Lecture#18/ Fracture Mechanics / Micro & Macroscopic

Fracture is a separation of an object into two or more pieces in response to active stresses below the melting temperature of the material.

A fracture is described as **ductile or brittle** depending on the amount of deformation (i.e., dislocation motion) that precedes it. **Ductile** materials demonstrate **large amounts of plastic deformation** while **brittle** materials show **little or no plastic deformation before fracture**.

The tensile stress-strain curve represents the degree of plastic deformation exhibited by both brittle and ductile materials before fracture. **Toughness** is the energy absorbed in fracturing. The stress necessary to cause fracture may be reached before there has been much plastic deformation to absorb energy. Ductility and toughness of a given material are lowered by factors that inhibit plastic flow; **these factors include decreased temperatures, increased strain rates, and the presence of notches**.

But, in general, properties which influence the plastic deformation of a material are:

- Modulus of elasticity
- Crystal structure
- Temperature (for many BCC metals and alloys)

**Ductile fracture**

Fracture usually starts by nucleation of voids in the center of the neck, where the hydrostatic tension is the greatest. As deformation continues, these internal voids grow and link up.

*Section through a necked tensile specimen of copper, showing an internal crack formed by linking voids.*

With continued elongation, this internal fracture grows outward until the outer rim can no longer support the load and the edges fail by sudden shear. The final shear failure at the outside also occurs by void formation and growth. This overall failure is often called a **cup and cone** fracture.
The formation and growth of voids during shear, and their growth and linking up by necking of the ligaments between them.

(a) Initial necking  (b) Cavity formation  (c) Cavities form a crack  (d) Crack propagation  (e) Final shear occurs at an angle of 45°, where shear stress is at a maximum.

A typical cup and cone fracture in a tension test of a ductile manganese bronze.

In ductile fractures, voids form at inclusions because either the inclusion-matrix interface or the inclusion itself is weak. Ductility is strongly dependent on the inclusion content of the material. With increasing numbers of inclusions, the distance between the voids decreases, so it is easier for them to link together and lower the ductility. Ductile fracture by void coalescence can occur in shear as well as in tension testing. Mechanical working tends to produce directional microstructures. Grain boundaries and weak interfaces are aligned by the working. Inclusions are elongated and sometimes
broken up into strings of smaller inclusions. Loading in service parallel to the direction along which the interfaces and inclusions are aligned has little effect on the ductility.

Microstructure of steel plate at a higher magnification showing elongated manganese sulfide inclusions. These inclusions are the major cause of the directionality of ductility in steel.

In steels, elongated manganese sulfide inclusions are a major cause of directionality. Reduced ductility perpendicular to the rolling direction can cause problems in sheet forming. This effect can be eliminated by reducing the sulfur content and adding small amounts of Ca, Ce, or Ti. These elements are stronger sulfide formers than Mn. At hot working temperatures, their sulfides are much harder than the steel and therefore do not elongate as the steel is rolled.

**Brittle fracture**

*Cleavage:* In some materials, fracture may occur by cleavage. Cleavage fractures occur on certain crystallographic planes (cleavage planes) that are characteristic of the crystal structure. This results in a relatively flat surface at the break. It is significant that **fcc metals do not undergo cleavage.**

It is thought that cleavage occurs when the normal stress, $\sigma_n$, across the cleavage plane reaches a critical value, $\sigma_c$.

In three dimensions the cleavage planes in one grain of a polycrystal will not link up with the cleavage planes in a neighboring grain. Therefore fracture cannot occur totally by cleavage. Some other mechanism must link up the cleavage fractures in different grains.

* **Grain boundary fracture:**

Some polycrystals have brittle grain boundaries, which form easy fracture
paths. The brittleness of grain boundaries may be inherent in the material or may be caused by segregation of impurities to the grain boundary or even by a film of a brittle second phase. For example: Molten FeS in the grain boundaries of steels at hot working temperatures would cause failure along grain boundaries. Such loss of ductility at high temperatures is called hot shortness. Hot shortness is prevented in steels by adding Mn, which reacts with the sulfur to form MnS. Manganese sulfide is not molten at hot working temperatures and does not wet the grain boundaries.

• **Role of grain size:** With brittle fracture, toughness depends on grain size. Decreasing the grain size increases the toughness and ductility. Perhaps this is because cleavage fractures must reinitiate at each grain boundary (i.e., providing a high density of crack arrest/deflection points) and with smaller grain sizes there are more grain boundaries. Also, even if a grain cracks, then the stress concentration at the end of the crack decreases with decreasing crack size [= grain size]. Decreasing grain size, unlike most material changes, increases both yield strength and toughness.

