Exponential dis, التوزيع الأسي

we said X is r.v continuos has exponential dis if p.f.d. is:

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \quad , x \geq 0 \; , \; \theta > 0$$

$$= 0 \quad \text{otherwise}$$

and $\lambda = \frac{1}{\theta}$ Then $f(x) = \lambda e^{-\lambda x}$, $x > 0$, $\lambda > 0$

$\theta = \frac{1}{\lambda}$ is paramebe of dis

$X \sim Exp(\lambda)$ i.e $X \sim Exp(\frac{1}{\theta})$

① $\int_0^\infty f(x)\, dx = 1$

$$\int_0^\infty f(x)\, dx = \int_0^\infty \frac{1}{\theta} e^{-x/\theta} dx = -e^{-x/\theta} \Big]_0^\infty = -(0-1) = 1$$

② $F(x) = \int_0^x f(t)\,dt = \int_0^x \frac{1}{\theta} e^{-t/\theta} dt = -e^{-t/\theta} \Big]_0^x$

$$= -(e^{-x/\theta} - e^0) = -(e^{-x/\theta} - 1) = 1 - e^{-x/\theta} \quad , x \geq 0$$

$$\therefore F(x) = \begin{cases} 1 - e^{-x/\theta} & , x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

mean of Ex. dis.

$$E(x) = \int x f(x)\, d_x = \int_0^\infty x \frac{1}{\theta} e^{-x/\theta} dx$$

$$E(x) = -x e^{-x/\theta} \Big|_0^{\infty} + \int_0^{\infty} e^{-x/\theta} dx$$

$$= -x e^{-x/\theta} \Big|_0^{\infty} - \theta e^{-x/\theta} \Big|_0^{\infty}$$

$$= -[0-0] - [0 - \theta e^{0}]$$

$$= -(0 - \theta) = \theta$$

varanue of Exp. dis,

$$V(X) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \int_x x^2 f(x) dx = \int_0^{\infty} x^2 \frac{1}{\theta} e^{-x/\theta} dx$$

$$= -x^2 e^{-x/\theta} \Big|_0^{\infty} - \int_0^{\infty} -2x e^{-x/\theta} dx$$

$$= -x^2 e^{-x/\theta} \Big|_0^{\infty} + 2\int_0^{\infty} x e^{-x/\theta} d\theta$$

$$\therefore E(x^2) = -x^2 e^{-x/\theta} \Big|_0^{\infty} + 2\left[-\theta x e^{-x/\theta} \Big|_0^{\infty} - \int_0^{\infty} -\theta e^{-x/\theta} dx\right]$$

$$= -x^2 e^{-x/\theta} \Big|_0^{\infty} - 2x\theta e^{-x/\theta} \Big|_0^{\infty} + 2\int_0^{\infty} \theta e^{-x/\theta} dx = -x^2 e^{-x/\theta} \Big|_0^{\infty} - 2x\theta e^{-x/\theta}$$
$$- 2\theta^2 e^{-x/\theta} \Big|_0^{\infty}$$

$$= -[0-0] - 2[0-0] - 2\theta^2 (0 - e^{0}) = -2\theta^2(-1) = 2\theta^2$$

$$\therefore V(x) = E(x^2) - [E(x)]^2 = 2\theta^2 - \theta^2 = \theta^2 = \frac{1}{\lambda^2}$$

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Moment generating fun

$$M_x(t) = E(e^{tx}) = \int_x e^{tx} f(x)\,dx = \int_0^\infty e^{tx} \frac{1}{\theta} e^{-x/\theta}\,dx = \frac{1}{1-\theta t}$$

$$E(x) = \frac{d}{dt} M_x(t)\Big|_{t=0} = \frac{d}{dt}\left(\frac{1}{1-\theta t}\right)\Big|_{t=0} = \frac{\theta}{(1-\theta t)^2}\Big|_{t=0} = \theta$$

$$E(x^2) = \frac{d^2}{dt^2} M_x(t)\Big|_{t=0} = \frac{d^2}{dt^2}\left(\frac{1}{1-\theta t}\right)\Big|_{t=0} = \frac{d}{dt}\left(\frac{\theta}{(1-\theta t)^2}\right)\Big|_{t=0} = \frac{2\theta^2}{(1-\theta t)^3}\Big|_{t=0} = 2\theta^2$$

$$\left(\frac{-\theta(2(1-\theta t)(-\theta))}{(1-\theta t)^4}\right)_{t=0} = \frac{2\theta^2(1-\theta t)}{(1-\theta t)^4}\Big)_{t=0}$$

$$var(x) = E(x^2) - [E(x)]^2 = 2\theta^2 - \theta^2 = \theta^2$$

Ex: If $x \sim Exp(4)$, find mean, variance & ~~mod~~ moment generating fun.

$$E(x) = \mu_x = \frac{1}{\lambda} = \frac{1}{4} = \theta, \quad var(x) = \frac{1}{\lambda^2} = \frac{1}{4^2} = \frac{1}{16} = \theta^2$$

$$M_x(t) = \frac{\lambda}{\lambda - t} = \frac{4}{4-t} = \frac{1}{1-\theta t}$$

for $\lambda = 0.5, 1$ and 2
The shaps of The exponetial density curve are