

Turing Machines: (TM)

Turing machine is a simple mathematical model of a computer. Turing machine models the computing capability of a general-purpose computer. This model will enable us not only to study some theoretical limitations on the tasks that computers can perform, it will also be a model that we can use to show that certain operations "can" be done by computer.

The languages accepted by F.A. are called "regular" and they can be defined by regular expression. The languages accepted by PDA are called CFGs. The languages accepted by TM are called type θ , or phrase-structure or recursively enumerable language.

a_1	a_2	...	a_i	...	a_n	B	B	...
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The figure above is basic Turing machine, has a finite control, an input tape that is divided into cells, and a tape head that scan one cell of the tape at a time. The tape has a leftmost cell but is infinite to the right. Each cell of the tape may hold exactly one symbol. Initially, the n leftmost cells hold the input. The remaining infinity of cells each hold the blank.

In one move the Turing machine, depending upon the symbol scanned by the tape head and the state of the finite control,
1- changes state,

2- prints a symbol on the tape cell scanned, replacing what was written there, and

3- moves its head left or right one cell

Note that the difference between a Turing machine and a two-way finite automation lies in the ability to change symbols on its tape.

Formally, a Turing machine (TM) is denoted:

$$M = (Q, \Sigma, \Gamma, t, q_0, B, F)$$

Where

Q is the finite set of "states",

Γ is the finite set of allowable "tape symbols",

B , a symbol of Γ , is the "blank",

Σ , a subset of Γ not including B , is the set of "input symbols"

T , is the next move function, a mapping from $Q \times \Gamma$ to

$$Q \times \Gamma \times \{L, R\},$$

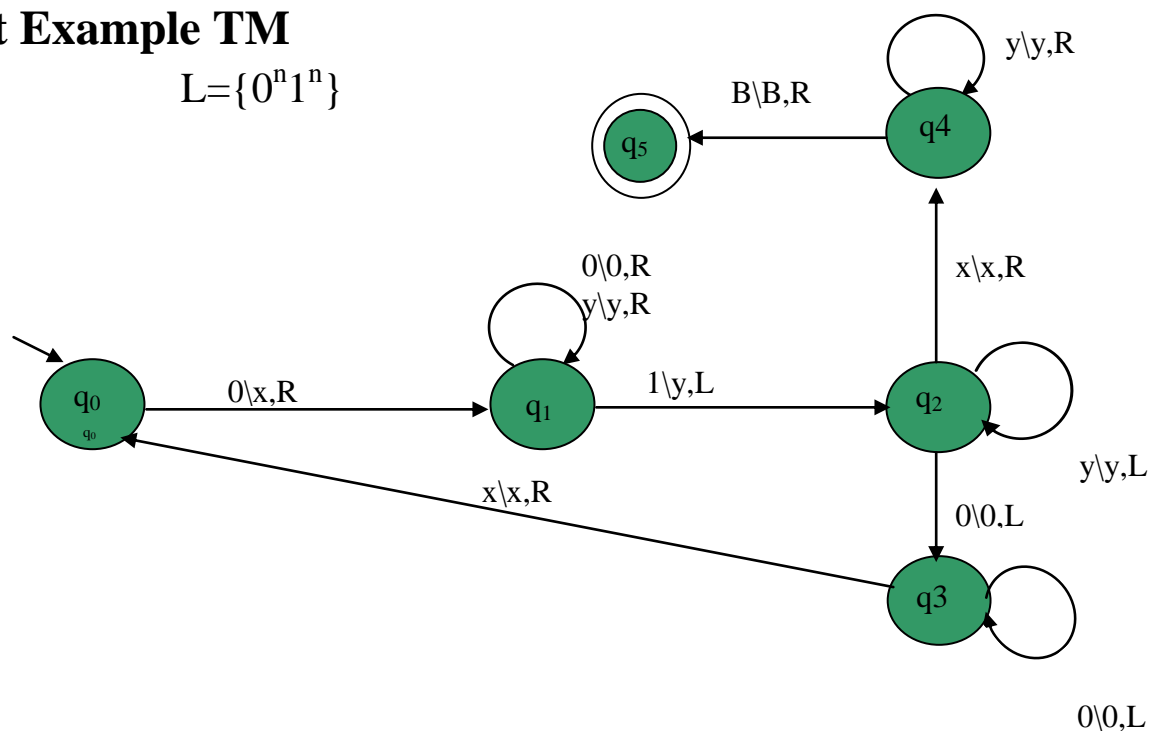
q_0 , in Q is the "start state",

$F \subseteq Q$ is the set of "final states", or called "HALT states" that cause execution to terminate when we enter them.

