Channels of Information theory:

Examples

1. A noiseless channel

We have :

$$I(X;Y) = H(X) - H(X/Y)$$

The occurrence of Y uniquely specifies the input X. Consequently, $H(X/Y) = 0$ and

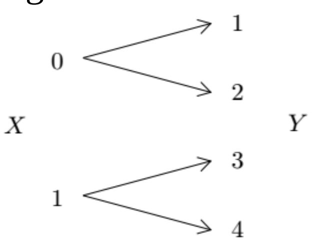
$$C = \underset{p(X)}{Max}\, H(X)$$

As X is a random variable taking on 4 different values, the maximum of $H(X)$ is $(\log_2 4)$bits. This value is achieved for a uniform probability distribution on the input alphabet.

Finally, we get :

$$\boxed{C = \log_2 4 = 2\text{ bits}}$$

2. Noisy channel with nonoverlapping :

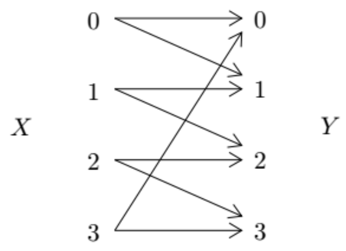


so that if $X = 0$ then $Y = \begin{cases} 1 & w.p. 1/2 \\ 2 & w.p. 1/2 \end{cases}$, and similarly for $X = 1$. Since given Y we can determine completely X, we expect to be able to communicate 1 bit of information per transmission. This time we expand $I(X;Y)$ a little differently when computing C:

$$I(X;Y) = H(X) - H(X|Y) = H(X) \leq 1$$

Here we note that X is a function of Y, so $H(X|Y) = 0$, and again the upper bound can be achieved with $X \sim \text{Bern}(1/2)$. Thus $C = 1$ bit per transmission. Note that the transition probabilites above can be arbitrary and the same result holds.

3- Noisy typewriter.



Consider the diagram above, where each branch is labeled with probability 1/2. Note that if we restrict ourselves to using only 0 or 2 (or 1 and 3), then this situation looks exactly like the previous example, which allows us to communicate at 1 bit per transmission through this channel. It turns out that this is the best we can do.

Symmetric channels:

A channel is said to be **symmetric** if the set of output symbols can be partitioned into subsets in such a way that for each subset, the probability transition matrix has the following two properties

- each row is a permutation of each remaining row,
- each column (if there are more than one) is a permutation of each remaining column.

In mathematics, the notion of **permutation** relates to the act of **arranging** all the members of a set into some sequence or order, or if the set is already ordered, **rearranging** (reordering) its elements, a process called **permuting**. These differ from combinations, which are selections of some members of a set where order is disregarded. For example, written as tuples, there are six permutations of the set {1,2,3}, namely: (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), and (3,2,1).

Example 1 Let us consider a channel with the probability transition matrix :

$$
\begin{array}{c}
\\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \underbrace{\hphantom{AB}}_{\mathbf{X}}
\end{array}
\begin{array}{c}
\overbrace{\begin{array}{cccc} \mathbf{D} & \mathbf{E} & \mathbf{F} & \mathbf{G} \end{array}}^{\mathbf{Y}} \\
\begin{bmatrix} 0.1 & 0.5 & 0.1 & 0.3 \\ 0.5 & 0.1 & 0.1 & 0.3 \\ 0.1 & 0.1 & 0.5 & 0.3 \end{bmatrix} \\
\\
\end{array}
$$

The output symbols {D, E, F, G} can be partitioned into two subsets {D, E, F} and {G}. The two probability transition matrices are :

$$T_1 = \begin{pmatrix} 0.1 & 0.5 & 0.1 \\ 0.5 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.5 \end{pmatrix} \text{ and } T_2 = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.3 \end{pmatrix}$$

Each of them meets the required properties to make the channel symmetric.

