Chapter Two

2.1 Introduction:

Logic gates are “elementary bricks” used in the construction of digital circuits. While the binary numeration system studied in the precedent chapter was an interesting mathematical abstraction, we have not yet seen its practical application to electronics. This chapter is devoted to practically apply the concept of binary digits to circuits. A logic gate is a special type of circuit designed to accept (inputs) and generate (outputs) voltages signals corresponding to binary digits (1 and 0).

2.2 Digital signals and gates:

Let us consider the following circuit:

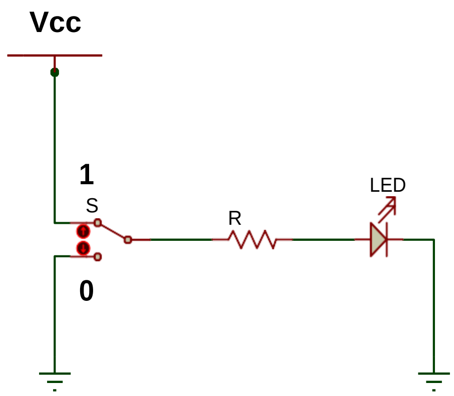


Figure 2.1: Logic circuit.

When the switch is connected to the ground (0V), the light emitting diode (LED) does not shine. If we were using this circuit to represent binary digits, we would say that the input signal is a binary “0” and that the output is a binary “0” or that the output is at the low logic level. Moving the switch to the other position (Vcc), we apply a binary “1” to the input and receive a binary “1” at the output. The output is also said to be at the high logic level.

The gate shown by this simple circuit is a “buffer” or “yes” gate, because the logic state of its input is identical to that of its output. Many types of gates are used in digital electronics: single input gates like the buffer and the NOT gates; multiple inputs gates like AND, NAND, OR, NOR, and XOR gates. The aim of this chapter is to study the functioning of each of those logic gates and also how they can be combined to design a simple logic function.

2.3 The NOT gate:

The NOT gate or Inverter is a logic gate which functions in such a way that the logic state of the output is exactly the opposite of that of the input.

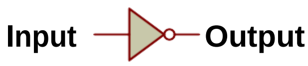
Remark 2.1: The truth table

A truth table is a standard way of representing the Inputs/outputs relationships of a digital circuit, listing all the possible input logic level combinations with their respective output logic levels.

- **The NOT gate truth table:**

Input	Output
0	1
1	0

- **Symbol**



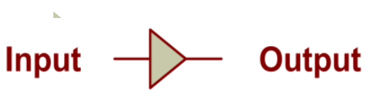
Remark 2.2: the buffer gate

If we were to connect two inverter gates together so that the output of one fed into the input of another, the two inversion functions would “cancel” each other out so that there would be no inversion from input to final output.

Figure 2.2: Principle of the buffer gate

A buffer is a special logic gate manufactured to perform the same function as two inverters connected together. Buffer gates serve to amplify signals, taking a weak signal source that is not capable of providing much current, and boosting the current capacity of the signal so as to be able to drive a load.

- **Symbol of a buffer gate:**



- **Truth table of the buffer gate:**

Input	Output
0	0
1	1

2.4 Multiple input gates:

With a single input gate such as the inverter or buffer, there can only be two possible input states: either 1 or 0. With multiple input gates, many possibilities are available for input states. The number of possible input states is equal to two to the power of the number of inputs. So, if a gate has n inputs, therefore there are 2^n possible input combinations.

2.4.1 The AND gate:

The output of the AND gate is high if and only if all inputs are high. If any input is low, the output is guaranteed to be in a low state as well.

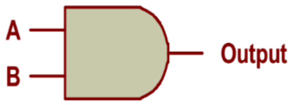
- **Truth table:**

Let us draw the truth table of a two inputs AND gate.

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

As you can notice on the truth table above, the output is high only when all the two inputs are high.

- **Symbol**

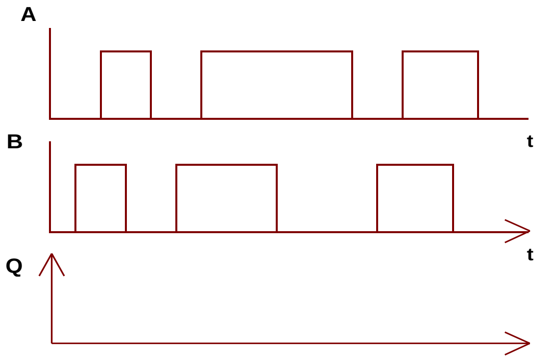


Exercise 2.1:

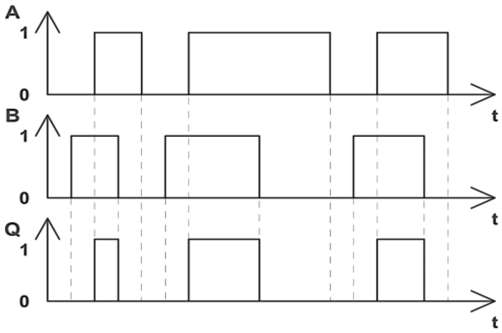
Draw the truth table of a three inputs AND gate.

