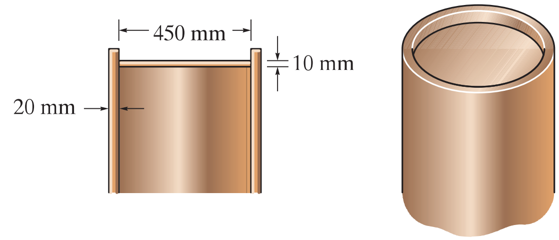
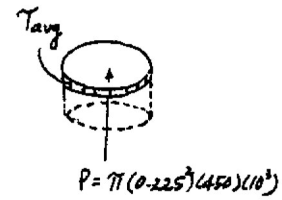
***8–12.** A pressure-vessel head is fabricated by gluing the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the glue and the state of stress in the wall of the vessel.



$+\uparrow \Sigma F_y = 0; \quad \pi(0.225)^2 450(10^3) - \tau_{avg}(2\pi)(0.225)(0.01) = 0;$

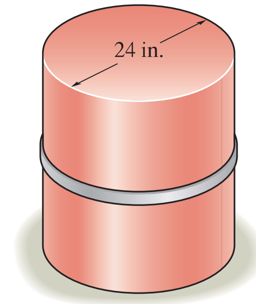


$\tau_{avg} = 5.06$ MPa **Ans.**

$$\sigma_1 = \frac{p\,r}{t} = \frac{450(10^3)(0.225)}{0.02} = 5.06 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{450(10^3)(0.225)}{2(0.02)} = 2.53 \text{ MPa}$$ **Ans.**

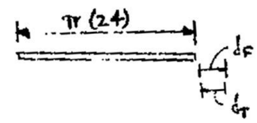
8–13. An A-36-steel hoop has an inner diameter of 23.99 in., thickness of 0.25 in., and width of 1 in. If it and the 24-in.-diameter rigid cylinder have a temperature of 65° F, determine the temperature to which the hoop should be heated in order for it to just slip over the cylinder. What is the pressure the hoop exerts on the cylinder, and the tensile stress in the ring when it cools back down to 65° F?



$$\delta_T = \alpha \, \Delta T L$$

$$\pi(24) - \pi(23.99) = 6.60(10^{-6})(T_1 - 65)(\pi)(23.99)$$

$$T_1 = 128.16^\circ \ F = 128^\circ$$ **Ans.**



Cool down:

$$\delta_F = \delta_T$$

$$\frac{FL}{AE} = \alpha \, \Delta \, TL$$

$$\frac{F(\pi)(24)}{(1)(0.25)(29)(10^6)} = 6.60(10^{-6})(128.16 - 65)(\pi)(24)$$

$$F = 3022.21 \text{ lb}$$

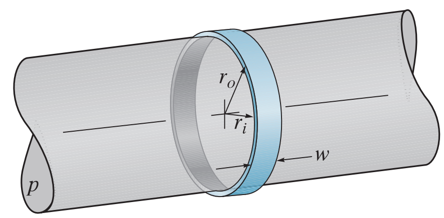
$$\sigma_1 = \frac{F}{A}; \qquad \sigma_1 = \frac{3022.21}{(1)(0.25)} = 12\,088 \text{ psi} = 12.1 \text{ ksi}$$ **Ans.**

$$\sigma_1 = \frac{pr}{t}; \qquad 12\,088 = \frac{p(12)}{(0.25)}$$

$$P = 252 \text{ psi}$$ **Ans.**

Ans:
$T_1 = 128^\circ$, $\sigma_1 = 12.1$ ksi, $p = 252$ psi

8–14. The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in the internal radius of the ring after this pressure is applied. The modulus of elasticity for the ring is E.



Equilibrium for the Ring: From the FBD

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law

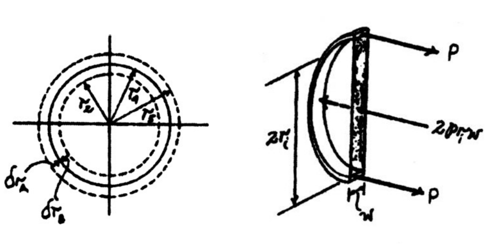
$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad (1)$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}.$

Then, from Eq. (1)

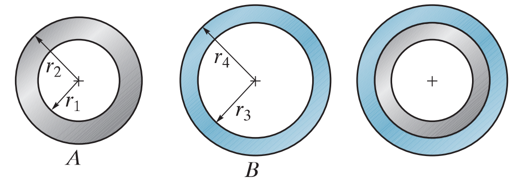
$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)} \qquad \textbf{Ans.}$$



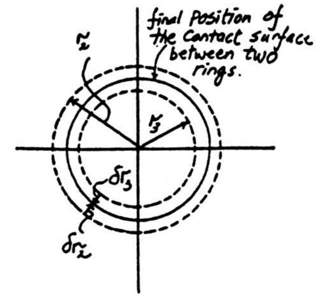
Ans:
$\delta r_i = \dfrac{pr_i^2}{E(r_o - r_i)}$

8–15. The inner ring A has an inner radius r_1 and outer radius r_2. Before heating, the outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.



