"Introduction to dynamics".

We begin the study of dynamics, the part of mechanics that deals with the analysis of bodies in motion.

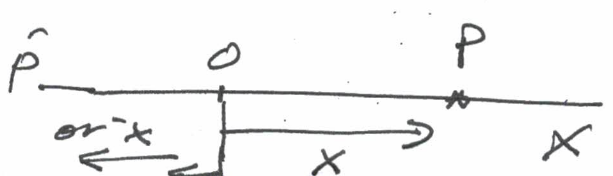
Dynamics includes:

1. Kinematics which is the study of geometry of motion. It is used to relate displacement, velocity acceleration and time without reference to the cause of the motion.

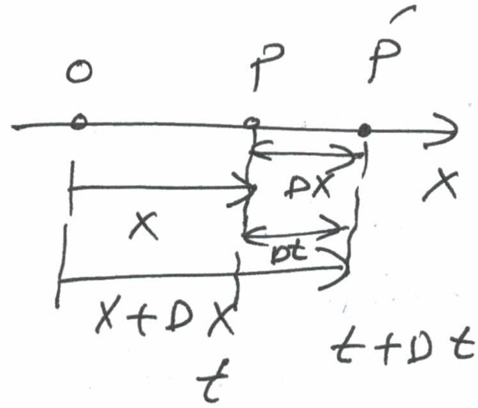
2. Kinetics which is the study of the relation existing between the forces acting on the body, mass of the body and motion. Kinetics is used to predict the motion caused by given forces.

1. Rectilinear motion of particles

A particle moving alone a straight line is said to be in rectilinear motion. At any given instant t, the particle will occupy a certian position on the striaght line



Average velocity $= \dfrac{\Delta x}{\Delta t}$



Instantaneous velocity $= v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$

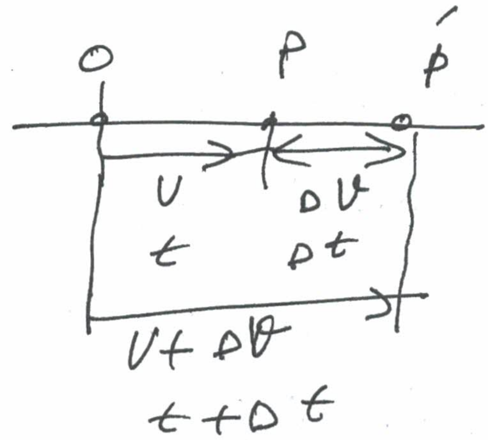
$$v = \frac{dx}{dt}$$

Average acceleration $= \dfrac{\Delta v}{\Delta t}$

instantaneous acceleration, $a = \dfrac{dv}{dt}$

but $v = \dfrac{dx}{dt}$ so $\Rightarrow$

$$a = \frac{d^2 x}{dt^2} = \frac{d^2 x}{dt^2}$$



* distance, velocity and acceleration are vector values which have the magnitude and sence at same time.

<u>Note</u> $v = \dfrac{dx}{dt} \Rightarrow dt = \dfrac{dx}{v} \Rightarrow$

acceleration, $a = \dfrac{v \cdot dv}{dx}$

There are three common classes of motion.

1. $a = f(t)$. The acceleration is a function of t.

$$a = \frac{dv}{dt} \Rightarrow dv = a \cdot dt$$
$$= f(t)\,dt$$

$$\int dv = \int f(t)\,dt$$

$$v\Big|_{v_0}^{v} = \int_0^t f(t)\,dt$$

$$v - v_0 = \int_0^t f(t)\,dt \qquad \&$$

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

$$\int dx = \int v\,dt$$

$$x\Big|_{x_0}^{x} = \int_{v_0}^{v} v\,dt$$

2. $a = f(x)$. The acceleration is a function of position x

$$a = \frac{v \cdot dv}{dx} \Rightarrow v\,dv = f(x)\,dx$$

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx \qquad \&$$

$$v = dx/dt \Rightarrow \int_{v_0}^{v} v\,dt = \int_{x_0}^{x} dx$$

3. $a = f(v)$. acceleration is a function of velocity

$$a = f(v) = \frac{dv}{dt} = v \cdot \frac{dv}{dx}$$

$$dt = \frac{dv}{f(v)} \quad , \quad dx = \frac{v \cdot dv}{f(v)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} \quad , \quad \int_{x_0}^{x} dx = \int \frac{v \cdot dv}{F(v)}$$

<u>Example ①</u> The position of a particle moves along a straight line is $x = t^3 - 6t^2 - 15t + 40$ where x in feets & t in seconds. Determine

ⓐ the time at which the velocity will be zero.

ⓑ the position and distance travelled by the Particle at that time. ⓒ acceleration at that time?!