Exercise 2.2:

Complete the chronogram of the output Q of a two inputs AND gate.



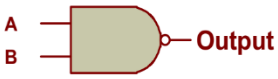
The following solution can be given for the exercise 2.2 above:



2.4.2 The NAND gate:

The word NAND is a verbal contraction of the words NOT and AND. Essentially, a NAND gate behaves the same as an AND gate with a not gate connected to the output terminal.

- **Symbol**



- **Truth table:**

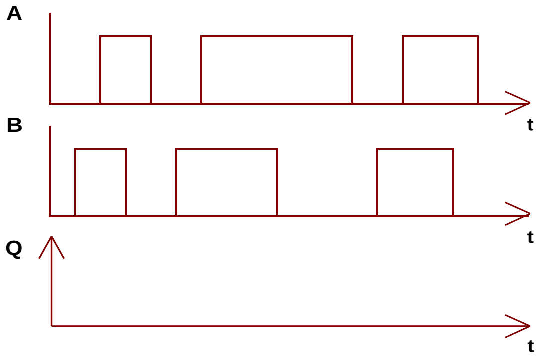
Let us draw the truth table of a two inputs NAND gate.

A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

As with AND gates, NAND gates can be made with more than two inputs.

Exercise 2.3:

Complete the chronogram of the output Q of a two inputs NAND gate.



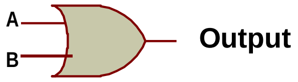
2.4.3 The OR gate:

The output of the OR gate is high if any of the inputs is high. The output of an OR gate goes low if and only if all inputs are low.

- **Truth table:**

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

- **Symbol:**

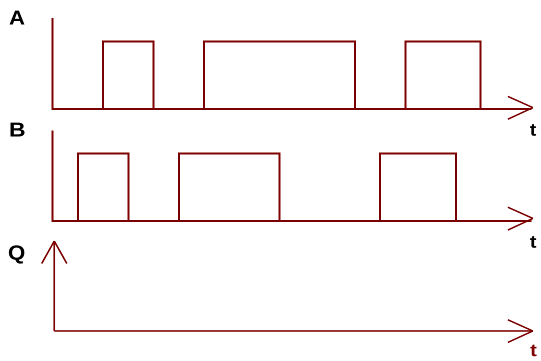


Exercise 2.4:

Draw the truth table of a three inputs OR gate.

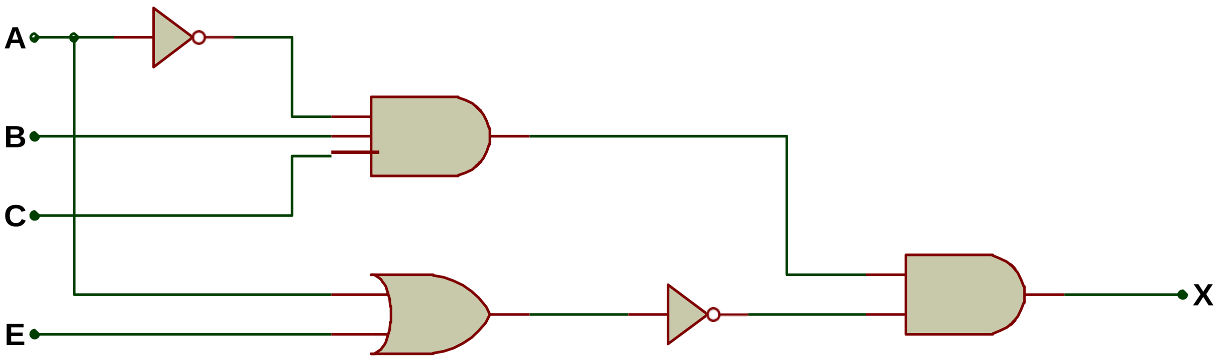
Exercise 2.5:

Complete the chronogram of the output Q of a two inputs OR gate.



Exercise 2.6:

Let us consider the following digital circuit:



a. Give the expression of the output X.

b. Draw the truth table of the digital circuit.

