Second semester, First exam, 1st year, Classical Mechanics, time:1.5 h

Q.1: Runner A is initially 4m west of a flagpole and is running with a constant velocity of 6 m/s due east. Runner B is initially 3m east of the flagpole and is running with a constant 5m/s due west. How far are the runners from the flagpole when they meet? **(10M)**

Q.2: Given the vectors $\vec{U} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $\vec{V} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$, find

(A) $\vec{U} + \vec{V}$ and $\vec{U} - \vec{V}$

(B) $\vec{U}.\vec{V}$ and $\vec{U} \times \vec{V}$

(C) Find the angle between two vector using the dot product definition

(D) Verify $\vec{U} \times \vec{V}$ perpendicular to $\vec{U}$ and $\vec{V}$ **(10M)**

Q.3: A mountain climber stands at the top of a 50m cliff that overhangs a calm pool of water. She throws two stones vertically downward 1 s apart and observes that they cause a single splash. The first stone had an initial velocity 2 m/s. (a) How long after release of the first stone did the two stones hit the water? (b) What initial velocity must the second stone have had, given that they the water simultaneously? (c) What was the velocity of each stone at the instant it hit the water? **(10M)**

Examiner: Prof.Dr. Fouad A. Majeed

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Q.1: A hockey player is standing on his skates on a frozen pond when an opposing player, moving with a uniform speed of 12 m/s, skates by with the puck. After 3 s, the first player makes up his mind to chase his opponent. If he accelerates uniformly at 4 m/s^2, (a) how long does it take him to catch his opponent, and (b) how far has he traveled in that time? (Assume that the player with the puck remains in motion at constant speed.) **(10 M)**

Q.2: Let $\vec{A} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and $\vec{B} = 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

a) $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$

b)The magnitude of $\vec{A}$ and $\vec{B}$

c) The angle between $\vec{A}$ and $\vec{B}$

d) Show that $\vec{A}.(\vec{A} \times \vec{B}) = 0$ and $\vec{B}.(\vec{A} \times \vec{B}) = 0$ **(10 M)**

Q.3: Two students are on a balcony 19.6 m above the street. One student throws a ball vertically downward at 14.7 m/s; at the same instant, the other student throws a ball vertically upward at the same speed. The second ball just misses the balcony on the way down. (a) What is the difference in the two balls' time in the air? (b) What is the velocity of each ball as it strikes the ground? (c) How far apart are the balls 0.800 s after they are thrown? **(10M)**

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حلول اسئلة النموذج الاول

Q1/ Solution

For Runner A : $x_A = -4m$, $v_0 = 6\,{}^{m}/_{s}$, $a = 0$

For Runner B $x_B = 3m$, $v_0 = -5\,{}^{m}/_{s}$, $a = 0$

$$x_f = x_i + v_0 t + \frac{1}{2} a t^2$$

$$x_{fA} = x_{fB}$$

$$x_{fA} = -4 + 6t + \frac{1}{2}(0)t^2 \Longrightarrow x_{fA} = -4 + 6t$$

$$x_{fB} = 3 - 5t + \frac{1}{2}(0)t^2 \Longrightarrow x_{fB} = 3 - 5t$$

$$\because x_{fA} = x_{fB} \Longrightarrow -4 + 6t = 3 - 5t$$

$$t = \frac{7}{11}\ \text{Sec}$$

$$x_{fA} = x_i + v_0 t + \frac{1}{2} a t^2$$

$$x_{fA} = -4 + 6\left(\frac{7}{11}\right) + \frac{1}{2}(0)\left(\frac{7}{11}\right)^2 \Longrightarrow x_{fA} = -0.18m = x_{fB}$$

Q2/ Solution

$$\vec{U} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$$

$$\vec{V} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$$

$$\vec{U} + \vec{V} = 2\hat{\imath} - \hat{\jmath} + 6\hat{k}\,,\ \vec{U} - \vec{V} = -3\hat{\jmath} - 2\hat{k}$$

$$\vec{U}.\vec{V} = 1(1) - 2(1) + 4(2) = 1 - 2 + 8 = 7$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 4 \\ 1 & 1 & 2 \end{vmatrix}$$

$$\vec{U} \times \vec{V} = \hat{\imath}[-2(2) - 4(1)] - \hat{\jmath}[1(2) - 4(1)] + \hat{k}[1(1) - (-2)(1)]$$

$$\boxed{\vec{U} \times \vec{V} = -8\hat{\imath} + 2\hat{\jmath} + 3\hat{k}}$$

$\vec{U}.\vec{V} = |\vec{U}||\vec{V}| \cos\theta = AB\cos\theta$

$|\vec{U}| = \sqrt{U_x^2 + U_y^2 + U_z^2} = \sqrt{(1)^2 + (-2)^2 + (4)^2} = \sqrt{21}$

