1-11 The ∇ Operator

The ∇ is a differential operator can be written in it's component as in the following:

$$\nabla = \hat{i}\nabla_x + \hat{j}\nabla_y + \hat{k}\nabla_z$$

1-11-1 Scalar Product of ∇ operator

If $\vec{A}$ *is a vector*

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Then

$$\nabla . \vec{A} = (\hat{i}\nabla_x + \hat{j}\nabla_y + \hat{k}\nabla_z) . (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$$= \nabla_x A_x + \nabla_y A_y + \nabla_z A_z \quad = \frac{\partial A_x}{\partial t} + \frac{\partial A_y}{\partial t} + \frac{\partial A_z}{\partial t}$$

The quantity $\nabla . \vec{A}$ called the divergence $\vec{A}$ (div $\vec{A}$)

1-11-2 Vector Product of ∇ Operator

For $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$; $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ We define the vector product of $\nabla x \ \vec{A}$ as:

$$\nabla x \ \vec{A} = \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

The vector quantity $\nabla x \ \vec{A}$ called the curl $\vec{A}$

Example(1)

What is the angle between $\vec{A} = 3\hat{i} + 7\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ (dot product)?

Solution:

$$A_x = 3 \qquad B_x = -1$$
$$A_y = 0 \qquad B_y = 2$$
$$A_z = 7 \qquad B_z = 1$$

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= 3(-1)+0(2)+7(1)$$
$$= 4$$

$$|A| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

$$= \sqrt{3^2 + 0^2 + 7^2}$$
$$= \sqrt{58}$$

$$|B| = \sqrt{(B_x)^2 + (B_y)^2 + (B_z)^2}$$
$$= \sqrt{-1^2 + 2^2 + 1^2}$$
$$= \sqrt{6}$$

$$\vec{A}.\vec{B} = |A|\,|B|\cos\theta$$

$$\text{Cos } \theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{4}{\sqrt{58}\sqrt{6}}$$

$\theta = 77.6^0$ { the angle between two vector }

Question (1)

For vectors $\vec{A} = 3\hat{i} - 4\hat{j} + 7\hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$ find $\vec{A} + \vec{B}$, $\vec{A} - \vec{B}$, $\vec{A} x \vec{B}$, $\vec{A}.\vec{B}$??