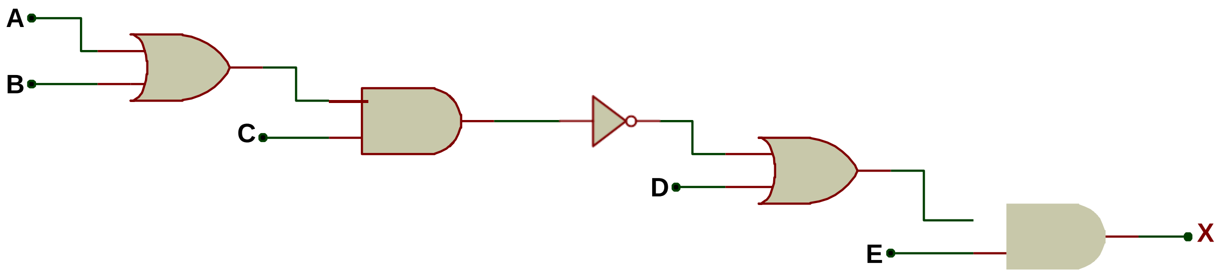
Exercise 2.7:

Draw the truth table of the digital circuit described by the following equation:

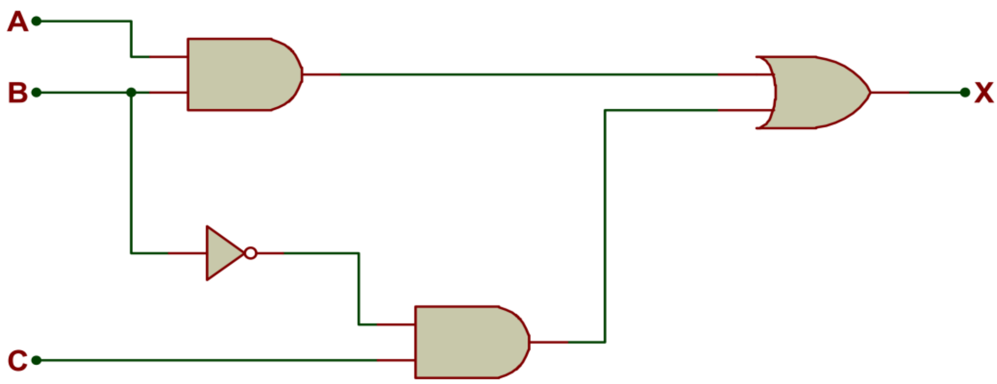
$$X = AB + AB\overline{C} + \overline{AC}$$

Exercise 2.8:

Let us consider the following digital circuit:



a. Give the expression of the output X.
b. Draw the truth table of the circuit.
c. Answer the two previous questions considering the following digital circuit:



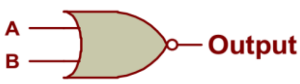
2.4.4 The NOR gate:

The NOR gate is an OR gate with its output inverted.

- **Truth table:**

A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

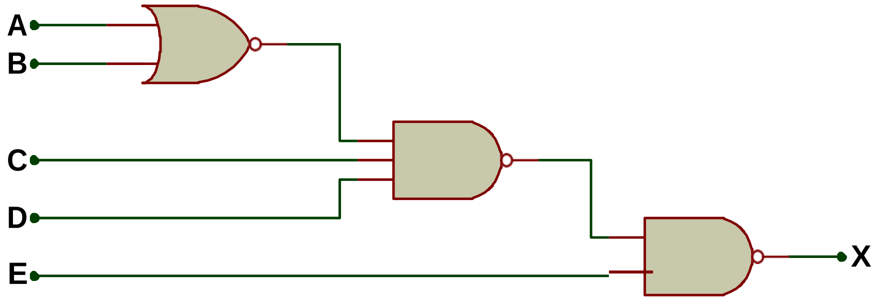
- **Symbol:**



The NOR gate can also be manufactured with more than two inputs.

Exercise 2.9:

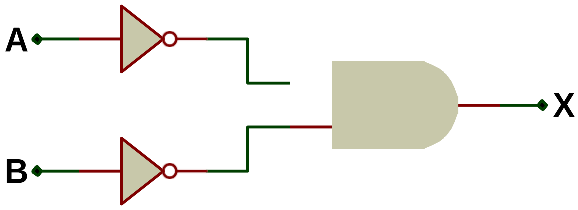
Let us consider the following digital circuit:



a. Give the expression of the output X.
b. Draw the truth table of the circuit.

Remark 2.3: The negative AND gate, the negative OR gate.

Let us consider the following digital circuit:

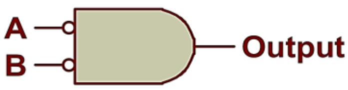


a. Draw the truth table of this circuit.
b. Show that this circuit is equivalent to a NOR gate.

