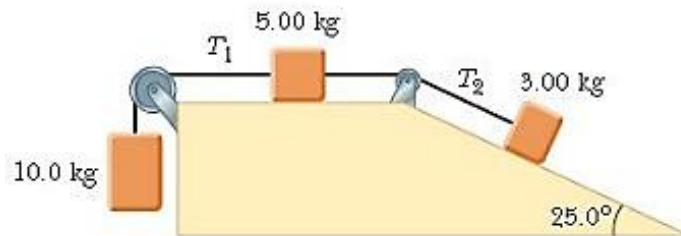


**Second semester, Second exam, 1<sup>st</sup> year, Classical Mechanics, time: 1.5 h**

**Q.1:** Three blocks of masses 10(kg), 5(kg), and 3(kg) are connected by light strings that pass over frictionless pulleys as shown in Figure below. The acceleration of the 5(kg) block is  $2(\text{m/s}^2)$  to the left, and the surfaces are rough Find (a) the tension in each string and (b) the coefficient of kinetic friction between blocks and surfaces. (Assume the same  $\mu_k$  for both blocks are in contact with surface.)

(10M)



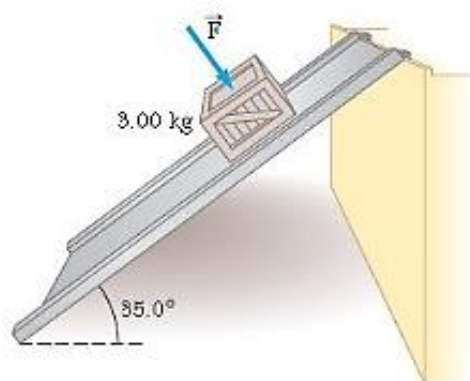
**Q.2:** A Stone is thrown from the top of a building upward at an angle of  $30^\circ$  to the horizontal and with an initial speed of  $20 \left(\frac{\text{m}}{\text{s}}\right)$ . If the height of the building is  $45\text{m}$ ,

- a) How long is it before the stone hits the ground
- b) What the speed of the stone just before it strikes the ground?

(10M)

**Q.3:** The coefficient of static friction between the 3(kg) crate and the  $35^\circ$  incline of Figure is 0.3. What minimum force must be applied to the crate perpendicular to the incline to prevent the crate from sliding down the incline?

(10M)



*Examiner: Prof. Dr. Fouad A. Majeed*

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**Problem 1:** A dockworker loading crates on a ship finds that a 20-kg crate, initially at rest on a horizontal surface, requires a 75-N horizontal force to set it in motion. However, after the crate is in motion, a horizontal force of 60 N is required to keep it moving with a constant speed. Find the coefficients of static and kinetic friction between crate and floor.

**Solution:** When the block is on the verge of moving, the static friction force has a magnitude  $f_s = (f_s)_{max} = \mu_s n$ .

Since equilibrium still exists and the applied force is 75 N, we have

$$\Sigma F_x = 75 \text{ N} - f_s = 0 \text{ or } (f_s)_{max} = 75 \text{ N} .$$

In this case, the normal force is just the weight of the crate, or  $n = mg$ . Thus, the coefficient of static friction is

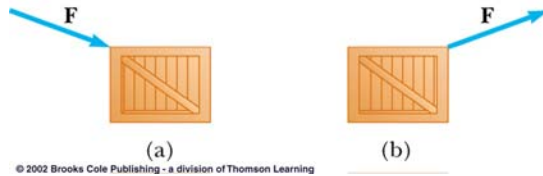
$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{(f_s)_{max}}{mg} = \frac{75 \text{ N}}{(20 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.38} .$$

After motion exists, the friction force is that of kinetic friction,  $f_k = \mu_k n$ .

