Examples for how to Translate English Sentences into First-Order Logic

If you would like to practise, there are some more problems (with sample solutions) in Nilsson, Section 15.6.2, and ((without sample solutions) in Russell and Norvig, Exercise 7.2. The following problems were inspired by them. In the following, it is important to remember the precedence of the operators, which are (from highest to lowest): $\neg$ (NOT), $\wedge$ (AND), $\vee$ (OR), $\Rightarrow$ (IMPLIES), $\Leftrightarrow$ (EQUIV). Notice also that there are always several (equivalent) sentences in first-order logic that correspond to a given English sentence. We give only one example.

All students are smart.
$\forall$ x (Student(x) $\Rightarrow$ Smart(x))

There exists a student.
$\exists$ x Student(x).

There exists a smart student.
$\exists$ x (Student(x) $\wedge$ Smart(x))

Every student loves some student.
$\forall$ x (Student(x) $\Rightarrow$ $\exists$ y (Student(y) $\wedge$ Loves(x,y)))

Every student loves some other student.
$\forall$ x (Student(x) $\Rightarrow$ $\exists$ y (Student(y) $\wedge$ $\neg$ (x = y) $\wedge$ Loves(x,y)))

There is a student who is loved by every other student.
$\exists$ x (Student(x) $\wedge$ $\forall$ y (Student(y) $\wedge$ $\neg$(x = y) $\Rightarrow$ Loves(y,x)))

Bill is a student.
Student(Bill)

Bill takes either Analysis or Geometry (but not both)
Takes(Bill, Analysis) $\Leftrightarrow$ $\neg$ Takes(Bill, Geometry)

Bill takes Analysis or Geometry (or both).
Takes(Bill, Analysis) $\vee$ Takes(Bill, Geometry)

Bill takes Analysis and Geometry.
Takes(Bill, Analysis) $\wedge$ Takes(Bill, Geometry)

Bill does not take Analysis.
$\neg$ Takes(Bill, Analysis).

No student loves Bill.
$\neg$ $\exists$ x (Student(x) $\wedge$ Loves(x, Bill))

Bill has at least one sister.
$\exists$ x SisterOf(x,Bill)

Bill has no sister.
$\neg$ $\exists$ x SisterOf(x,Bill)

Bill has at most one sister.
$\forall$ x, y (SisterOf(x, Bill) $\wedge$ SisterOf(y, Bill) $\Rightarrow$ x = y)

Bill has exactly one sister.
$\exists$ x (SisterOf(x, Bill) $\wedge$ $\forall$ y (SisterOf(y, Bill) $\Rightarrow$ x = y))

Bill has at least two sisters.
$\exists$ x, y (SisterOf(x, Bill) $\wedge$ SisterOf(y, Bill) $\wedge$ $\neg$ (x = y))

Every student takes at least one course.
$\forall$ x (Student(x) $\Rightarrow$ $\exists$ y (Course(y) $\wedge$ Takes(x,y)))

Only one student failed History.
$\exists$ x (Student(x) $\wedge$ Failed(x, History) $\wedge$ $\forall$ y (Student(y) $\wedge$ Failed(y, History) $\Rightarrow$ x = y))

No student failed Chemistry but at least one student failed History.
$\neg$ $\exists$ x (Student(x) $\wedge$ Failed(x, Chemistry)) $\wedge$ $\exists$ x (Student(x) $\wedge$ Failed(x, History))

Every student who takes Analysis also takes Geometry.
$\forall$ x (Student(x) $\wedge$ Takes(x, Analysis) $\Rightarrow$ Takes(x, Geometry))

No student can fool all the other students.
$\neg$ $\exists$ x (Student(x) $\wedge$ $\forall$ y (Student(y) $\wedge$ $\neg$ (x = y) $\Rightarrow$ Fools(x,y)))