The "language accepted" by M , denoted $L(M)$, is the set of those word in Σ^* that cause M to enter a final state.

First Example TM

$$L = \{0^n 1^n\}$$



$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1, B\}, \{x, y\}, \{q_4\})$ and Transition functions are:

$$t(q_0, 0) = (q_1, X, R)$$

$$t(q_2, 0) = (q_3, 0, L)$$

$$t(q_1, Y) = (q_1, Y, R)$$

$$t(q_2, X) = (q_4, X, R)$$

$$t(q_1, 0) = (q_1, 0, R)$$

$$t(q_2, Y) = (q_2, Y, L)$$

$$t(q_1, 1) = (q_2, Y, L)$$

$$t(q_3, 0) = (q_3, 0, L)$$

$$t(q_3, x) = (q_0, x, R)$$

$$t(q_4, Y) = (q_4, Y, R)$$

$$t(q_4, B) = (q_5, B, R)$$

A computation of M on input 0011 is:

$$q_0 0011 \Rightarrow X q_1 011 \Rightarrow X 0 q_1 11 \Rightarrow X q_2 0Y1 \Rightarrow$$

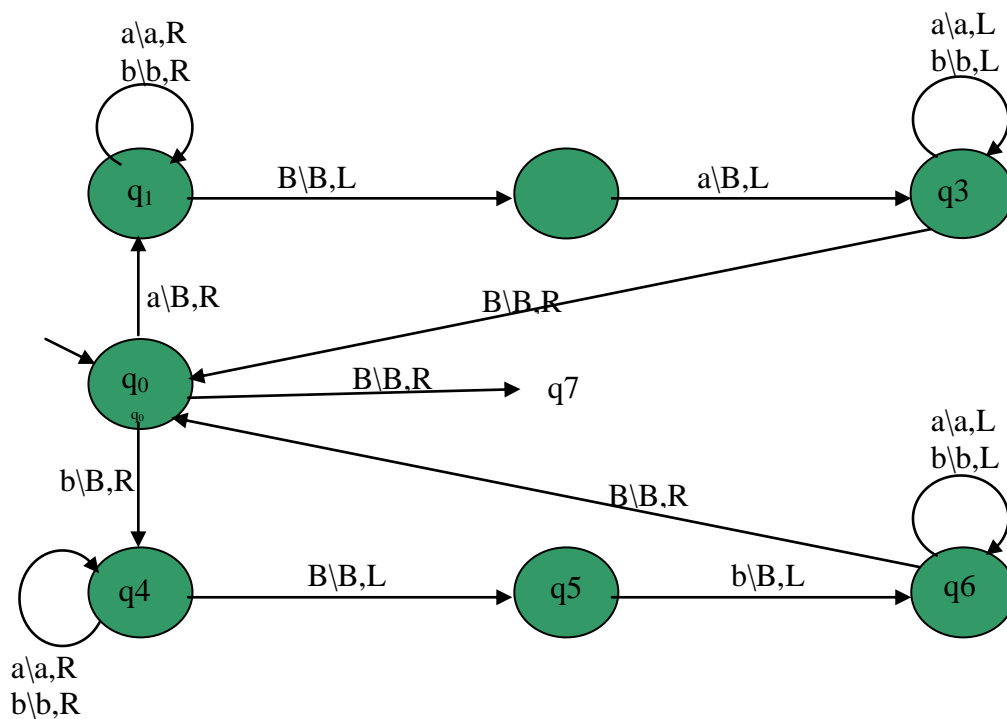
$$q_3 X 0 Y 1 \Rightarrow X q_0 0 Y 1 \Rightarrow X X q_1 Y 1 \Rightarrow X X X q_1 1 \Rightarrow$$

$$X X q_2 Y Y \Rightarrow X q_2 X Y Y \Rightarrow X X q_4 X Y \Rightarrow X X Y q_4 Y \Rightarrow$$

$$X X Y Y q_4 \Rightarrow X X Y Y B q_5$$

Second Example TM

$$L = \{ W W^r \mid W = \{a, b\}^+ \}$$



My Best Wishes
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