## Permuntation

- When a group of objects or people are arranged in a certain order, this arrangement is called a permutation
- For permutations, the order is important (for combinations, order is unimportant.
- You can find the number of permutations by making use of the fundamental counting principal and multiplying the choices for each category together
- Or you can use the following formula:

The number of permutations of **n** distinct objects taken **r** at a time is given by:

P(n,r) = n! / (n – r)!

Can't use if choices may be repeated!

### Example

Eight people enter the Best Pic contest. How many ways can blue, red, and green ribbons be awarded?

Since each winner will receive a different ribbon, order is important. You must find the number of permutations of 8 things taken 3 at a time.

$$\boldsymbol{p_r^n} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!} \quad \text{Permutation formula}$$

$$= \frac{\mathbf{8}!}{(\mathbf{8}-\mathbf{3})!} \qquad \boldsymbol{n} = \mathbf{8}, \boldsymbol{r} = \mathbf{3}$$

$$=\frac{8!}{5!} = \frac{8\times7\times6\times5!}{5!}$$

$$=336$$

The ribbons can be awarded in 336 ways.

**Your turn**

**Example**

Ten people are competing in a swim race where 4 ribbons will be given. How many ways can blue, red, green, and yellow ribbons be awarded?

Answer : 5040 ways

**Example**

From a class of 20 students we need to select 3 for a committee, one to be president, another one to be vice-president and the third one to be secretary.

In this case n = 20, and r = 3. The order is important, so we apply the formula

$$\boldsymbol{p_3^{20}} = \frac{20!}{(20-3)!}$$

$$= \frac{20 \times 19 \times 18 \times 17!}{17!}$$

$$= 20 \times 19 \times 18$$

= 6840

## Permutations of duplicate items:

The number of permutations of $n$ items, where n1 items are identical, n2 items are identical, n3 items are identical, and so on, is given by:

$$\frac{n!}{n1!\ n2!\ n3! \cdots}$$

Example

In how many distinct ways can the letters of the word MISSISSIPPI be arranged?

The word contains 11 letters ($n$ = 11) where four I are identical (n1 = 4), four S are identical (n2 = 4) and $n_2$ P are identical (n3 = 2). The number of distinct permutations is:

$$\frac{11!}{4!\ 4!\ 2!} = \frac{11\,.10\,.9\,.8\,.7\,.6\,.5\,.4!}{4!\ .4.3\,.2.\,1.2\,.1} = 34{,}650$$

Example

How many different ways can the letters of the word BANANA be arranged?

$$\frac{6!}{3!2!} = \frac{6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{3 \bullet 2 \bullet 1 \bullet 2 \bullet 1} \text{ or } 60$$

There are 60 ways to arrange the letters.

**Your turn**

How many different ways can the letters of the word ALGEBRA be arranged?

Answer : 2520 ways

## combinations

A **combination** is a way of selecting several things out of a larger group, where (unlike permutations) order does not matter.

A combination of items occurs when

- The items are selected from the same group.
- No item is used more than once.
- The order of items makes no difference.

**Remarks**

**Permutation** problems involve situations in which **order matters.**

**Combination** problems involve situations in which the **order** of items **makes no difference**.

The number of possible combinations if ***r*** items are taken from ***n*** items is:

C(n,r) = n! /r! (n – r)!

Example

From a class of 20 students we need to select 3 for a committee.
In this case n = 20, and r = 3. The order is not important, so we apply the formula

$$C_3^{20} = \frac{20!}{3!\,(20-3)!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{3 \cdot 2 \cdot 1 \cdot 17!}$$

=1,140

Example

Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose three people to go?

Since the order they choose is not important, you must find the number of combinations of 5 cousins taken three at a time.

$$\begin{aligned} C_r^n &= \frac{n!}{r!\,(n-r)!} \\ &= \frac{5!}{3!\,(5-3)!} \qquad\qquad n = 5\,, r = 3 \\ &= \frac{5!}{3!\,2!} = 10 \end{aligned}$$

There are 10 ways to choose three people from the five cousin.

**Your turn**

Six friends at a party decide that three of them will go to pick up a movie. How many ways can they choose three people to go?

Answer :20 ways

## Multiple Events

In more complicated situations, you can use the Fundamental Counting Principle in conjunction with permutations/combinations to determine the number of possibilities

For example, suppose I have a class with 15 boys and 10 girls and I want to send 2 boys and 2 girls to represent our Algebra class at the U.N.

Do C(15,2) for the boys and C(10,2) for the girls, then multiply these answers together!

Example

A soccer club has 8 femal and 7 male members for today's match,the coach want to have 6 femal and 5 meal players on the grass.How many possible configuration are there?

$$C(8,6) \,.\, C(7,5) = \frac{8!}{6!\,2!} \times \frac{7!}{5!\;2!}$$

$$= 28 \times 21$$

$$= 588$$