Solution

$$x = t^3 - 6t^2 - 15t + 40$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15$$

$$a = \frac{dv}{dt} = 6t - 12$$

$\boxed{a}$ Time at which $v = 0 \Rightarrow$ Set $v = 0 \Rightarrow$

$$3t^2 - 12t - 15 = 0$$

$$3(t^2 - 4t - 5) = 0$$

$$(t-5)(t+1) = 0 \Rightarrow t = -1 \text{ sec. or } t = 5 \text{ seconds.}$$

$\boxed{b}$ position and distance when $v = 0 \Rightarrow t = 5$ sec.

$X(5\text{ sec}) = 5^3 - 6 * 5^2 - 15 * 5 + 40$

The $= -60$ ft

position at $t = 0$ equal to initial position

$X_{(0)} = 40$ ft. so that

Distance travled $= X_{(5)} - X_{(0)}$

$= -60\text{ ft} - 40\text{ ft}$

$= -100$ ft.

$= 100$ ft in negative direction.

$\boxed{c}$ acceleration when $v = 0$, $t = 5$ sec.

$a(5) = 6 * 5 - 12$

$= 30 - 12 = 18\text{ ft/sec}^2$

$\boxed{\text{Example 2}}$ The position of a particle which is confined to move along a straight line is $X = 2t^3 - 24t + 6$

Determine $\boxed{a}$ the time required for a particle to reach velocity of 72 m/s from its initial condition at $t = 0$. $\boxed{b}$ the acceleration of particle when $v = 30$ m/sec. and $\boxed{c}$ net displacement of the particle during $t = 1$ sec to $t = 4$ sec.?

$\boxed{\text{Solution}}$ $X = 2t^3 - 24t + 6$

$v = (6t^2 - 24)$ m/sec.

$a = 12t \ m/sec^2$

a). for $\vartheta = 72$ m/sec $\Rightarrow$

$72 = 6t^2 - 24 \Rightarrow t = \pm 4$ sec. $= 4$ sec.

$t = 4$ seconds

b). for $\vartheta = 30$ m/sec. $\Rightarrow$

$30 = 6t^2 - 24$ for $\Rightarrow$

$t = 3$ sec.

$a = 12 * 3\text{sec} = 36 \ m/sec^2$

c). for net displacement for $t=1$ to $t=4$ sec.

$\Delta X = X_{(4)} - X_{(1)}$

$= \{2 * 4^3 - 24 * 4 + 6\} - \{2 * 1^3 - 24 * 1 + 6\}$

$= 54$ meter.

Example 3 A particle moves along the x-axis with an initial velocity $V_x = 50$ m/s at the origin when $t=0$ for first 4 seconds, It has no acceleration and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10 \ m/s^2$. Calculate the velocity and x-coordinate of the particle for $t = 8$ sec and $t = 12$ sec. and find the

maximum positive x-coordinate reached by the particle.

$\boxed{\text{solution}}$ $a = \frac{dv}{dt} \Rightarrow dv = a\,dt$

$$\int_{50}^{v_x} dv = \int_4^t a\,dt = -10\int_4^t dt \Rightarrow$$

$V_x = 90 - 10t$ m/sec.

at $t = 8$ sec $\Rightarrow V_x = 90 - 10 * 8 = 10$ m/s.

at $t = 12$ sec $\Rightarrow V_x = 90 - 10 * 12 = -30$ m/s.

$$v = \frac{dx}{dt} \Rightarrow dx = v.dt$$

$$\int dx = \int v\,dt$$

$$x - x_0 = \int (90 - 10t)\,dt$$

$$x - (50 * 4_{sec}) = \int_{4sec}^{t} (90 - 10t)\,dt$$

$$x = 50 * 4 + \int_4^t (90 - 10t)\,dt = -5t^2 + 90t - 80$$

for $t = 8$ sec $\Rightarrow x = -5 * 8^2 + 90 * 8 - 80$
$= 320$ m

for $t = 12$ sec $\Rightarrow x = -5 * 12^2 + 90 * 12 - 80$
$= 280$ m

the maximum x-positive coordinates is given when $v = 0 \Rightarrow t = 9$ sec.