Since the crate moves with constant velocity when the applied force is 60 N, we find that  $\Sigma F_x = 60 \text{ N} - f_k = 0$  or  $f_k = 60 \text{ N}$ . Therefore, the coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{n} = \frac{f_k}{mg} = \frac{60 \text{ N}}{(20 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.31} .$$

**Problem 2:** A 1000-N crate is being pushed across a level floor at a constant speed by a force  $\mathbf{F}$  of 300 N at an angle of  $20.0^\circ$  below the horizontal as shown in the figure **a**. (a) What is the coefficient of kinetic friction between the crate and the floor? (b) If the 300-N force is instead pulling the block at an angle of  $20.0^\circ$  above the horizontal as shown in figure **b**, what will be the acceleration of the crate? Assume that the coefficient of friction is the same as found in (a).



**Solution:** (a) Since the crate has constant velocity,  $a_x = a_y = 0$ .

Applying Newton's second law:

$$\Sigma F_x = F \cos 20.0^\circ - f_k = ma_x = 0, \text{ or } f_k = (300 \text{ N}) \cos 20.0^\circ = 282 \text{ N}$$

$$\text{and } \Sigma F_y = n - F \sin 20.0^\circ - w = 0, \text{ or}$$

$$n = (300 \text{ N}) \sin 20.0^\circ + 1000 \text{ N} = 1.10 \times 10^3 \text{ N}.$$

$$\text{The coefficient of friction is then } \mu_k = \frac{f_k}{n} = \frac{282 \text{ N}}{1.10 \times 10^3 \text{ N}} = \boxed{0.256}.$$

$$(b) \text{ In this case, } \Sigma F_y = n + F \sin 20.0^\circ - w = 0,$$

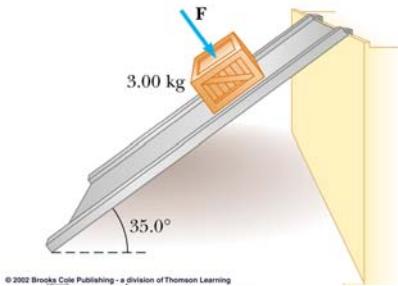
$$\text{so } n = w - F \sin 20.0^\circ = 897 \text{ N}.$$

$$\text{The friction force now becomes } f_k = \mu_k n = (0.256)(897 \text{ N}) = 230 \text{ N}.$$

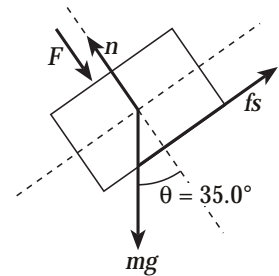
$$\text{Therefore, } \Sigma F_x = F \cos 20.0^\circ - f_k = ma_x = \left( \frac{w}{g} \right) a_x \text{ and the acceleration is}$$

$$a = \frac{(F \cos 20.0^\circ - f_k) g}{w} = \frac{[(300 \text{ N}) \cos 20.0^\circ - 230 \text{ N}](9.80 \text{ m/s}^2)}{1000 \text{ N}} = \boxed{0.509 \text{ m/s}^2}$$

**Problem 3:** The coefficient of static friction between the 3.00-kg crate and the 35.0° incline of the figure below is 0.300. What mini-mum force **F** must be applied to the crate perpendicular to the incline to prevent the crate from sliding down the incline?



**Solution:** The normal force acting on the crate is given by  $n = F + mg \cos \theta$ . The net force tending to move the crate down the incline is  $mg \sin \theta - f_s$ , where  $f_s$  is the force of static friction between the crate and the incline. If the crate is in equilibrium, then  $mg \sin \theta - f_s = 0$ , so that  $f_s = mg \sin \theta$ .



But, we also know  $f_s \leq \mu_s n = \mu_s (F + mg \cos \theta)$ .