Example 2

Let the probability transition matrix be :

$$\mathbf{T} = \begin{pmatrix} \mathbf{0.1} & \mathbf{0.6} & \mathbf{0.3} \\ \mathbf{0.4} & \mathbf{0.1} & \mathbf{0.5} \\ \mathbf{0.5} & \mathbf{0.2} & \mathbf{0.3} \end{pmatrix}$$

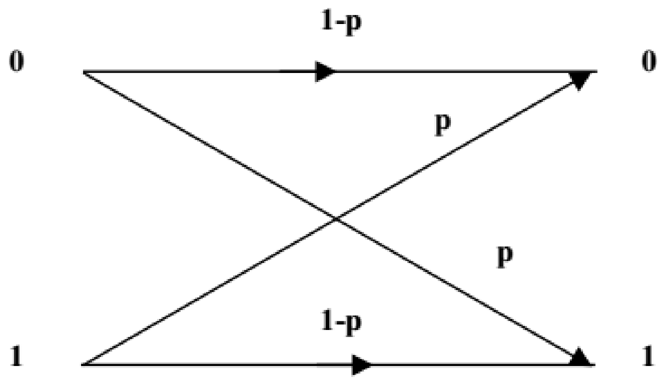
As not one of the three columns has the same value on its three rows, there is no partition containing one input symbol for which the symmetry properties are met. Neither does the global probability transition matrix meet the properties. Consequently, the channel is not symmetric.

Theorem

For a symmetric channel, the capacity is achieved for a uniform input probability distribution.

Example 1

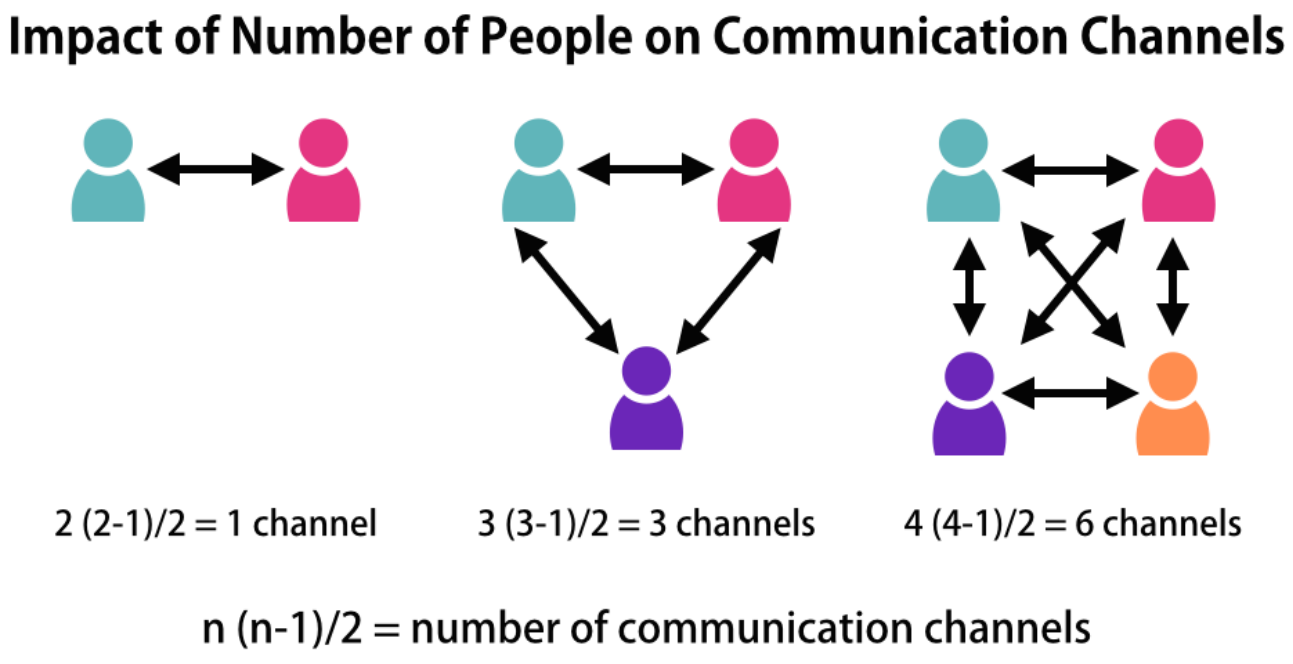
Let us consider a Binary Symmetric Channel :