• **Weakly bonded second phase particles** tend to promote fracture by acting as initiation sites for cracks.

• **Voids** decrease toughness by introducing nucleation sites for cracks. Residual stresses introduced by thermal cycling or deformation tend to decrease toughness also because they mean that there must be a region with elevated tensile stress (balanced by compressive stresses elsewhere).

**Temperature Effect:**
Temperature affects plasticity in many materials. Higher temperatures promote deformation whereas low temperatures promote fracture. In many materials, a ductile-to-brittle transition can be detected as you lower the temperature.

• It is common to define a **transition Temperature** in a range over which there is a large change of energy absorption and fracture appearance. At temperatures below the transition temperature the fracture is brittle and absorbs little energy in a Charpy test. Above the transition temperature the fracture is ductile and absorbs a large amount of energy. Figure shows typical results for steel.

• The transition temperature **does not** indicate a structural change of the material. This ductile–brittle transition temperature (DBTT) depends greatly on the type of test being made.
• As the temperature is decreased, it becomes too difficult to move dislocations (as quantified by the critical resolved shear stress) relative to the stress required to propagate a crack (as quantified by the tensile breaking stress).
• Most materials exhibit a cress that increases rapidly with decreasing temperature. This is generally caused by an increase in the Peierls stress, i.e. the lattice friction.

Environment, Loading:
• Environment: certain interstitial elements are particularly deleterious, e.g. hydrogen in steels. This may be because atomic hydrogen lowers the cohesive energy between the planes that tend to cleave in brittle fracture. Hydrogen can be readily introduced in welding. Ammonia is notorious as a promoter of corrosion fatigue, e.g. cracking in brass. Similarly chloride ions (salt) in iron alloys (even stainless steel!).

• Type of loading: multiaxial stresses involving tension promote fracture whereas stresses involving compression promote deformation, especially if deviatoric stresses are maximized. Monotonic loading is generally less severe than cyclic loading. Specimen design is also critical – notches promote fracture over deformation.

Crack Initiation and Propagation:
The process of fracture can be considered to be made up of two components:
  - crack initiation
  - crack propagation
• Cracks usually initiate at some point of stress concentration (if the materials is perfect free and this is seldom true). Typical for that includes: scratches, fillets, threads, dents, and pre-existing internal cracks which play a major role.

• Crack propagation may be transgranular or intergranular
  • Transgranular - The fracture travels through the grain of the material.
  - The fracture changes direction from grain to grain due to the different lattice orientation of atoms in each grain.
  - when the crack reaches a new grain, it may have to find a new path or plane of atoms to travel on because it is easier to change direction for the crack than it is to rip through.
  - Cracks choose the path of least resistance. You can tell when a crack has changed in direction through the material, because you get a slightly bumpy crack surface.

• Intergranular:
  - The crack path follows the grain boundaries, and not through the actual grains.
  - Usually occurs when the grain boundary is weak and brittle (cementite in Iron's grain boundaries).
Fracture Mechanics:
Use of fracture analysis to determine the critical stress at which a crack will propagate and lead to failure. The stress at which fracture occurs is termed fracture strength.

Materials, when exposed to external forces, will deform or fail. This may occur by:-

1- **Shear**:- When a material breaks under a “parallel” stress and the fracture path is parallel to the applied stress. Consumes more energy as shear distorts specimens. Bond breakage is sequential. Theoretical shear stress $\frac{G b}{2\pi a}$

2- **Cleavage**:- When a material breaks under a perpendicular (i.e., normal) stress and the fracture path is perpendicular to the applied stress. Consumes little energy and produces a brittle fracture. All bonds break instantaneously. $\sigma_{max} = \frac{E}{\pi}$

It can be concluded that in order to have a high theoretical cleavage strength, a materials must have high E and surface energy and small distance between atomic planes. This strength is a function of the cohesive forces between the atoms. Experimental values lie between 10 and 1000 times below this value. This is due to the presence of small flaws occurring throughout the material, referred to as stress raisers.

- Real materials always have cracks (or potential cracks): therefore one must always be concerned about whether the cracks will grow catastrophically, and answer a practical question “At what value of the external load will a crack start to grow?”

