Motion in One Dimension (1 D)

2.1 Position and particle

The position of a particle on an x-axis locate the particle with respect to the origin, or zero point of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at origin. The positive direction on an axis is the direction of increasing positive numbers, the opposite direction is the negative direction. The moving object is either a particle (by which we mean a point like object such as electron) or an object that moves like a particle such that move in the same direction and at the same rate). In our study we describe the moving object as a particle regardless of its size.

2.2 Displacement, average velocity and average speed

The motion of a particle is completely known if the particle position in space is known at all times. If a particle is moving we can easily determine its change in position, a charge in position of a particle is called a displacement. When a particle moves from one position x_1 to another position x_2 its displacement is written as:

$$\Delta x = x_2 - x_1 \qquad (1)$$

The symbol Δ (delta), represents a change in quantity and it means the final value of that quantity minus the initial value.

Average speed of a particle $\bar{s}$ is defined as total distance travelled divided by the total time it takes to travel that distance

$$\bar{s} = \frac{distance\ travelled}{time\ taken} \Rightarrow \bar{s} = \frac{d}{t} \qquad > 0 \quad always \qquad (2)$$

The average velocity ($\bar{v}$) of a particle is defined as a particle's displacement Δx divided by the time interval Δt during which that displacement occurred when a particle has moved from position x_1 to x_2 during a time interval $\Delta t(t_2 - t_1)$, its average velocity is:

$$\bar{v} = \frac{change\ in\ position}{change\ in\ time} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad \left(\frac{m}{s}\right) \qquad (3)$$

From equation (3) we see that average velocity doesn't depend on the actual distance a particle moves, but depends on its original and final positions.

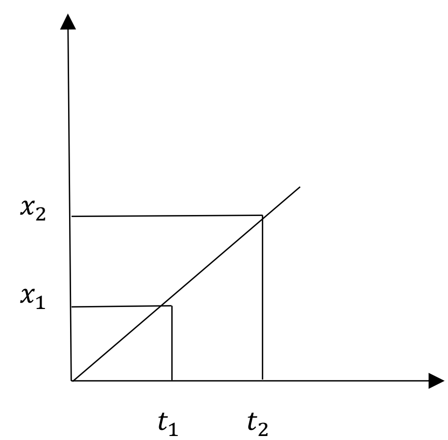
$\bar{v} > 0$ motion to right along x-axis

$\bar{v} < 0$ motion to left along x-axis

$\Delta x = x_2 - x_1$

$\bar{v} = \frac{d}{t}$

$\bar{v} = \frac{\Delta x}{\Delta t}$



2.3 Instantaneous velocity

Often we need to know the velocity of a particle at particular instant in time, rather than over a finite time interval. The velocity of any instant of time, or point in space, is called instantaneous velocity which is given by the slope of tangent to the position-time graph at that time expressed mathematically:

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (4)$$

v (Instantaneous Velocity) ≡ Velocity

Equation (4) means that velocity is the general time derivative of the position function.

2.4 Constant velocity motion

Let a particle moves with a position- time dependence which is a straight line, slope of $x(t) = constant$

$\bar{v} = constant = v_0$, also $v(t) = v_0 \Rightarrow \bar{v} = v$ or average = instantaneous

Motion at constant velocity is called uniform linear motion.

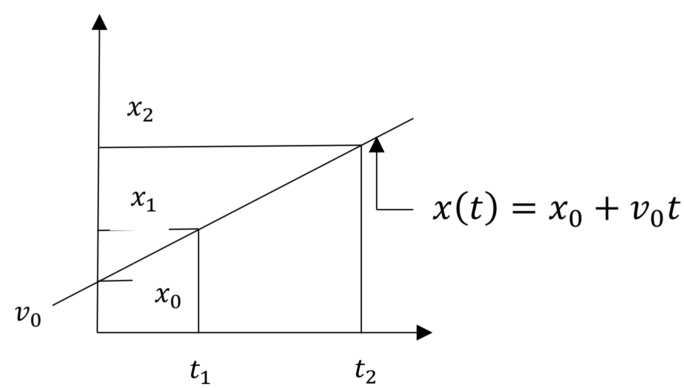
Let $\bar{v} = v(t) = v_0$; a constant

suppose at time t=0 the position of the particle is at $x = x_0$. Then at any time t is position at $x(t)$

$$\therefore \bar{v} = v_0 = \frac{x(t) - x}{t - 0} \Longrightarrow x(t) = x_0 + v_0 t \qquad (5)$$