Equilibrium for the Ring: From the FBD

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2P - 2pr_iw = 0 \qquad P = pr_iw$$



Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_iw}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's law

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad (1)$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}.$

Then, from Eq. (1)

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

Compatibility: The pressure between the rings requires

$$\delta r_2 + \delta r_3 = r_2 - r_3 \qquad (2)$$

From the result obtained above

$$\delta r_2 = \frac{pr_2^2}{E(r_2 - r_1)} \qquad \delta r_3 = \frac{pr_3^2}{E(r_4 - r_3)}$$

Substitute into Eq. (2)

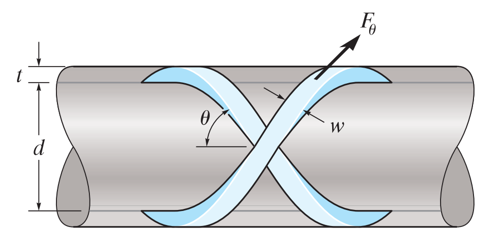
$$\frac{pr_2^2}{E(r_2 - r_1)} + \frac{pr_3^2}{E(r_4 - r_3)} = r_2 - r_3$$

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}} \qquad \textbf{Ans.}$$

Ans:

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}}$$

***8–16.** A closed-ended pressure vessel is fabricated by cross winding glass filaments over a mandrel, so that the wall thickness t of the vessel is composed entirely of filament and an expoxy binder as shown in the figure. Consider a segment of the vessel of width w and wrapped at an angle θ. If the vessel is subjected to an internal pressure p, show that the force in the segment is $F_\theta = \sigma_0 wt$, where σ_0 is the stress in the filaments. Also, show that the stresses in the hoop and longitudinal directions are $\sigma_h = \sigma_0 \sin^2\theta$ and $\sigma_1 = \sigma_0 \cos^2\theta$, respectively. At what angle θ (optimum winding angle) would the filaments have to be would so that the hoop and longitudinal stresses are equivalent?



The Hoop and Longitudinal Stresses: Applying Eq. 8–1 and Eq. 8–2

$$\sigma_1 = \frac{pr}{t} = \frac{p\left(\frac{d}{2}\right)}{t} = \frac{pd}{2t}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{p\left(\frac{d}{2}\right)}{2t} = \frac{pd}{4t}$$

The Hoop and Longitudinal Force for Filament:

$$F_1 = \sigma_1 A = \frac{pd}{2r}\left(\frac{w}{\sin\theta}t\right) = \frac{pdw}{2\sin\theta}$$

$$F_1 = \sigma_2 A = \frac{pd}{4t}\left(\frac{w}{\cos\theta}t\right) = \frac{pdw}{4\cos\theta}$$

Hence,

$$F_\theta = \sqrt{F_b^2 + F_t^2}$$

$$= \sqrt{\left(\frac{pdw}{2\sin\theta}\right)^2 + \left(\frac{pdw}{4\cos\theta}\right)^2}$$

$$= \frac{pdw}{4}\sqrt{\frac{4}{\sin^2\theta} + \frac{1}{\cos^2\theta}}$$

$$= \frac{pdw}{4}\sqrt{\frac{4\cos^2\theta + \sin^2\theta}{\sin^2\theta}}$$

$$= \frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5}$$

$$\sigma_\theta = \frac{F_\theta}{A} = \frac{\frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5}}{wt}$$

$$= \frac{pd}{2\sqrt{2}t}\left(\frac{\sqrt{3\cos 2\theta + 5}}{\sin 2\theta}\right) \qquad \textit{(Q.E.D)}$$

$\frac{d\sigma_\theta}{d\theta} = 0$ when σ_θ is minimum.

$$\frac{d\sigma_\theta}{d\theta} = \frac{pd}{2\sqrt{2}r}\left[-\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) - \frac{3}{\sqrt{3\cos 2\theta + 5}}\right] = 0$$

8–16. (Continued)

$$\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) + \frac{3}{\sqrt{3\cos 2\theta + 5}} = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left(\frac{2\cos 2\theta}{\sin^2 2\theta} + \frac{3}{3\cos 2\theta + 5}\right) = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left[\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)}\right] = 0$$