Therefore, we may write  $mg \sin \theta \leq \mu_s (F + mg \cos \theta)$ , or

$$F \geq \left( \frac{\sin \theta}{\mu_s} - \cos \theta \right) mg = \left( \frac{\sin 35.0^\circ}{0.300} - \cos 35.0^\circ \right) (3.00 \text{ kg})(9.80 \text{ m/s}^2) =$$

$$\boxed{32.1 \text{ N}}$$

**Problem 4:** A hockey puck is hit on a frozen lake and starts moving with a speed of 12.0 m/s. Five seconds later, its speed is 6.00 m/s. (a) What is its average acceleration? (b) What is the average value of the coefficient of kinetic friction between puck and ice? (c) How far does the puck travel during this 5.00-s interval?

**Solution:** (a)  $a_x = \frac{v_f - v_i}{t} = \frac{6.00 \text{ m/s} - 12.0 \text{ m/s}}{5.00 \text{ s}} = \boxed{-1.20 \text{ m/s}^2}$ .

(b) From Newton's second law,  $\Sigma F_x = -f_k = ma_x$ , or  $f_k = -ma_x$ .

The normal force exerted on the puck by the ice is  $n = mg$ , so the coefficient of friction is

$$\mu_k = \frac{f_k}{n} = \frac{-m(-1.20 \text{ m/s}^2)}{m(9.80 \text{ m/s}^2)} = \boxed{0.122}$$

(c)  $\Delta x = \bar{v}t = \left( \frac{v_f + v_i}{2} \right) t = \left( \frac{12.0 \text{ m/s} + 6.00 \text{ m/s}}{2} \right) (5.00 \text{ s}) = \boxed{45.0 \text{ m}}$ .

**Problem 5:** The coefficient of static friction is 0.800 between the soles of a sprinter's running shoes and the level track surface on which she is running. Determine the maximum acceleration she can achieve. Do you need to know that her mass is 60.0 kg?

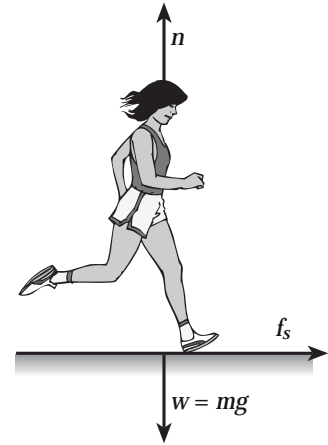
**Solution:** From Newton's third law, the forward force of the ground on the sprinter equals the magnitude of the friction force the sprinter exerts on the ground. If the sprinter's shoe is not to slip on the ground, this is a static friction force and its maximum magnitude is

$$(f_s)_{max} = \mu_s mg.$$

From Newton's second law applied to the sprinter,  $(f_s)_{max} = \mu_s mg = ma_{max}$  where  $a_{max}$  is the maximum forward acceleration the sprinter can achieve. From this, the acceleration is seen to be  $a_{max} = \mu_s g$ . Note that the mass has canceled out.

If  $\mu_s = 0.800$ ,

$$a_{max} = (0.800)(9.80 \text{ m/s}^2) = \boxed{7.84 \text{ m/s}^2 \text{ independent of the mass}}.$$



**Problem 6:** A car is traveling at 50.0 km/h on a flat highway. (a) If the coefficient of friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and the coefficient of friction is 0.600?

**Solution:** (a) The force of friction is found as  $f = \mu_k n = \mu_k (mg)$ .

Choose the positive direction of the  $x$ -axis in the direction of motion and apply the second law. We have  $-f = ma_x$ , or  $a_x = \frac{-f}{m} = -\mu_k g$ .

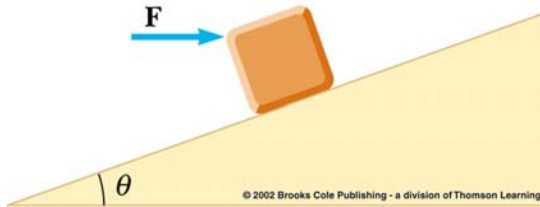
From  $v^2 = v_i^2 + 2a(\Delta x)$ , with  $v = 0$ ,  $v_i = 50.0 \text{ km/h} = 13.9 \text{ m/s}$ , we find

$$0 = (13.9 \text{ m/s})^2 + 2(-\mu_k g)(\Delta x), \text{ or } \Delta x = \frac{(13.9 \text{ m/s})^2}{2\mu_k g}. \quad (1)$$

With  $\mu_k = 0.100$ , this gives  $\Delta x = \boxed{98.6 \text{ m}}$ .

