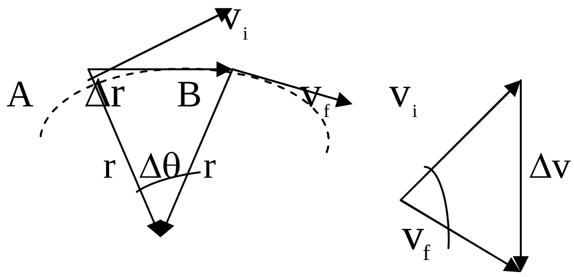
3-4 Uniform Circular Motion

A particle at point A at time t_i, it's velocity is v_i, it's at point B at time t_f. It's velocity at that time is v_f.

v_i, v_f differ only in direction and their magnitudes are the same. To calculate the acceleration of the particle, let us begin with the defining equation of average acceleration.



$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

(1) (2)

Consider the triangle (1) in figure above which has sides Δr and r, this triangle (1) and triangle (2) which has sides Δv and v are similar, then we can write a relation ship between then.

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \Rightarrow \Delta v = \frac{v.\Delta r}{r}$$

Then

$$\vec{a} = \frac{v.\Delta r}{r.\Delta t}$$

The magnitude of the acceleration is

$$a_r = \lim_{\Delta t \to 0} \frac{v\,\Delta v}{r\,\Delta t} = \frac{v}{t} \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{v^2}{r} \;.$$

In uniform circular motion, the acceleration is directed toward the center of the circle and has a magnitude given by $\frac{v^2}{r}$ where r is the radius of the circle and v is the speed of the particle . this acceleration is called a "centripetal acceleration " and is along the radial direction.

3-5 Tangential and Radial Acceleration

Now. Let us consider a particle moving along a curve path where the velocity change both in direction and in magnitude.

The acceleration can be solved into two component vectors; a radial component vector a_r and a tangential component vector a_t, thus a can be written as:

$$a = a_r + a_t$$

The tangential acceleration a_t causes the change in the speed of the particle, it's parallel to the instantaneous velocity. And it's magnitude is

$$\vec{a} = \frac{d|v|}{dt}$$

The radial acceleration arises from the change in direction if the velocity vector.

$$a_r = \frac{v^2}{r}$$

Where r is radius of curvature of the path.

$$a = \sqrt{a_r^2 + a_t^2}$$

in uniform circular motion, where v is constant, a_t =0 and the acceleration is completely radial.

Example(1)

If the poison of particle (r) moving in xy-plane is given by:

$$\vec{r} = (3t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$$

Calculated the position ,velocity and acceleration at t=2sec?

Solution:

$$\vec{r} = (3t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$$

$$\vec{r} = (3(2)^3 - 5(2))\hat{i} + (6 - 7(2)^4)\hat{j}$$

$$= 4\hat{i} + 106\hat{j}$$

$$\vec{v} = \frac{dr}{dt} = (9t^2 - 5)\hat{i} + (-28t^3)\hat{j}$$

$$= (9(2)^2 - 5)\hat{i} + (-28(2)^3)\hat{j}$$

$$= 31\hat{i} - 224\hat{j}$$

$$\vec{a} = \frac{dv}{dt} = (18t)\hat{i} + (-28 * 3t^2)\hat{j}$$

$$= (18(2))\hat{i} + (-28 * 3(2)^2)\hat{j}$$

$$= 36\hat{i} - 336\hat{j}$$