJECTIVE TYPE QUESTIONS

1. The coefficient of performance of vapour compression refrigeration system is quite .... as compared to air refrigeration system.
   (*a*) low (*b*) high
2. During a refrigeration cycle, heat is rejected by the refrigerant in a
   (*a*) compressor (*b*) condenser (*c*) evaporator (*d*) expansion valve
3. In a vapour compression refrigeration system, the condition of refrigerant before entering the compressor is
   (*a*) saturated liquid (*b*) wet vapour
   (*c*) dry saturated liquid (*d*) superheated vapour
4. The highest temperature during the cycle, in a vapour compression refrigeration system, occurs after
   (*a*) compression (*b*) condensation (*c*) expansion (*d*) evaporation
5. In a vapour compression refrigeration system, the lowest temperature during the cycle occurs after
   (*a*) compression (*b*) condensation (*c*) expansion (*d*) evaporation
6. In a vapour compression refrigeration system, the effect of superheating the vapour before suction to compression
   (*a*) increases the work of compression
   (*b*) increases the heat rejection in the condenser
   (*c*) may increase or decrease C.O.P. depending upon the refrigerant used
   (*d*) all of the above
7. In a domestic vapour compression refrigerator, the refrigerant commonly used is
   (*a*) CO_2 (*b*) Ammonia (*c*) Freon - 12 (*d*) all of these
8. The subcooling is a process of cooling the refrigerant in vapour compression refrigeration system
   (*a*) before compression (*b*) after compression
   (*c*) before throttling (*d*) after throttling
9. In a vapour compression refrigeration system, subcooling the liquid refrigerant is to .... coefficient of performance.
   (*a*) increase (*b*) decrease
10. The process of undercooling is generally brought about by
    (*a*) circulating more quantity of cooling water through the condenser
    (*b*) using water colder than the main circulating water
    (*c*) employing a heat exchanger
    (*d*) any one of the above

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*d*)	**4.** (*a*)	**5.** (*d*)
6. (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*d*)