$\bar{v} = v_0 = \frac{x(t)-x_0}{t-0}$

$x(t) = x_0 + v_0 t$



2.5 Acceleration

The conventional definition of acceleration is as follows acceleration is the rate of change of velocity with time. This definition implies that:

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad \left(\frac{m}{s^2}\right) \qquad (6)$$

Instead of an average acceleration over some time interval Δt, we want to be able to calculate the instantaneous acceleration at any time t. it is defined as the limiting process for $\Delta t \to 0$

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (7)$$

=derivative of the velocity with respect to time since $v(t) = \frac{dx}{dt}$

$$a = \frac{dv(t)}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad (8)$$

=the second derivative of the derivative of the position with respect to time

2.6 Constant acceleration

For a particle moving along the x-axis with uniform acceleration (a constant in magnitude and direction), the average acceleration and instantaneous acceleration are equal and we can write Eq. (6) as:

$$\bar{a} = a = \frac{v(t) - v_0}{t - 0} \qquad (9)$$

$$\bar{v} = v = \frac{x(t) - x_0}{t - 0}$$

where v_0 is the velocity at t=0

v is the velocity at any time (t)

Equation (9) can be written as:

$$v(t) = v_0 + at \qquad (10)$$

If particle is at x_0 at time t =0, after an elapsed time (t) it will be at

$$x = x_0 + vt \qquad (11)$$

since v increase uniformly with

$$\bar{v} = \frac{1}{2}[v_0 + v(t)]$$

$$= \frac{1}{2}[v_0 + v_0 + at]$$

$$\bar{v} = v_0 + \frac{at}{2} \tag{12}$$

$$\therefore x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \tag{13}$$

Equation (10) and (13) give $v(t)$ and $x(t)$ as a function of time. From eq. (10)

$$t = \frac{v - v_0}{a} \tag{14}$$

substitute eq. (14) in eq.(13)

$$x = x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

after some algebra

$$v^2 - {v_0}^2 = 2a(x - x_0) \tag{15}$$

using calculus

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v(t) = \frac{dx}{dt} = v_0 + at$$

$$a(t) = \frac{dv}{dt} = a \qquad [\,a\ is\ constant]$$

[If $a \equiv 0$, uniform straight line motion]

2.7 Acceleration due to gravity

Important class of constant acceleration problem involves gravity body released near the surface of the earth is acceleration downwards under influence of gravity. All object near the earth accelerate at the same constant rate when other external effects are excluded: wind, etc. we use the y-axis for vertical motion if the y-axis point up ward, the acceleration due to gravity is $a = -g$ where $g = 9.8\ \frac{m}{s^2}$ the equation of the motion with constant a become,

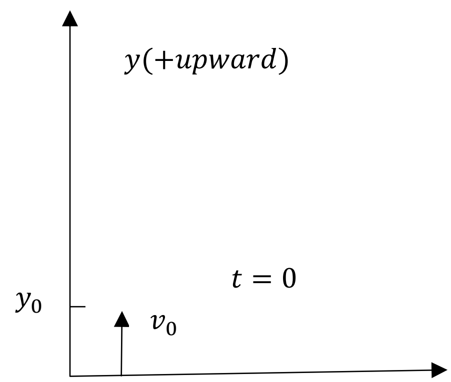
$$a = -g$$

$$v = v_0 - gt$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2$$

$$v^2 - {v_0}^2 = -2g(y - y_0)$$

$$y = y_0 + \bar{v}t$$



Example (1): ideal particle moving in a straight line with position given by:-

$$x = 2.1t^2 + 2.80 \quad (m)$$

a. what is the average velocity between $t_1 = 3\ s$ and $t_2 = 5\ s$
b. instantaneous velocity

Solution

$$t_1 = 3\ s \implies x_1 = 2.1(3)^2 + 2.8 = 21.7m$$

$$t_2 = 5\ s \implies 2.1(5)^2 + 2.8 = 55.3m$$

a. $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{55.3 - 21.7}{5 - 3} = 16.8\ \frac{m}{s}$

b. Instantaneous velocity $v(t) = \frac{dx}{dt} = 4.2\ t$

Example (2): $v(t) = \frac{1}{2}\beta t^2$, where β constant

What is $\bar{a}$ between $t_1 = 1\ s$ and $t_2 = 3\ s$?

