What is noise?

Noise is any undesired information that contaminates an image. Noise appears in image from a variety of source. The digital image a acquisition process, which converts an optical image into a continuous electrical signal that is then sampled, is the primary process by which noise appears in digital images. At every step in the process there are fluctuations caused by natural phenomena that add a random value to exact brightness value for a given pixel. In typical image the noise can be modeled with one of the following distribution:

1. Gaussian ("normal") distribution.
2. Uniform distribution.
3. Salt _and _pepper distribution.

(a) original

(b) with Gaussian noise (variance=0.005)

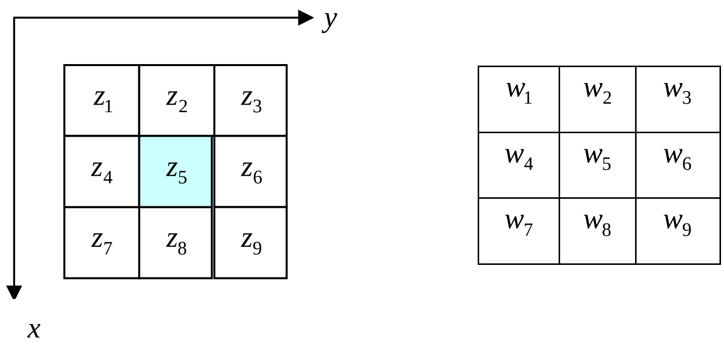
(c) with salt and Pepper noise (p=1%)

Spatial masks

Many image enhancement techniques are based on spatial operations performed on local neighborhoods of input pixels.

The image is usually convolved with a finite impulse response filter called spatial mask. The use of spatial masks on a digital image is called spatial filtering.

Suppose that we have an image $f(x, y)$ of size N^2 and we define a neighbourhood around each pixel. For example let this neighbourhood to be a rectangular window of size 3×3



If we replace each pixel by a weighted average of its neighbourhood pixels then the response of the linear mask for the pixel z_5 is $\sum_{i=1}^{9} w_i z_i$. We may repeat the same process for the whole image.

A 3×3 spatial mask operating on an image can produce:

(a) a smoothed version of the image (which contains the low frequencies) or

(b) it can enhance the edges and suppress essentially the constant background information.

The behavior is basically dictated by the signs of the elements of the mask.

Noise Removal using Spatial Filters:

Spatial filtering is typically done for:

1. Remove various types of noise in digital images.
2. Perform some type of image enhancement.

Note: These filters are called spatial filter to distinguish them from frequency domain filter.

The three types of filters are:

1. Mean filters
2. Median filters (order filter)
3. Enhancement filters

Mean and median filters are used primarily to conceal or remove noise, although they may also be used for special applications. For instance, a mean filter adds "softer" look to an image. The enhancement filter high lights edges and details within the image.

Spatial filters are implemented with convolution masks. Because convolution mask operation provides a result that is weighted sum of the values of a pixel and its neighbors, it is called a linear filter.

Overall effects the convolution mask can be predicated based on the general pattern. For example:

- If the coefficients of the mask sum to one, the average brightness of the image will be retained.
- If the coefficients of the mask sum to zero, the average brightness will be lost and will return a dark image.
- If the coefficients of the mask are alternatively positive and negative, the mask is a filter that returns edge information only.
- If the coefficients of the mask are all positive, it is a filter that will blur the image.

Mean Filters:

The mean filters, are essentially **averaging filter**. They operate on local groups of pixel called neighborhoods and replace the centre pixel with an average of the pixels in this neighborhood.

Example: 3x3 average

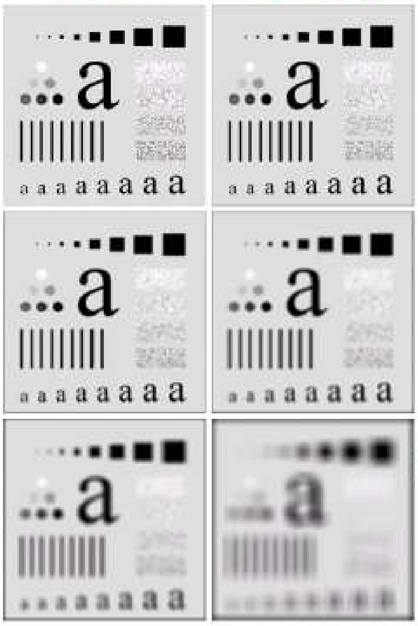
100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

100	100	100	100	100
100	144	167	145	100
100	167	200	168	100
100	144	166	144	100
100	100	100	100	100

a. Original image

b. Mean filtered image

Example



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Weighted Averaging Filter

- Instead of averaging all the pixel values in the window, give the closer-by pixels higher weighting, and far-away pixels lower weighting.

$$g(m,n) = \sum_{l=-L}^{L} \sum_{k=-L}^{L} h(k,l)s(m-k,n-l)$$

- This type of operation for arbitrary weighting matrices is generally called “2-D convolution or filtering”. When all the weights are positive, it corresponds to weighted average.
- Weighted average filter retains low frequency and suppresses high frequency = low-pass filter

This replacement is done with a convolution mask such as the following 3X3 mask Arithmetic mean filter smoothing or low-pass filter.