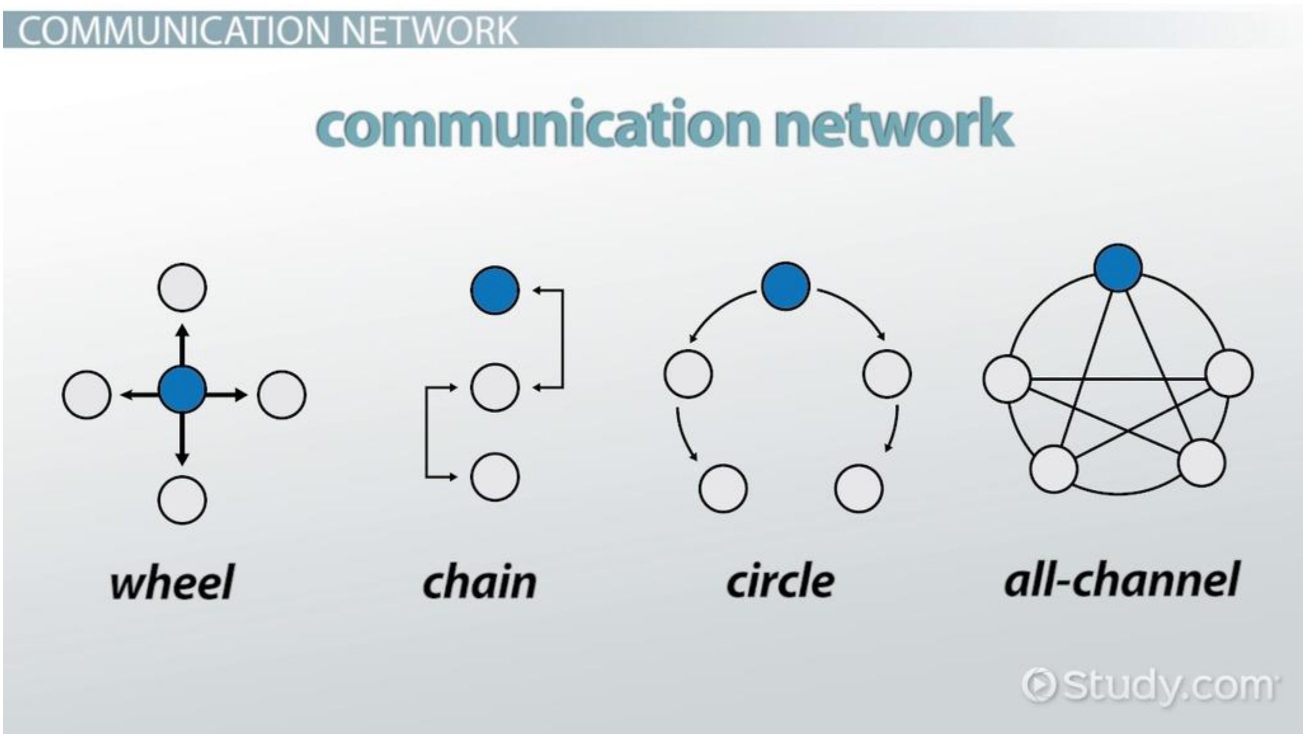


The probability transition matrix is :

$$
\begin{array}{c c}
 & \begin{array}{cc} 0 & 1 \end{array} \quad \leftarrow Y \\
\begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix} \\
\uparrow & \\
X &
\end{array}
$$

This matrix meets the requirements to make the channel symmetric. Thus, the capacity is achieved for $P\{X=0\}=P\{X=1\}=\frac{1}{2}$





What Is WiFi

A wireless or WiFi network uses a radio frequency signal instead of wires to connect your devices - such as computers, printers and smartphones - to the Internet and each other. The WiFi signal can be picked up by any wireless-capable device such as a laptop or tablet within a certain distance in all directions.

Why Do I Need WiFi?

Considering whether or not to set up WiFi in your home? Here are just a few reasons to set up a wireless network in your home:

- Freedom to access the Internet from anywhere within the signal range and move your devices around your home - anywhere within the WiFi signal range - without losing your connection. Take any mobile device, like a laptop, into any room and still have Internet access, no extra work required.
- Ability to access other devices connected to your network, for example, multiple computers can use one printer without a directly wired connection.
- Access to the Internet on devices like smartphones and tablets to download books, music, movies and apps, or surf the web.
- Freedom from the hassle of installing wired connections in different rooms.

Is WiFi Secure?

Because WiFi devices use a broadcast signal instead of wires to connect to the Internet and teach other, it is possible for unauthorized users to access your network. This could reduce the speed of your connection or make you vulnerable to things like identity theft. Yet there are multiple ways to ensure that your wireless home network is secure.