(b) With  $\mu_k = 0.600$ , Equation (1) above gives  $\Delta x = \boxed{16.4 \text{ m}}$ .

**Problem 7:** A 2.00-kg block is held in equilibrium on an incline of angle  $\theta = 60.0^\circ$  by a horizontal force  $\mathbf{F}$  applied in the direction shown in the figure. If the coefficient of static friction between block and incline is  $\mu_s = 0.300$ , determine (a) the minimum value of  $\mathbf{F}$  and (b) the normal force of the incline on the block.



**Solution:** When the minimum force  $\mathbf{F}$  is used, the block tends to slide down the incline so the friction force,  $f_s$  is directed up the incline.

While the block is in equilibrium, we have

$$\Sigma F_x = F \cos 60.0^\circ + f_s - (19.6 \text{ N}) \sin 60.0^\circ = 0 \quad (1)$$

and

$$\Sigma F_y = n - F \sin 60.0^\circ - (19.6 \text{ N}) \cos 60.0^\circ = 0 \quad (2)$$

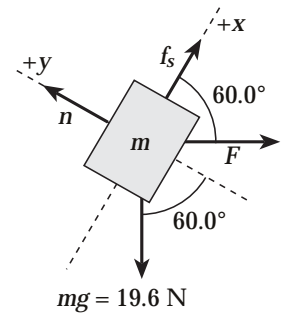
$$\text{For minimum } F \text{ (impending motion), } f_s = (f_s)_{\max} = \mu_s n = (0.300)n. \quad (3)$$

$$\text{Equation (2) gives } n = 0.866F + 9.80 \text{ N}. \quad (4)$$

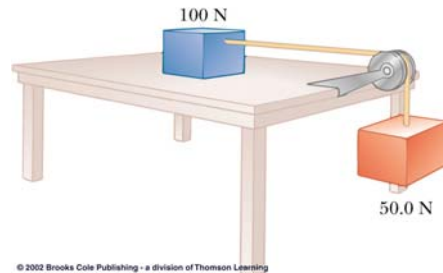
(a) Equation (3) becomes:  $f_s = 0.260F + 2.94 \text{ N}$ , so Equation (1) gives

$$0.500F + 0.260F + 2.94 \text{ N} - 17.0 \text{ N} = 0, \text{ or } F = \boxed{18.5 \text{ N}}.$$

(b) Finally, Equation (4) gives the normal force  $n = \boxed{25.8 \text{ N}}$ .



**Problem 8:** (a) What is the minimum force of friction required to hold the system of the figure below in equilibrium? (b) What coefficient of static friction between the 100-N block and the table ensures equilibrium? (c) If the coefficient of kinetic friction between the 100-N block and the table is 0.250, what hanging weight should replace the 50.0-N weight to allow the system to move at a constant speed once it is set in motion?



**Solution:** (a) For the suspended block,  $\Sigma F_y = T - 50.0 \text{ N} = 0$ , so the tension in the rope is  $T = 50.0 \text{ N}$ . Then, considering the horizontal forces on the 100-N block, we find  $\Sigma F_x = T - f_s = 0$ , or  $f_s = T = \boxed{50.0 \text{ N}}$ .

(b) If the system is on the verge of slipping,  $f_s = (f_s)_{max} = \mu_s n$ . Therefore,

$$\text{the required coefficient of friction is } \mu_s = \frac{f_s}{n} = \frac{50.0 \text{ N}}{100 \text{ N}} = \boxed{0.500}.$$

(c) If  $\mu_k = 0.250$ , then the friction force acting on the 100-N block is

$$f_k = \mu_k n = (0.250)(100 \text{ N}) = 25.0 \text{ N}.$$

Since the system is to move with constant velocity, the net horizontal force on the 100-N block must be zero, or  $\Sigma F_x = T - f_k = T - 25.0 \text{ N} = 0$ .

