POLYNOMIALS IN MATLAB

A polynomial of degree $n \geq 1$ has the form:

$$(x)=a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where the $a_i's$ are constants and $a_n \neq 0$.

Examples: the following are polynomials:

1. $3x + 5$
2. $x^2 - 2x + 3$
3. $2x^3 + x^2 - 4x + 6$
4. $2x^4 - x^2 + 5$

To enter the polynomials above in MATLAB, we enter

```
>> p1=[3   5]

>> p2=[1   -2    3]

>> p3=[2   1   -4    6]

>> p4=[2   0   -1    0   5]
```

sym2poly and **poly2sym**

There are built-in functions **sym2poly** and **poly2sym** that convert from symbolic expressions to polynomial vectors and vice versa, for example:

```
>> s1= poly2sym(p1)
    s1=
           3*x+5

>> s2= poly2sym(p2)
    s2=
          x^2-2*x+3

>> s3= poly2sym(p3)
     s3=
          2*x^3+x^2-4*x+6

>> s4= poly2sym(p4)
     s4=
```

```
    2*x^4 – x^2+5

>> sym2poly (s1)
   ans=
       3    5

>> sym2poly (s2)
   ans=
      1   -2    3

>> sym2poly (s3)
   ans=
      2   1   -4   6

>> sym2poly (s1)
   ans=
      2    0    -1    0    5
```

roots Function

The roots function calculates the roots of a polynomial. For example to solve the equation $f(x) = 0$, where:

$$f(x) = 4x^3 - 2x^2 - 8x + 3$$

We enter:

```
>> r1= roots([4   -2   -8   3])
   r1 =
     -1.3660
      1.5000
      0.3660

>> p=[1   0    -2    -5];
>> r2= roots(p)
   r2=
       2.0946
      -1.0473 + 1.1359i
      -1.0473  - 1.1359i
```

poly Function

The function poly returns to the polynomial coefficients.

```
>> poly (r1)
    ans=
         4   -2   -8   3

>> poly (r2)
    ans=
        1   0    -2    -5
```

polyval Function

The polyval function evaluates a polynomial at a specified value. To evaluate p at *s* = 5, use

```
>> p = [1    0   -2   -5];
>> s=5;
>> polyval(p, 5)
    ans=
        110

>> p = [-2   1   4];
>> polyval(p,3)
    ans =
         -11
```

The argument *x* can be a vector, for example:

```
>> polyval(p,1:3)
    ans =
        3   -2    -11
>> polyval(p, [5 7])
    ans =
        -41     -87
```

conv and deconv Function

Polynomial multiplication and division correspond to the operations convolution and deconvolution. The functions conv and deconv implement these operations.

1. w = conv(u,v) returns the convolution of vectors u and v. If u and v are vectors of polynomial coefficients, convolving them is equivalent to multiplying the two polynomials.
2. [q,r] = deconv(u,v) deconvolves a vector v out of a vector u using long division, and returns the quotient q and remainder r such that u = conv(v,q)+r. If u and vare vectors of polynomial coefficients, then deconvolving them is equivalent to dividing the polynomial represented by u by the polynomial represented by v.

Examples

1. Create vectors u and v containing the coefficients of the polynomials $x^2 + 1$ and $2x + 7$. Use convolution to multiply the polynomials. Write the resulting polynomial as a mathematical expression

```
>> u = [1   0   1];
>> v = [2   7];
>> w = conv(u,v)
    w =
        2    7    2    7
```

w contains the polynomial coefficients for $2x^3 + 7x^2 + 2x + 7$.

2. Create two vectors u and v containing the coefficients of the polynomials $2x^3 + 7x^2 + 4x + 9$ and $x^2 + 1$, respectively. Divide the first polynomial by the second by deconvolving v out of u. Write the resulting polynomials as a mathematical expressions.

```
>> u = [2   7   4   9];
>> v = [1   0   1];
>> [q,r] = deconv(u,v)
    q =
        2    7

    r =
        0    0    2    2
```

The results are $q = 2x + 7 \quad and \quad r = 2x + 2$

polyder Function

The polyder function computes the derivative of any polynomial. To obtain the derivative of the polynomial, we have three possibilities:

1. k = polyder(p) returns the derivative of the polynomial represented by the coefficients in p,

$$k(x) = \frac{d}{dx}p(x).$$

2. k = polyder(a,b) returns the derivative of the product of the polynomials a and b,

$$k(x) = \frac{d}{dx}[a(x)b(x)].$$

3. [q,d] = polyder(a,b) returns the derivative of the quotient of the polynomials a and b,

$$\frac{q(x)}{d(x)} = \frac{d}{dx}\left[\frac{a(x)}{b(x)}\right].$$

```
>> p = [1   0   -2   -5]
>> q = polyder(p)
    q =
       3   0   -2
```

Examples

1. Create two vectors to represent the polynomials

$$a(x) = x^4 - 2x^3 + 11$$
$$b(x) = x^2 - 10x + 15$$

Calculate

$$k(x) = \frac{d}{dx}p(x).$$

```
>> a = [1   -2    0    0    11];
>> b = [1   -10       15];

>> q = polyder(a,b)

    q =
         6   -60   140   -90    22   -110
```

The result is

$$q(x) = 6x^5 - 60x^4 + 140x^3 - 90x^2 + 22x - 110$$

2. Create two vectors to represent the polynomials in the quotient,

$$\frac{x^4 - 3x^2 - 1}{x + 4}$$

Calculate

$$\frac{q(x)}{d(x)} = \frac{d}{dx}\left[\frac{a(x)}{b(x)}\right].$$

Write the result in mathematical expression.

```
>> p = [1   0   -3    0    -1];
>> v = [1    4];
>> [q,d] = polyder(p,v)
  q =
      3    16    -3   -24     1
  d =
      1     8    16
```

The result is:

$$\frac{q(x)}{d(x)} = \frac{3x^2 + 16x^3 - 3x^2 - 24x + 8}{x^2 + 8x + 16}$$

H.W.:

1. Consider the polynomials:

$$a(x) = x^2 + 2x + 3 \quad \text{and} \quad b(x) = 4x^2 + 5x + 6$$

Find the following and write the results as mathematical expressions:

a. $c(x) = a(x) * b(x)$.

b. $\frac{c(x)}{a(x)}$

c. $\frac{c(x)}{b(x)}$

d. Solve $a(x) = 0, b(x) = 0$ and $c(x) = 0$.

e. $k(x) = \frac{d}{dx} a(x)$.

f. $k(x) = \frac{d}{dx} b(x)$.

g. $k(x) = \frac{d}{dx} c(x)$.

h. $k(x) = \frac{d}{dx} [a(x)b(x)]$.

i. $\frac{q(x)}{d(x)} = \frac{d}{dx} \left[\frac{a(x)}{b(x)}\right]$.

j. $a(4)$, $b(5)$ and $c(2)$

2. Find the polynomials whose roots are:

a. 1.5352 and -0.8685

b. -1.0000 + 0.0000i
0.0000 + 1.0000i
0.0000 - 1.0000i
1.0000 + 0.0000i

c. 8

d. 2 and -2