Quantile:

The qth quantile of a random variable X or of its corresponding distribution is denoted by x_q and is defined as the smallest number x_q satisfying $F(x_q) \geq q, \; 0 \leq q \leq 1.$

If X is a continuous random variable, then the qth quantile of X is given as the smallest number x_q satisfying $F(x_q) = q$.

Median:

Then if $q = \frac{1}{2}$, then the mediam of a random variable X is defined as the 5th quantile denoted by med(X) or med_X and if X is a continuous random variable, then the median of X is given as the smallest number x_q satisfying $F\left(x_{\frac{1}{2}}\right) = \frac{1}{2}$. That is it satisfies $\int_{-\infty}^{\text{med}(X)} f(u)du = \int_{\text{med}(X)}^{\infty} f(u)du = \frac{1}{2}$.

Example: Find the median of a r.v. X that has the p.d.f . defined

$f(x;\lambda) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$.

<u>Solution</u> $\because \int_{\text{med}(X)}^{\infty} f(u)du = \frac{1}{2}$,then

$$\int_{\text{med}(X)}^{\infty} \lambda e^{-\lambda x} du = \frac{1}{2} \Longrightarrow -e^{-\lambda x}\Big]_{\text{med}(X)}^{\infty} = \frac{1}{2} \Longrightarrow -e^{-\lambda \text{med}(X)} + 1 = \frac{1}{2}$$

$$\Rightarrow e^{-\lambda \text{med}(X)} = \frac{1}{2} \Longrightarrow, -\lambda \text{med}(X) = ln\frac{1}{2} \Longrightarrow \text{med}(X) = \frac{ln2}{\lambda}$$

Introduction to the Theory of Statistics

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