The required tension in the rope is  $T = 25.0 \text{ N}$ . Now, considering the forces acting on the suspended block when it moves with constant velocity,  $\Sigma F_y = T - w = 0$ , giving the required weight of this block as

$$w = T = \boxed{25.0 \text{ N}}.$$

**Problem 9:** An 80-kg stuntman jumps from the top of a building 30 m above a catching net. Assuming that air resistance exerts a 100-N force on the stuntman as he falls, determine his velocity just before he hits the net.

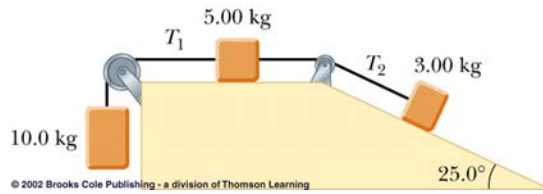
**Solution:** Taking the downward direction as positive, applying the second law to the falling person yields  $\Sigma F_y = mg - f = ma_y$ , or

$$a_y = g - \frac{f}{m} = 9.80 \text{ m/s}^2 - \left( \frac{100 \text{ N}}{80 \text{ kg}} \right) = 8.6 \text{ m/s}^2.$$

Then,  $v_y^2 = v_{iy}^2 + 2 a_y (\Delta y)$  gives the velocity just before hitting the net as

$$v_y = \sqrt{v_{iy}^2 + 2 a_y (\Delta y)} = \sqrt{0 + 2(8.6 \text{ m/s}^2)(30 \text{ m})} = \boxed{23 \text{ m/s}}.$$

**Problem 10:** The three blocks of masses 10.0 kg, 5.00 kg, and 3.00 kg are connected by light strings that pass over frictionless pulleys as shown in the figure. The acceleration of the 5.00-kg block is  $2.00 \text{ m/s}^2$  to the left, and the surfaces are rough. Find (a) the tension in each string and (b) the coefficient of kinetic friction between blocks and surfaces. (Assume the same  $\mu_k$  for both blocks in contact with surfaces.)



**Solution:** The magnitude of the acceleration is  $a = 2.00 \text{ m/s}^2$  for all three blocks and applying Newton's second law to the 10.0-kg block gives

$$(10.0 \text{ kg})(9.80 \text{ m/s}^2) - T_1 = (10.0 \text{ kg})(2.00 \text{ m/s}^2), \text{ or } T_1 = 78.0 \text{ N}.$$

Applying the second law to the 5.00-kg block gives:

$$T_1 - T_2 - \mu_k [(5.00 \text{ kg})(9.80 \text{ m/s}^2)] = (5.00 \text{ kg})(2.00 \text{ m/s}^2).$$

$$\text{With } T_1 = 78.0 \text{ N}, \text{ this simplifies to: } T_2 = 68.0 \text{ N} - (49.0 \text{ N})\mu_k \quad (1)$$

For the 3.00-kg block, the second law gives  $T_2 - \mu_k n - [mg \sin 25.0^\circ] = ma$ .

With  $m = 3.00 \text{ kg}$ ,  $a = 2.00 \text{ m/s}^2$ ,  $g = 9.80 \text{ m/s}^2$ , and  $n = mg \cos 25.0^\circ$ , this reduces to:

$$T_2 - (26.6 \text{ N})\mu_k = 18.4 \text{ N} \quad (2)$$

Solving Equations (1) and (2) simultaneously, and using the value of  $T_1$  from above, we find that

$$(a) \quad T_1 = \boxed{78.0 \text{ N}}, \quad T_2 = \boxed{35.9 \text{ N}}, \quad \text{and} \quad (b) \quad \mu_k = \boxed{0.656}$$