$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{6}$

$\cos\theta = \frac{\vec{U}.\vec{V}}{|\vec{U}||\vec{V}|} = \frac{7}{\sqrt{21}\sqrt{6}}$

$\cos\theta = 0.6236 \Longrightarrow \theta = \cos^{-1}0.6236 \Longrightarrow \theta = 51.41$

To verify that $\vec{U} \times \vec{V}$ is orthogonal to $\vec{U}$

$(-8\hat{\imath} + 2\hat{\jmath} + 3\hat{k}).(\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) = -8 - 4 + 12 = 0$ is orthogonal to $\vec{U}$

To verify that $\vec{U} \times \vec{V}$ is orthogonal to $\vec{V}$

$(-8\hat{\imath} + 2\hat{\jmath} + 3\hat{k}).(\hat{\imath} + \hat{\jmath} + 2\hat{k}) = -8 + 2 + 6 = 0$ is orthogonal to $\vec{V}$

Q3/ Solution

We set $y_i = 0$**,** at the top of the cliff, and find the time interval required for the first stone to reach the water using the particle under constant acceleration model:

(a) $y_f = y_i + v_{yi}t + \frac{1}{2}at^2 \Rightarrow y_f - y_i = v_{yi}t - \frac{1}{2}at^2$

$y_f = -50m$, $v_{yi} = -2m/s$, $a = 9.8m/s^2$

$-50 - 0 = -2t - \frac{1}{2}(9.8)t^2$

$4.9t^2 + 2t + 50 = 0$

$t = \frac{-2 \pm \sqrt{4 - 4(4.9)(50)}}{2(4.9)} \Longrightarrow t = 3sec$

(b) For the second stone, the time of travel is t = 3 − 1 = 2s.

$$y_f - y_i = v_{yi}t - \frac{1}{2}at^2 \Longrightarrow v_{yi} = \frac{y_f - y_i + \frac{1}{2}at^2}{t}$$

$v_{yi} = \frac{-50 - 0 + \frac{1}{2}(9.8)2^2}{2} \Longrightarrow v_{yi} = -15.2m/s$

(c) For the first stone,

$$v_{1f} = v_{1i} - at \Rightarrow v_{1f} = -2 - 9.8(3) \Rightarrow v_{1f} = -31.4m/s$$

$$v_{2f} = v_{2i} - at \Rightarrow v_{2f} = -15.2 - 9.8(2) \Rightarrow v_{2f} = -34.8m/s$$

حلول أسئلة النموذج الثاني

Q1/ Solution

We know the initial velocities and accelerations of the two players,

$$a_1 = 4\,{}^{m}/_{s^2}, a_2 = 0$$

$$v_{i1} = 0, v_{i2} = 12m/s$$

$$x_{i1} = x_{i2} = 0$$

The position of the second player

$$x_2 = x_{i2} + v_{i2}t + \frac{1}{2}at^2 \Rightarrow x_2 = 0 + 12t + \frac{1}{2}(0)t^2 \Rightarrow x_2 = 12t$$

$$x_1 = x_{i1} + v_{i1}(t-3) + \frac{1}{2}(4)(t-3)^2 \Rightarrow x_1 = 0 + 0 + \frac{1}{2}(4)t^2$$

$$\Rightarrow x_2 = 2(t^2 - 6t + 9)$$

$$x_1 = x_2 \Rightarrow 12t = 2(t^2 - 6t + 9) \Rightarrow 12t = 2t^2 - 12t + 18$$

$$2t^2 - 24t + 18 = 0 \Rightarrow t^2 - 12t + 9 = 0$$

$$either\ t = 11.2s\ or\ t = 0.8s$$

however, the equation for the position of player number 1 is not valid for $t > 3$ s, so we keep only tx,