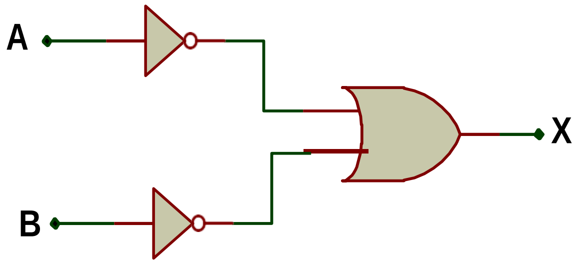
The expression of the output X can be written as follow: $X = \bar{A}.\bar{B}$. Therefore, the truth table of the circuit can be easily deduced:

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

We can notice that the truth table of this circuit is identical to that of a NOR gate. The gate described in this exercise is called the negative AND gate and its symbol is given as follow:



Let us consider the following gate circuit:



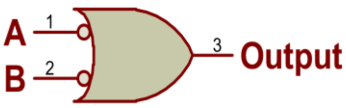
a. Draw the truth table of the circuit.

b. Show that the circuit is equivalent to a NAND gate.

The expression of the output X can be written as follow: $X = \bar{A} + \bar{B}$. Therefore, the truth table of the circuit can be easily deduced:

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

We can notice that the truth table of this circuit is identical to that of a NAND gate. The circuit described in this exercise is called the negative OR gate. Its symbol is given as follow:



Remark 2.4:

The previous remark leads us to two important theorems of the Boolean algebra (the Boolean algebra will be studied in detail in the next chapter). Those theorems are called **De Morgan's** theorems:

$$\overline{A+B} = \overline{A}..\overline{B}$$

$$\overline{A.B} = \overline{A} + \overline{B}$$

Where A and B are two Boolean variables (A Boolean variable is that which can only take values 0 and 1).

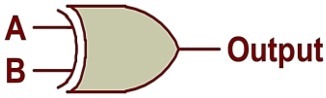
2.4.5 The exclusive-OR gate:

The exclusive-OR gate outputs a high level only if the inputs are at different logic levels, either 0 and 1 or 1 and 0. Conversely, its output is low if the inputs are at the same logic levels. The exclusive-OR gate is sometimes called XOR gate.

- **Truth table:**

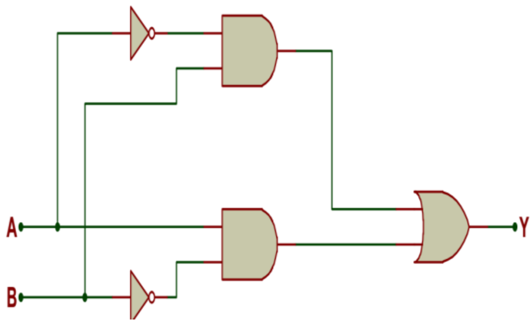
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

- **Symbol:**



Exercise 2.10:

Let us consider following gate circuit:



a. Determine the expression of the output.
b. Deduce the truth table.
c. Conclude.

Remark 2.5:

From the exercise above the following property can be deduced:

$\bar{A}.B + A.\bar{B} = A \oplus B$

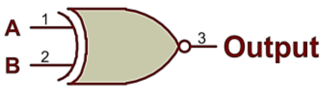
2.4.6 The exclusive-NOR gate:

The exclusive-NOR gate is equivalent to an exclusive OR gate with an inverted output. The truth table is exactly opposite as for the exclusive-OR gate. The exclusive-NOR gate also known as the XNOR gate.

- **Truth table:**

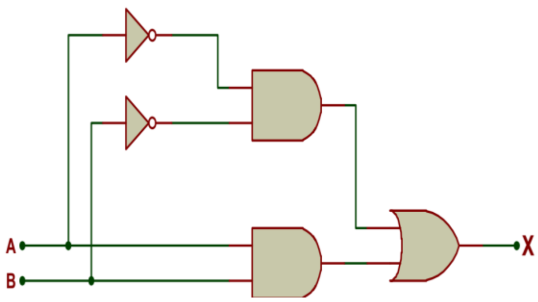
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

- **Symbol:**



Exercise 2.11:

Let us consider the following gate circuit:



a. Determine the expression of the output.

b. Deduce the truth table.

c. Conclude.

Remark 2.6:

From the previous exercise, the following property can be deduced:

$$\overline{A}.\overline{B} + A.B = \overline{A \oplus B}$$

The exclusive-OR and exclusive-NOR gates are very useful for circuits where two or more binary numbers are to be compared bit-for-bit, and also for error detection (parity check).