**Modes of crack displacement:**

(a) Mode I = opening (or tensile) mode  (b) Mode II = sliding mode  (c) Mode III = tearing mode
Stress Concentration:

\[ \sigma_{\text{max}} = \sigma_a (1 + 2a/b) \]

The term \((1 + 2a/b)\) is called the stress concentration factor. The radius of curvature at the end of an ellipse is, \(\rho = b^2/a\) so \(a/b = \sqrt{a/\rho}\).

\[ \sigma_{\text{max}} = \sigma_a (1 + 2 \sqrt{a/\rho}) \]

Because \(a/\rho\) is usually very large \((a/\rho >> 1)\)

So \[ \sigma_{\text{max}} = 2\sigma_a \sqrt{a/\rho} \]

For a circular crack \((a = b)\), and \[ \sigma_{\text{max}} = 3\sigma_a \]

Fracture will occur when the level of \(\sigma_{\text{max}}\) reaches the theoretical fracture strength, so the average stress, \(\sigma_a\), at fracture is much lower than the theoretical value.

Griffith Criterion:

Griffith pointed out that, an existing crack will propagate if the elastic strain energy released by doing so is greater than the surface energy created by the two new crack surfaces.

![Diagram of plate with crack](image)

Plate of thickness \(t\) that contains a crack of length \(2a\) under plane stress.

The volume in which the strain energy is relaxed is \(2\pi a^2 t\), where \(\pi (2a)a\) is the area of the ellipse.

The total strain energy released is thus \[
\frac{\sigma^2}{2E} \left(2\pi a^2 t\right) = \frac{\pi \sigma^2 a^2 t}{E} = \frac{\pi \sigma^2 a^2}{E} \]
per unit thickness

The decrease in strain energy, \(U_e\), when a crack propagates is balanced by an increase in the surface energy, \(U_s\), produced by the creation of the two new crack surfaces. The increase in surface energy equals:

\[ U_s = (2at)(2\gamma_s) = 4a\gamma_s \] per unit thickness, where \(\gamma_s\) is the specific surface energy, i.e., the energy per unit area. The change in potential energy of the plate,
\[ \Delta U = U_s - U_e = 4a\gamma_s - \frac{\pi\sigma^2 a^2}{E} \]

The crack becomes stable when these energy components balance each other. If they are not in balance, we have an unstable crack (i.e., the crack will grow). We can obtain the equilibrium condition by equating to zero the first derivative of the potential energy \(\Delta U\) with respect to the crack length. Thus:

\[ 2\gamma_s = \frac{\pi\sigma^2 a}{E} \]

OR \( \sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}} \) = the critical stress required for the crack to propagate in plain stress situation.

For the plane-strain situation: \( \sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a(1 - \nu^2)}} \)

So:
- \( \sigma_c \sqrt{\pi a} \), which is called fracture toughness depends only on materials constants
- \( \sigma_c \) plane strain > \( \sigma_c \) plain stress

**Crack Propagation with Plasticity**

If the material in which a crack is propagating can deform plastically, the form of the crack tip changes because of plastic strain. A sharp crack tip will be blunted. The amount of plastic deformation that can occur at the crack tip will depend on how fast the crack is moving. So, a certain amount of plastic work \( \gamma_p \) is done during crack propagation, in addition to the elastic work done in the creation of two fracture surfaces. \( \gamma_p \) depends on the crack speed, temperature, and the nature of the material.

For an inherently brittle material, at low temperatures and at high crack velocities \( \gamma_p \) is relatively small \( \gamma_p < 0.1 \gamma_s \). In such a case, the crack propagation would be continuous.
and elastic. These cases are usefully treated by means of linear elastic fracture mechanics (LEFM). In other cases:

\[ \sigma_c = \sqrt{\frac{2E}{\pi a} \left( \gamma_p + \gamma_s \right)} \]  
(Plain stress)

\[ \sigma_c = \sqrt{\frac{2E}{\pi a(1-\nu^2)} \left( \gamma_p + \gamma_s \right)} \]  
(Plain strain)

For \( \gamma_p >> \gamma_s \), for ductile materials: \[ \sigma_c = \sqrt{\frac{2E}{\pi a} \gamma_p} \]

Thus, the plastic deformation around the crack tip makes it blunt and serves to relax the stress concentration by increasing the radius of curvature of the crack at its tip. Localized plastic deformation at the crack tip improves the fracture toughness of the material. It is difficult to measure \( \gamma_p \) and \( \gamma_s \), so Irwin proposed that fracture occurs at a stress that corresponds to a critical value of the crack extension force, \( G \) (J/m² or \( \frac{in.lb}{in^2} \)).

\[ G = \text{rate of change of energy with crack length} = \text{strain energy release rate} = \frac{U/a}{E} = \frac{\pi a \sigma^2}{E} \]

At fracture, \( G = G_c \), and \( \sigma_c = \sqrt{\frac{EG_c}{\pi a}} \) (Plain stress)

and \[ \sigma_c = \sqrt{\frac{EG_c}{\pi a(1-\nu^2)}} \] (Plain strain)

**Linear Elastic Fracture Mechanics:**
Nonductile materials \( \Rightarrow \) No plastic deformation \( \Rightarrow \) Brittle fracture (sudden crack propagation, rapid, and unstable) \( \Rightarrow \) LEFM is used.