Solution:

$$\Delta t = t_2 - t_1 = 3 - 1 = 2\ s$$

$$v_2(3) = \frac{1}{2}\beta(3)^2 = 4.5\ \beta$$

$$v_1(1) = \frac{1}{2}\beta(1)^2 = 0.5\beta$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{(4.5-0.5)\beta}{2} = 2\beta\ \frac{m}{s}$$

What is a?

$$a = \frac{dv}{dt} = \beta t$$

Example (3): A player tosses a baseball up along *y*-axis with initial speed of $\left(12\ \frac{m}{s}\right)$

1. How long does the ball take to reach its maximum height?
2. What is the ball's maximum height above its release point?
3. How long does the ball take to reach a point 5 m above its release point?

Solution:

$v_0 = 12\ \frac{m}{s}$

$v = 0$ (the velocity at the maximum height must be)

1. $v = v_0 - gt$

$$t = \frac{v - v_0}{-g} = \frac{0-12}{-9.8} = 1.2\ s$$

2. $v^2 = {v_0}^2 - 2g(y - y_0)$

$$y = \frac{v^2 - {v_0}^2}{-2g} = \frac{0 - 12^2}{-2*9.8} = 7.3\ m$$

3. $y - y_0 = v_0 t - \frac{1}{2} g t^2$

$y - y_0 = 5m$

$5 = 12 \times t - \frac{1}{2} \times 9.8 \times t^2$

$4.9t^2 - 12t + 5 = 0$

Either $t = 0.53\ s$ or $t = 1.95\ s$

Example (4): The position of a particle moving on an x-axis is given by $x = 7.8 + 9.2t - 2.1t^3$ where x in meter, and t in second what is the velocity at $t = 3.5\ s$, and its acceleration and $t = 2\ s$

Solution:

1. $v = \frac{dx}{dt} = \frac{d}{dt}(7.8 + 9.2t - 2.1t^3) = 9.2 - 6.3t^2$

   at $t = 3.5 \Longrightarrow v = 9.2 - 6.3 \times (3.5)^2 = -68\ \frac{m}{s}$

2. $a = \frac{dv}{dt} = \frac{d}{dt}(9.2 - 6.3t^2) = -12.6t$

   at $t = 2 \Rightarrow a = -12.6 \times 2 = -25.2\ \frac{m}{s^2}$

Example (5): A particle moving on straight line with acceleration $a = 4 - t^2\ (\frac{m}{s^2})$, find the velocity and the displacement as a function of time?

$v = 2\ \frac{m}{s}, t = 3\ s$ and $x = 9\ m$

$a = \frac{dv}{dt} \Longrightarrow dv = adt$

$\int dv = \int adt \Longrightarrow v = \int (4 - t^2)dt$

$\therefore v = \int 4dt - \int t^2 dt$

$$v = 4t - \frac{t^3}{3} + C_1 \Rightarrow 2 = 4(3) - \frac{3^3}{3} + C_1 \Longrightarrow C_1 = -1$$

$$v = 4t - \frac{t^3}{3} - 1$$

$$x = \int v dt \Longrightarrow x = \int (4t - \frac{t^3}{3} - 1) dt$$

$$x = 2t^2 - \frac{t^4}{12} - t + C_2 \Longrightarrow 9 = 2(3)^2 - \frac{(3)^4}{12} - 3 + C_2$$

$$C_2 = \frac{3}{4} = 0.75$$

$$x = 2t^2 - \frac{t^4}{12} - t + 0.75$$