Chapter Two
Motion in One Dimension

Displacement, velocity and speed

Displacement

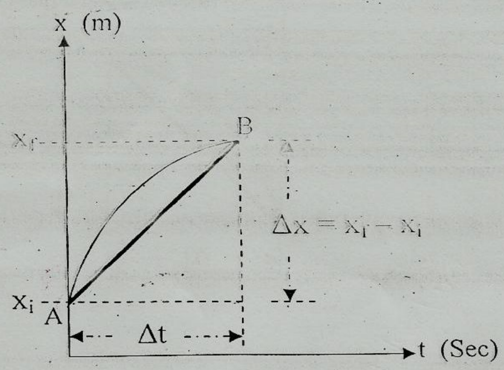
The displacement of a particle is defined as its change in position.

As its moves from as initial position x_i to a final position x_f , its displacement is given by:

$\Delta x = x_f - x_i$

Δx is positive if x_f is greater than x_i .

Δx is negative if x_f is less than x_i .



Average velocity $\overline{v}$:

معدل

Is the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\overline{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$\overline{v}$ is measured in (m/Sec) in SI units.

Average speed:

Is the total distance traveled by an particle divided by the total time it takes to travel that distance.

$$\text{Average Speed} = \frac{\text{total distance}}{\text{total time}}$$

Average speed is a scalar quantity

Instantaneous velocity and speed:

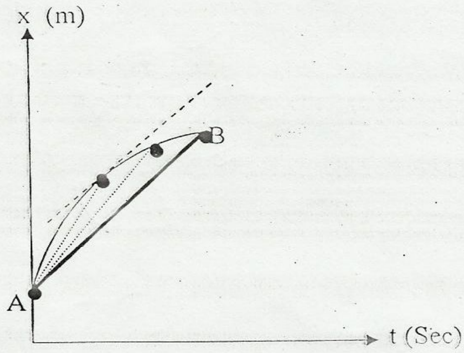
Instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero.

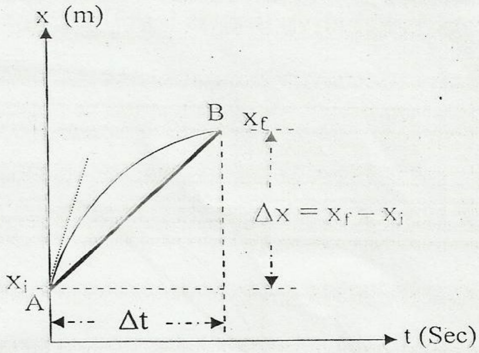
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

This limit is called the derivative of x with respect to t, written $(\frac{dx}{dt})$:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \begin{bmatrix} \text{the instantaneous velocity is the} \\ \text{1 - st derivative of displacement.} \end{bmatrix}$$

The instantaneous speed of a particle is defined as the magnitude of its velocity, and it has no direction associated with it.



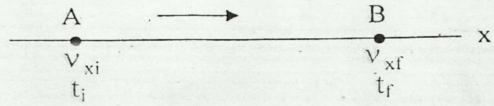


Acceleration:

By the same way we quantify changes in position as a function of time, it is easy to quantify changes in velocity as a function of time.

When the velocity of a particle changes with time, the particle is said to be accelerating.

Suppose a particle moving along the x-axis from A to B has a velocity v_{xi} at time t_i and a velocity v_{xf} at time t_f, the average acceleration of the particle is defined as the change in velocity Δv_x divided by the time interval Δt during which that change occurred:

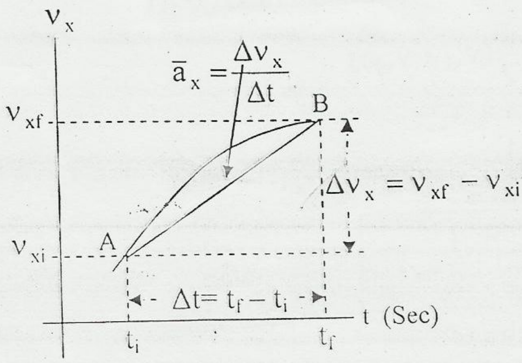


$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

The acceleration is measured in (m/Sec^2) in SI units.

In some situations, the value of the average acceleration may be different over different time intervals, it is therefore useful to define the (instantaneous acceleration) as the limit of the average acceleration as Δt approaches zero. We obtain the instantaneous acceleration:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$ [the instantaneous acceleration equals the derivative of the velocity with respect to time]



$\because \; v_x = \frac{dx}{dt}$, the acceleration a_x is:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$ [the acceleration equals the 2 - nd derivative of displacement with respect to time]

Motion with constant acceleration:

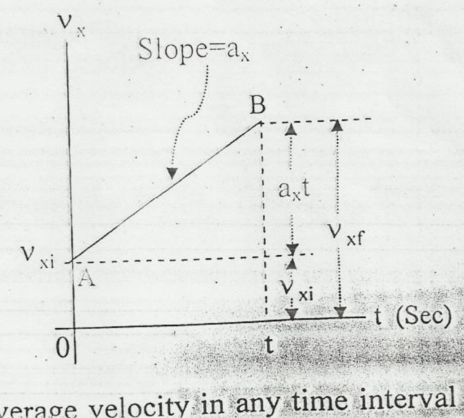
In this case, the average acceleration over any time intervals equals the instantaneous acceleration at any instant within the interval, and the velocity changes at the same rate throughout the motion.

From,

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Replace $\bar{a}_x$ by a_x , and take $t_i = 0$ and t_f to be any later time t , we find that :

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$



Or,

$$\boxed{v_{xf} = v_{xi} + a_x t}$$

At constant acceleration, the average velocity in any time interval is the mean of the initial velocity v_{xi} and the final velocity v_{xf} :