**Fracture Toughness:**
Stresses near the crack tip of a material can be characterized by the stress intensity factor, \( K \). A critical value of \( K \) exists, similar to the value \( \sigma_c \), known as fracture toughness given by:

\[ K_c = Y \sigma \sqrt{\pi a} \]  
\( \text{psi} \sqrt{in}(MPa \sqrt{m}) \)

\( K_c \) is the critical stress for crack propagation, \( a \) is the crack length, and \( Y \) is a dimensionless parameter that depends on both the crack and specimen geometries.

Note that the Equation is applicable in the region \( r << a \) (i.e., in the vicinity of the crack tip). For larger \( r \), higher order terms must be included.

**Fracture toughness** is a property that describes a material’s resistance to brittle fracture when a crack is present. \( K_c \) depends on the thickness of plate up to a certain point above which it becomes constant. This constant value is known as the plane strain fracture toughness denoted by:
\[ K_{ic} = Y \sigma \sqrt{\pi a} \]

The I subscript corresponds to a “mode I” crack displacement.

- Brittle materials have low \( K_{ic} \) values, leading to catastrophic failure
- Ductile materials usually have much larger \( K_{ic} \) values

**\( K_{ic} \)** depends on temperature, strain rate, and microstructure

- Increases as grain size decreases
- Decreases with increasing strain rate
- Decreases with decreasing temperature....

**Example 1:**
A thick wide plate (\( E = 3 \text{GPa} \) and \( \nu = 0.4 \)) contains a central sharp crack of length \( 2a = 40 \text{mm} \). The crack is found to propagate at stress \( \sigma = 4.2 \text{MPa} \). Find \( K_{ic} \). Will a crack of length \( 2 \text{mm} \) in a similar sheet fracture if \( \sigma = 10 \text{MPa} \).

Solution:

\[ K_{ic} = Y \sigma \sqrt{\pi a} \]

\[ K_{ic} = 4.2 \times 1 \times \sqrt{\pi \times \frac{0.4}{2}} = 1.05 \text{MPa} \cdot \sqrt{m} \]

\[ K = 10 \times 1 \times \sqrt{\pi \times \frac{0.002}{2}} = 0.56 \text{MPa} \cdot \sqrt{m} < K_{ic} \]

So: no tearing.

**Example 2:**
An aluminum alloy has a plane-strain fracture toughness \( K_{ic} \) of \( 50 \text{MPa} \cdot \text{m}^{1/2} \). A crack of 1-mm-long was detected in an automotive component made of this alloy. The component will be subjected to cyclic fatigue with \( \Delta \sigma = 100 \text{MPa} \) with \( R = 0 \). How many more cycles can this component endure? Take \( Y = 1.05 \) and \( da/dN(\text{mm/cycle}) = 1.5 \times 10^{-24} \Delta K^4 \text{ (MPa m}^{1/2})^4. \)

Solution:

\[ K_{ic} = Y \sigma \sqrt{\pi a_f} \]

\[ 50 = 1.05 \times 100 \times \sqrt{\pi a_f} \quad \Rightarrow \quad a_f = 72 \text{mm} \]

So: \( da/dN(\text{mm/cycle}) = 1.5 \times 10^{-24} \Delta K^4 \text{ (MPa m}^{1/2})^4. \)

\[ \Delta N = N_f - N_i = N_f = \frac{1 - (\frac{a_f}{a_i})^m}{c(Y \sigma \sqrt{\pi})^{m/2 - 1}} \left( \frac{1}{c_f m^{2-1}} \right) = 546 \text{ Cycles} \]
Example 2:
A steel tube (E = 200 GPa, yield strength = 1420MPa, the cleavage strength = 2200MPa and the fracture toughness is 90MPa.m^{1/2}) is to be used in a power plant. At the moment, a tube diameter of D = 1000mm with a wall thickness of t = 5mm is planned. The pressure within the tube is p = 12MPa. Ultrasonic measurements can limit the largest crack in the material to a size smaller than 2a = 3mm. The geometry factor can be assumed as Y = 1.

a) Can the tube be used with the intended material and dimensions? Check it against yielding, cleavage fracture, and crack propagation.
b) If the pressure is increased from zero until a failure criterion is met, which criterion is this? What is the corresponding pressure?
c) At what crack length are yield strength and fracture toughness reached simultaneously?