Example Weighting Mask

$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

All weights must sum to one

Note that the coefficient of this mask sum to one, so the image brightness will be retained , and the coefficients are all positive , so it will tend to blur the image . This type of mean filter smoothes out local variations within an image, so it essentially a low pass filter. So a low filter can be used to attenuate image noise that is composed primarily of high frequencies components.

Example: Weighted Average

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

100	100	100	100	100
100	144	167	145	100
100	167	200	168	100
100	144	166	144	100
100	100	100	100	100

$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

100	100	100	100	100
100	156	176	158	100
100	174	201	175	100
100	156	175	156	100
100	100	100	100	100

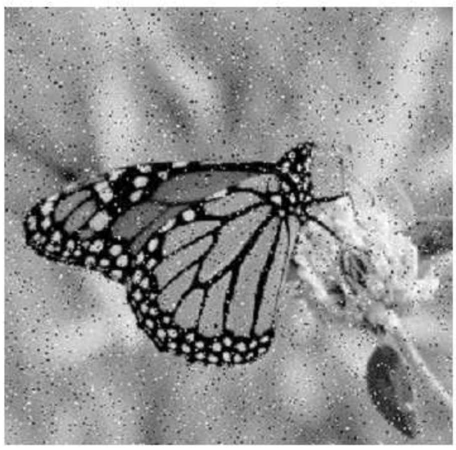
Median filter

The median filter is a non linear filter (order filter). These filters are based on as specific type of image statistics called order statistics. Typically, these filters operate on small sub image, "Window", and replace the centre pixel value (similar to the convolution process).

Order statistics is a technique that arranges the entire pixel in sequential order, given an NXN window (W) the pixel values can be ordered from smallest to the largest.

$I1 \leq I2 \leq I3 \ldots\ldots\ldots\ldots\ldots\ldots < I$

Where I1, I2, I3......., IN are the intensity values of the subset of pixels in the image.



a. Salt and pepper noise

b. Median filtered image (3x3)

Example:

Given the following 3X3 neighborhood

$$\begin{pmatrix} 5 & 5 & 6 \\ 3 & 4 & 5 \\ 3 & 4 & 7 \end{pmatrix}$$

We first sort the value in order of size (3,3,4,4,5,5,5,6,7) ; then we select the middle value , in this case it is 5. This 5 is then placed in centre location.

A median filter can use a neighbourhood of any size, but 3X3, 5X5 and 7X7 are typical. Note that the output image must be written to a separate image (a buffer); so that the results are not corrupted as this process is performed. (The median filtering operation is performed on an image by applying the sliding window concepts, similar to what is done with convolution).

The window is overlaid on the upper left corner of the image, and the median is determined. This value is put into the output image (buffer) corresponding to the centre location of the window. The window is then slide one pixel over, and the process is repeated.

When the end of the row is reached, the window is slide back to the left side of the image and down one row, and the process is repeated. This process continues until the entire image has been processed.

Note that the outer rows and columns are not replaced. In practice this is usually not a problem due to the fact that the images are much larger than the masks. And these “wasted” rows and columns are often filled with zeros (or cropped off the image). For example, with 3X3 mask, we lose one outer row and column, a 5X5 mask we lose two rows and columns. This is not visually significant for a typical 256X256 or 512X512 images.

maximum and minimum filters

The maximum and minimum filters are two order filters that can be used for elimination of salt- and-pepper noise. The maximum filter selects the largest value within an ordered window of pixels values; where as the minimum filter selects the smallest value.

The minimum filters works best for salt- type noise (High value), and the maximum filters work best for pepper-type noise.

In a manner similar to the median, minimum and maximum filter, order filter can be defined to select a specific pixel rank within the ordered set. For example we may find for certain type of pepper noise that selecting the second highest values works better than selecting the maximum value. This type of ordered selection is very sensitive to their type of images and their use it is application specific. It should note that, in general a minimum or low rank filter will tend to darken an image and a maximum or high rank filter will tend to brighten an image.

midpoint filter

The midpoint filter is actually both order and mean filter because it rely on ordering the pixel values , but then calculated by an averaging process. This midpoint filter is the average of the maximum and minimum within the window as follows:

$$\text{Order set} = I_1 \leq I_2 \leq I_3 \ldots\ldots\ldots\ldots\ldots\ldots \leq I_{N^2}.$$

$$\text{Midpoint} = (I_1 + I_{N^2})/2$$

The midpoint filter is most useful for Gaussian and uniform noise.