$0 = 90 - 10 * t \Rightarrow t = 9$ sec

So that

$$X_{max}\Big|_{\text{at } t=9\text{ sec}} = -5*9^2 + 90*9 - 80 = 325 \text{ meters.}$$

2. Uniform Rectilinear motion:

In this type the acceleration is zero & Velocity is Constant.

$$\frac{dx}{dt} = v = \text{Constant}$$

$$\frac{dv}{dt} = 0 = a$$

$$\frac{dx}{dt} = v \Rightarrow \int_{x_o}^{x} dx = \int_0^t v\,dt$$

$$x - x_o = v \cdot \int_0^t dt = v \cdot t \Rightarrow$$

$$x = x_o + v \cdot t$$

3. Uniform accelerated Rectilinear motion

The acceleration is Constant

$$\frac{dv}{dt} = a = \text{Constant}$$

$$\int_{v_o}^{v} dv = \int a\,dt = a \int_0^t dt$$

$$v - v_o = a \cdot t \Rightarrow v = v_o + a t.$$

$$\frac{dx}{dt} = v_o + at \Rightarrow$$

$$\int_{x_o}^{x} dx = \int (v_o + at)\,dt$$

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

and $a = \text{constant} = v\dfrac{dv}{dx} \Rightarrow$

$$v\,dv = a\,dx \Rightarrow \int_{v_0}^{v} v\,dv = a \cdot \int_{x_0}^{x} dx$$

$$\frac{1}{2}(v^2 - v_0^2) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

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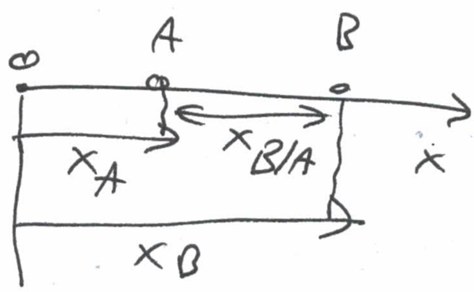
* Relative motion of two particles



$x_{B/A} = x_B - x_A$ or

relative distance.

$$x_B = x_A + x_{B/A}$$

and $\Rightarrow$

$v_{B/A} = v_B - v_A$ or $v_B = v_A + v_{B/A}$

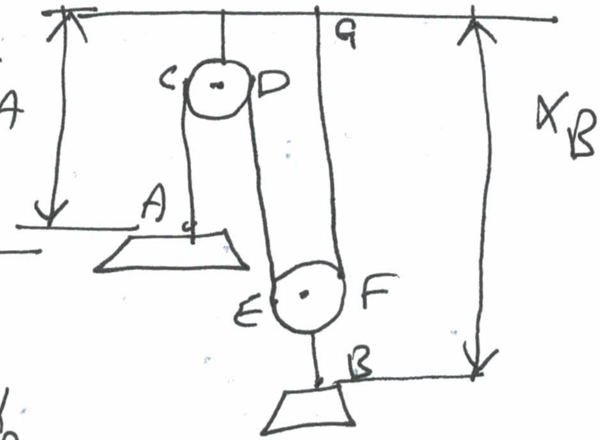
relative velocity

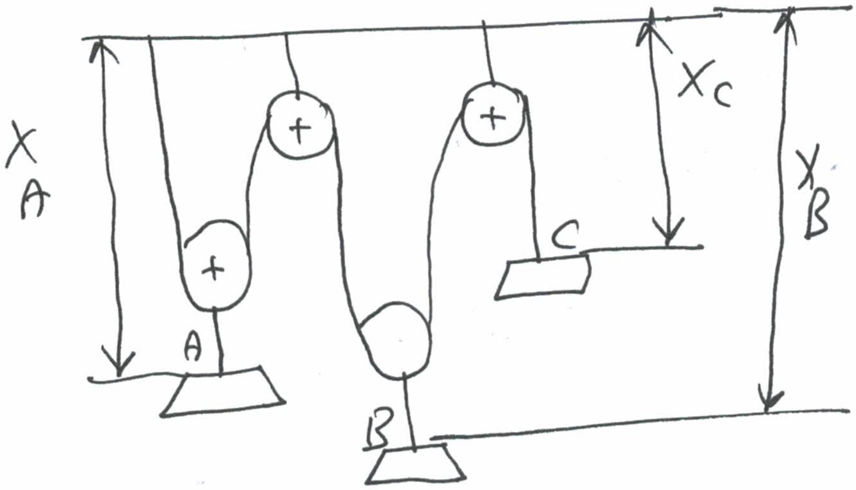
$a_{B/A} = a_B - a_A \Rightarrow a_B = a_A + a_{B/A}$

relative acceleration

** $\boxed{\text{dependent motion.}}$ Sometimes, the position of a particle will depend upon the position of another particle or several particles.

$$X_A + 2X_B = \text{Constant.}$$





$$X_C + 2X_B + 2X_A = \text{Constant}$$

$$\frac{dX_C}{dt} + \frac{2\,dX_B}{dt} + 2\frac{dX_A}{dt} = 0 \Rightarrow$$

$2v_A + 2v_B + v_C = 0$ for velocity and

for acceleration $\Rightarrow$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or}$$

$$2a_A + 2a_B + a_C = 0$$

<u>Example</u> (1) The motor M reels in the cable at a constant rate of 100 mm/s. Determine [a] the velocity of load L [b] the velocity of pulley B with respect to load L.?