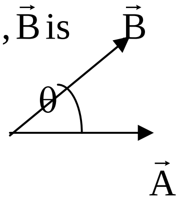
1-6 Multiplying of Vectors

(1) Scalar Product (dot product)

Let $\vec{A}$ and $\vec{B}$ are two vectors. The scalar product of $\vec{A},\vec{B}$ is

$$\vec{A}.\vec{B}=|A||B|\cos\theta \qquad \theta\leq 180$$



Where : $|A|$ is the magnitude of $\vec{A}$

$|B|$ is the magnitude of $\vec{B}$

θ is the angle between $\vec{A}$ and $\vec{B}$

The quantity obtained from the scalar product is a scalar quantity. For example, the work is a scalar quantity obtained from the product of tow vectors (displacement and the component of force in the direction of this displacement); $W=Fd\cos\theta$

Now, from: $\vec{A}.\vec{B}=|A||B|\cos\theta$

(1) if $\theta=0$ then $\cos(0)=1$ and $\vec{A}.\vec{B}=AB$

(2) if $\theta=90$ then $\cos(90)=0$ and $\vec{A}.\vec{B}=0$

(*) Properties of Scalar Product (Dot Product)

(a) $\vec{A}.\vec{B}=\vec{B}.\vec{A}$ (b) $\vec{A}.(\vec{B}+\vec{C})=\vec{A}.\vec{B}+\vec{A}.\vec{C}$

(c) $\vec{A}.\vec{A}=|A|^2$ (parallel vectors) (d) $\vec{A}.(-\vec{A})=-A^2$ (anti parallel vectors)

Using the unit vector $\hat{i},\hat{j},\hat{k}$ we obtain:

$\hat{i}.\hat{i}=|i|\,|i|\cos(0)=(1)(1)(1)=1$ where i=i , j=j , k=k , = 1

$\hat{i}.\hat{j}=|i|\,|j|\cos(90)=(1)(1)(0)=0$ where i.j , j.k , i.k ,= 0

1-7 The Scalar Product of $\vec{A}$ and $\vec{B}$ Analytically

For two vectors $\vec{A}$ and $\vec{B}$ in 3D-space

$$\vec{A}= A_x\hat{i}+A_y\hat{j}+A_z\hat{k} \quad , \quad \vec{B}= B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}.\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}).(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$=A_x\hat{i}.B_x\hat{i}+A_x\hat{i}.B_y\hat{j}+A_x\hat{i}.B_z\hat{k}+A_y\hat{j}.B_x\hat{i}+A_y\hat{j}.B_y\hat{j}+A_y\hat{j}.B_z\hat{k}$$

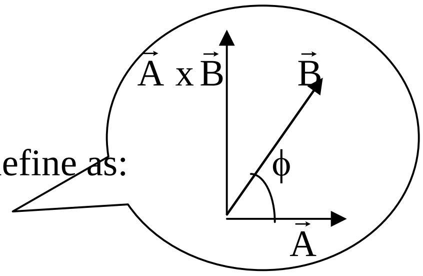
$$+A_z\hat{k}.B_x\hat{i}+A_z\hat{k}.B_y\hat{j}+A_z\hat{k}.B_z\hat{k}$$

$$=A_xB_x(\hat{i}.\hat{i})+\cancel{A_xB_y}(\hat{i}.\hat{j})+\cancel{A_xB_z}(\hat{i}.\hat{k})+\cancel{A_yB_x}(\hat{j}.\hat{i})+A_yB_y(\hat{j}.\hat{j})+\cancel{A_yB_z}(\hat{j}.\hat{k})$$

$$\cancel{A_zB_x}(\hat{k}.\hat{i})+\cancel{A_zB_y}(\hat{k}.\hat{i})+A_zB_z(\hat{k}.\hat{k})$$

$$= A_xB_x+ A_yB_y+ A_zB_z$$

(2) Cross Product (Vector Product)



The vector product for $\vec{A}$ *and* $\vec{B}$ vectors is define as:

$$\vec{A}\,x\,\vec{B}=|A||B|\sin\phi$$

Where ϕ is the angle between the two vectors. The direction of the result $\vec{A}\,x\,\vec{B}$ is obtained according to the right hand rule

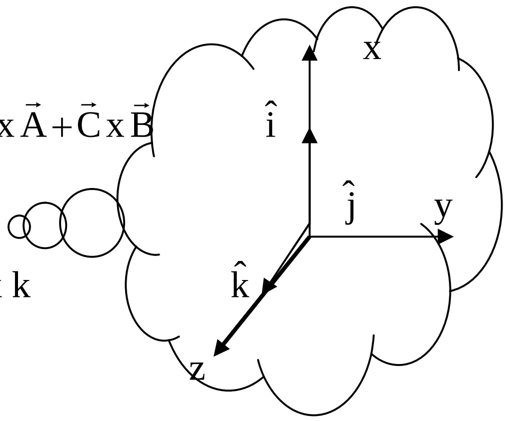
(**)*Properties of Vector Product*

(a) $\vec{A}\,x\,\vec{B}=-\vec{B}\,x\,\vec{A}$ (b) $\vec{C}\,x(\vec{A}+\vec{B})=\vec{C}\,x\,\vec{A}+\vec{C}\,x\,\vec{B}$

Using the unit vectors $\hat{i},\hat{j},\hat{k}$ we obtain:

$\hat{i}\,x\,\hat{i}=|i|\,|i|\sin(0)=(1)(1)(0)=0=j\,x\,j=k\,x\,k$

$\hat{i}\,x\,\hat{j}=\hat{k}\;,\;\hat{j}\,x\,\hat{k}=\hat{i}\;,\;\hat{k}\,x\,\hat{i}=\hat{j}$



1-8 The Vector Product Of $\vec{A}$ and $\vec{B}$ Analytically

For two vectors $\vec{A}$, $\vec{B}$ in 3D-space,where

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \quad , \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\,\hat{i} - (A_xB_z - A_zB_x)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$$

1-9 Triple Product

1-9-1 Triple Scalar Produce

For $\vec{A}, \vec{B}$ *,and* $\vec{C}$, the triple scalar product is define in the form:

$$(\vec{A} \times \vec{B}).\vec{C} = \vec{A}.(\vec{B} \times \vec{C})$$

$$\vec{A}.(\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = ???? \quad \text{(HW)}$$

1-9-2 Triple Vector Produce

This type of produce can be calculated by

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

1-10 Differentiation of Vector

The differential of vectors are the same that used for scalar quantities

Let $\vec{A}(t)$ is a vector as a function to the variable (t), where:

$$\vec{A}(t) = \hat{i}A_x(t) + \hat{j}A_y(t) + \hat{k}A_z(t)$$

Then the differential process due to (t) is:

$$\frac{d\vec{A}}{dt} = \hat{i}\,\frac{dA_x}{dt} + \hat{j}\,\frac{dA_y}{dt} + \hat{k}\,\frac{dA_z}{dt}$$

For two vectors $\vec{A}$ and $\vec{B}$, we can write:

(a) $\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$

(b) $\frac{d}{dt}(m\vec{A}) = \vec{A}\frac{dm}{dt} + m\frac{d\vec{A}}{dt}$

(c) $\frac{d}{dt}(\vec{A} . \vec{B}) = \vec{B} . \frac{d\vec{A}}{dt} + \vec{A} . \frac{d\vec{B}}{dt}$

(d) $\frac{d}{dt}(\vec{A} \, x \, \vec{B}) = \frac{d\vec{A}}{dt} \, x \, \vec{B} + \vec{A} \, x \, \frac{d\vec{B}}{dt}$