Example(2)

A particle with velocity $\vec{v}_o = -2\hat{i} + 4\hat{j}$ at t=0 undergoes a constant acceleration $\vec{a}$ of magnitude a=3 m/sec at angle $\theta = 130\,f$ from the positive direction of the x-axis, what is the particle's velocity $\vec{v}$ at t=5sec, in unit vector notation and in magnitude angle notation?

Solution:

$$v_x = v_{ox} + a_x t$$

$$v_y = v_{oy} + a_y t$$

$$a_x = a\cos\theta = 1\cos 130 = 1.93 m/sec^2$$

$$a_y = a\sin\theta = 3\sin 130 = 2.30 m/sec^2$$

At t=5sec

$$v_x = -2 + (-1.93)(5) = -11.65 m/sec$$

$$v_y = 4 + 2.30(5) = 15.5 m/sec$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = -11.65\hat{i} + 15.5\hat{j} =$$

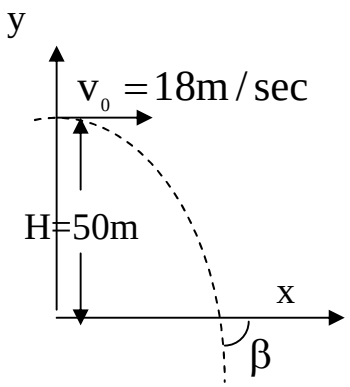
magnitude of $\vec{v}$

$$v = |v| = \sqrt{v_x^2 + v_y^2} \quad = 19 m/sec$$

$$\theta = \tan^{-1}(\frac{v_y}{v_x}) = 127^0$$

Example(3)

Ball kicked horizontally at 8m/sec off a 50m high cliff, find: (1)time to impact (2)speed at impact (3)impact point (4)angle at impact ?



Solution:

$$x(t) = v_0 \cos\theta\, t \qquad ,\theta = 0$$

$$= v_0\, t \qquad \text{.....(1)}$$

$$y(t) = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$= H - \frac{1}{2} g t^2 \quad \text{.....(2)} \qquad ,v_{0y} = 0,\ \ y_0 = H$$

At impact we must have $y = 0$.

Solving Eq.(2)

$$t = T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2*50}{9.81}} = 3.19\,\text{sec}$$

$$x(T) = 18*3.19 = 57.42$$

$$V_x(T)\frac{dx}{dt} = v_0 = 18m/sec \ \text{ (indpendent of time)}$$

$$V_y(T)\frac{dy}{dt} = -gt = -9.81*3.19 = -31.26m/sec$$

$$\tan\beta = \frac{V_y}{V_x} = -60.1^0$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 36.1m/sec \quad \text{(speed)}$$

Example(4)

Gun fired a bullet with velocity 200m/sec by an 40^0 with horizontal, find a velocity and position of a bullet after 20sec and find range and time required to return to ground?

Solution:

$$v_{0x} = v_0 \cos\theta = 200\cos 40 = 153.2 m/sec$$

$$v_{0y} = v_0 \sin\theta = 200\sin 40 = 128.6 m/sec$$

$$v_{0x} = v_x = 153.2 m/sec$$

$$v_y = v_{0x} - gt = 128.6 - 9.8t$$

$$At\ t = 20 sec$$

$$v_y = -67.4 m/sec$$

$$v = \sqrt{v_x^2 + v_y^2} = 167 m/sec$$

$$t = \frac{2v_{0y}}{g} = \frac{2(128.6)}{9.8}$$

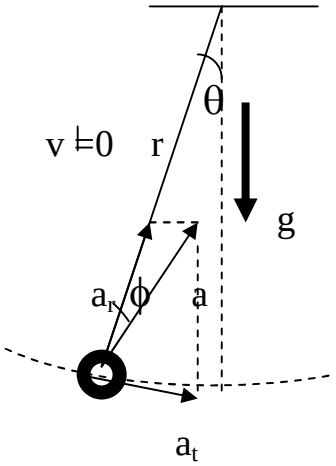
$$R = \frac{v_0^2 \sin 2\theta}{9.8} = \frac{(200)^2 \sin 2(40)}{9.8} = 4021 m$$

$$h = \frac{v_0^2 \sin^2 \theta}{2g} = \frac{(200)^2 (\sin 40)^2}{2*9.8} = 843.7 m$$

Example(5)

A ball tied to the end of a sting 0.50m in length swings in a vertical circle under the influence of gravity as show in figure down, when the string makes an angle $\theta = 20^0$ with the vertical, the ball has a speed of 1.5m/sec. (1) find the magnitude of the radial component of acceleration at this instant (2)what is the magnitude of the tangential acceleration (3) find the magnitude and direction of the total acceleration

Solution:



(1) $a_r = \frac{v^2}{r} = \frac{(1.5\text{m/sec})^2}{0.50\text{m}} = 4.5\text{m/sec}^2$

(2) $a = g\sin\theta = 9.8\sin 20^0 = 3.4\,\text{m/sec}^2$

(3) $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2}\ \text{m/sec}^2 = 5.6\text{m/sec}^2$

If the angle between a and the string, then

$$\phi = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left[\frac{3.4\text{m/sec}^2}{4.5\text{m/sec}^2}\right] = 37^0$$