However, $\sqrt{3\cos 2\theta + 5} \neq 0$. Therefore,

$$\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)} = 0$$

$$3\cos^2 2\theta + 10\cos 2\theta + 3 = 0$$

$$\cos 2\theta = \frac{-10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)}$$

$$\cos 2\theta = -0.3333$$

$$\theta = 54.7^\circ$$ **Ans.**

Force in θ Direction: Consider a portion of the cylinder. For a filament wire the cross-sectional area is $A = wt$, then

$$F_\theta = \sigma_0 wt \qquad \textit{(Q.E.D.)}$$

Hoop Stress: The force in hoop direction is $F_h = F_\theta \sin\theta = \sigma_0 wt \sin\theta$ and the area is $A = \frac{wt}{\sin\theta}$. Then due to the internal pressure p,

$$\sigma_h = \frac{F_h}{A} = \frac{\sigma_0\, wt \sin\theta}{wt/\sin\theta}$$

$$= \sigma_0 \sin^2\theta \qquad \textit{(Q. E. D.)}$$

Longitudinal Stress: The force in the longitudinal direction is $F_1 = F_\theta \cos\theta = \sigma_\theta\, wt \cos\theta$ and the area is $A = \frac{wt}{\sin\theta}$. Then due to the internal pressure p,

$$\sigma_t = \frac{F_h}{A} = \frac{\sigma_0\, wt\cos\theta}{wt/\cos\theta}$$

$$= \sigma_0 \cos^2\theta \qquad \textit{(Q. E. D.)}$$

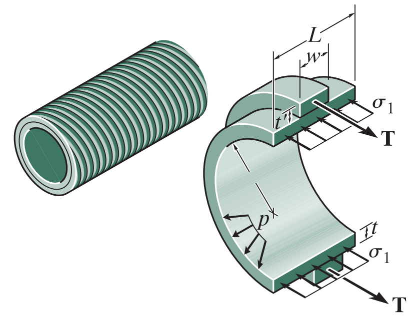
Optimum Wrap Angle: This requires $\frac{\sigma_h}{\sigma_l} = \frac{pd/2t}{pd/4t} = 2$. Then

$$\frac{\sigma_h}{\sigma_l} = \frac{\sigma_0 \sin^2\theta}{\sigma_0 \cos^2\theta} = 2$$

$$\tan^2\theta = 2$$

$$\theta = 54.7^\circ$$ **Ans.**

8–17. In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length L of the vessel.



Normal Stress in the Wall and Filament Before the Internal Pressure is Applied: The entire length L of wall is subjected to pretension filament force T. Hence, from equilibrium, the normal stress in the wall at this state is

$$2T - (\sigma')_w (2Lt) = 0 \qquad (\sigma')_w = \frac{T}{Lt}$$

and for the filament the normal stress is

$$(\sigma')_{fil} = \frac{T}{wt'}$$

Normal Stress in the Wall and Filament After the Internal Pressure is Applied: In order to use $\sigma_1 = pr/t$, developed for a vessel of uniform thickness, we redistribute the filament's cross-section as if it were thinner and wider, to cover the vessel with no gaps. The modified filament has width L and thickness $t'w/L$, still with cross-sectional area wt' subjected to tension T. Then the stress in the filament becomes

$$\sigma_{fil} = \sigma + (\sigma')_{fil} = \frac{pr}{(t + t'w/L)} + \frac{T}{wt'} \qquad \textbf{Ans.}$$

And for the wall,

$$\sigma_w = \sigma - (\sigma')_w = \frac{pr}{(t + t'w/L)} - \frac{T}{Lt} \qquad \textbf{Ans.}$$

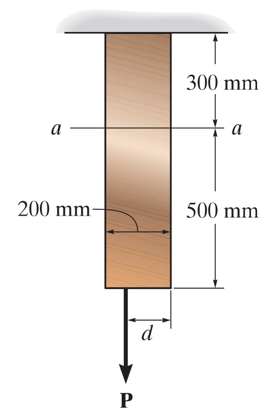
Check: $2wt'\sigma_{fil} + 2Lt\sigma_w = 2rLp$ OK

Ans:

$$\sigma_{fil} = \frac{pr}{t + t'w/L} + \frac{T}{wt'},$$

$$\sigma_w = \frac{pr}{t + t'w/L} - \frac{T}{Lt}$$

8–18. The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the shortest distance d to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the center line of this thickness.



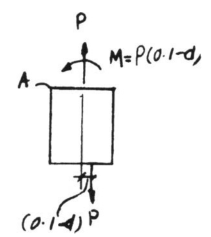
$$\sigma_A = 0 = \sigma_a - \sigma_b$$

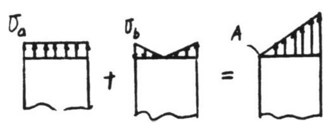
$$0 = \frac{P}{A} - \frac{M\,c}{I}$$

$$0 = \frac{P}{(0.2)(0.01)} - \frac{P(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

$$P(-1000 + 15000\,d) = 0$$

$$d = 0.0667 \text{ m} = 66.7 \text{ mm}$$ **Ans.**





Ans:
$d = 66.7$ mm