Solution:

a) Against yielding: The stress state is uniaxial with \( \sigma = PD/(2t) \). Thus, the condition \( \sigma = PD/(2t) < \sigma_{\text{yielding}} \) must be met: Since \( \sigma = 1200\text{MPa} < 1420\text{MPa} \), there is no yielding.

Against cleavage fracture: 1200MPa < 2200MPa. Cleavage fracture is not to be expected.

Against crack propagation: \( \sigma < K_{\text{lc}} / (\pi a)^{1/2} \), where a = 1.5mm is the maximum half crack length to be expected: 1200MPa < 1311MPa. There will be no crack propagation.

The tube can be used.

b) Yielding occurs at a pressure \( P = 2t \sigma_{\text{yielding}} / D = 14.2\text{MPa} \).

Cleavage fracture will be observed at a pressure \( P = 2t \sigma_{\text{cleavage}} / D = 22\text{MPa} \).

From the fracture toughness, the stress can be calculated using \( \sigma = K_{\text{lc}} / (\pi a)^{1/2} \)

The resulting failure pressure is \( P = 2t K_{\text{lc}} / (\pi a)^{1/2} D = 13.1\text{MPa} \).

Thus, the tube will fail by crack propagation at a pressure \( P = 13.1\text{MPa} \) if a crack of length a = 1.5mm is present.

c) The yield strength and the fracture toughness are reached simultaneously at a value \( a_c: \ a_c = (90\text{MPa.m}^{1/2}/1420\text{MPa})^2/\pi = 1.28\text{mm} \).
Example 3:
During a regular inspection, as done all 5000 cycles, a crack of length $a_0 = 1\text{mm}$ was detected in the lever gear of a machine. The following service parameters are known: at the service stress range of $\Delta \sigma = 100\text{MPa}$ the stress ratio $R = -1$, the critical crack length is $a_f = 10\text{mm}$, the geometry factor $= 1$, and $\frac{da}{dN} = 2 \times 10^{-12} \Delta K^2$

a) Calculate the fracture toughness $K_{ic}$.

b) Check whether unstable crack propagation is to be expected.

c) Estimate whether the component can stay in service if the crack propagation per cycle $da/dN$ is assumed to be constant.

d) Check whether the machine can stay in service until the next maintenance interval.

Solution:

a) For $R = \sigma_{min}/\sigma_{max} = -1$, the maximum stress is half of the stress range: $\sigma_{max} = \Delta \sigma/2$, as cyclic stress range $\Delta \sigma = \sigma_{max} - \sigma_{min}$. So:

$$K_{ic} = \sigma_{max} (\pi a_f)^{1/2} Y = 8.862\text{MPa.m}^{1/2}$$

b) At the current crack length:

$$K_{max} = \sigma_{max} (\pi a_0)^{1/2} Y = 2.8025\text{MPa.m}^{1/2}$$

Because the maximum stress intensity factor $K_{max}$ is clearly below $K_{ic}$, the component will not fail statically.

c) The crack growth per cycle at the current crack length can be calculated as:

$$\Delta K = 2K_{max} = 5.605 \text{MPa.m}^{1/2}$$

$$\frac{da}{dN} = 2 \times 10^{-12} \Delta K^2 = 2 \times 10^{-12} (5.605)^2 = 62.832 \times 10^{-12} \text{mm/cycle}$$

If the crack-growth rate would stay constant at this value, the critical crack length would be reached after:

$$N_f = (a_f - a_0)/(da/dN) = (0.01 - 0.001)/(62.832 \times 10^{-12}) = 1.4 \times 10^8 \text{cycles}$$

In this case, the component could be cleared for further use.

d) $\Delta N = \int_{a_i}^{a_f} \int_{c(\Delta K_i)}^{c(\Delta K_f)} \frac{da}{c(Y \Delta \sigma \sqrt{\pi a})^{m/2}} = \frac{1}{c(Y \Delta \sigma \sqrt{\pi})^{m/2}} \int_{a_0}^{a_f} \int_{c(\Delta K_i)}^{c(\Delta K_f)} \frac{da}{a}$

$$= \frac{1}{2 \times 10^{-12} (1 \times 100^2 \times \pi)} [\ln(0.01) - \ln(0.001)] = 36.64 \times 10^6$$

The component can be cleared until the next service interval because the number of cycles to failure is significantly larger than the interval time (when $N = 5000$).