$$t = 11.2s$$

(b) $x_2 = x_{i2} + v_{i2}t + \frac{1}{2}at^2 \Rightarrow x_2 = 0 + 12t + \frac{1}{2}(0)t^2 \Rightarrow x_2 = 12t$

$$x_2 = 12t \Rightarrow x_2 = 12 \times 11.2 = 134.4m$$

Q2/ Solution

$$\vec{a} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}\ ,\ \vec{b} = 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$$

a) $\vec{a} + \vec{b} = 7\hat{\imath} + \hat{\jmath}\ , \vec{a} - \vec{b} = -5\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$

b) $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \Rightarrow |\vec{A}| = \sqrt{1^2 + (-2)^2 + (-2)^2} \Rightarrow |\vec{A}| = 3$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} \Rightarrow |\vec{B}| = \sqrt{6^2 + 3^2 + 2^2} \Rightarrow |\vec{B}| = 7$$

c) $\cos\phi = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} \Rightarrow \cos\phi = \frac{6-6-4}{3\times 7} \Rightarrow \phi = cos^{-1}\left(\frac{-4}{21}\right) \Rightarrow \phi = 100.98$

d) $\left(\vec{A} \times \vec{B}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & -2 \\ 6 & 3 & 2 \end{vmatrix}$

$\left(\vec{A} \times \vec{B}\right) = \hat{\imath}[(-2)(2) - (-2)(3)] - \hat{\jmath}[1(2) - (-2)(6)] + \hat{k}[1(3) - (-2)(6)]$

$\left(\vec{A} \times \vec{B}\right) = 2\hat{\imath} - 14\hat{\jmath} + 15\hat{k}$

$$\vec{A}.\left(\vec{A} \times \vec{B}\right) = \left(\hat{\imath} - 2\hat{\jmath} - 2\hat{k}\right).\left(2\hat{\imath} - 14\hat{\jmath} + 15\hat{k}\right) = 2 + 28 - 30 = 0$$

$$\vec{B}.\left(\vec{A} \times \vec{B}\right) = \left(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}\right).\left(2\hat{\imath} - 14\hat{\jmath} + 15\hat{k}\right) = 12 - 42 + 30 = 0$$

Q3/Solution

(a)

$$h_i = 19.6m$$
$$h_f = 0m$$
$$v_0 = -14.7m/s$$
$$g = -9.8m/s^2$$
$$y_f - y_i = v_0 t - \frac{1}{2} g t^2$$
$$-19.6 = -14.7 * t - \frac{1}{2} * 9.8t^2$$

$$4.9t^2 + 14.7t - 19.6 = 0 \Rightarrow t = -4s \ or \ t = 1s$$

Similarly, for the ball thrown upward, we have:

$$h_i = 19.6m$$
$$h_f = 0m$$
$$v_0 = 14.7m/s$$
$$g = -9.8m/s^2$$

$$y_f - y_i = v_0 t - \frac{1}{2} g t^2$$
$$-19.6 = 14.7 * t - \frac{1}{2} * 9.8t^2$$

$$4.9t^2 - 14.7t - 19.6 = 0 \Rightarrow t = -1s \ or \ t = 4s$$

So, the difference in the two balls' time in the air is t = (4-1) s = 3s.

(b) For the ball thrown downward we have,

$$v_f = v_i - gt \Rightarrow v_f = -14.7 - 9.8 * 1 \Rightarrow v_f = -24.5m/s$$

Similarly, for the ball thrown upward we have,

$$v_f = v_i - gt \Rightarrow v_f = 14.7 - 9.8 * 4 \Rightarrow v_f = -24.5m/s$$

(c) Recall once again that

$$y_f = y_i + v_0 t - \frac{1}{2} g t^2$$

For the ball thrown downward, we have

$$y_{f\downarrow} = 19.6 + (-14.7)(0.8) - \frac{1}{2} * 9.8(0.8)^2$$

$$y_f = 4.7m$$

And for the ball thrown upward, we have

$$y_{f\uparrow} = 19.6 + (14.7)(0.8) - \frac{1}{2} * 9.8(0.8)^2$$

$$y_f = 28.2m$$

Thus, 0.8s after they are thrown, the two balls are $y_{f\downarrow}$ - $y_{f\uparrow}$ = (28.2 − 4.7)m = 23.5m apart.