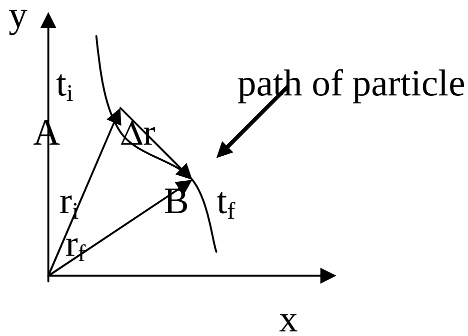
# Chapter Three
# Motion in two dimension

## 3-1 The Displacement, velocity and acceleration

A particle moving in xy-plane is located with the position vector r drawn from the origin to the particle. The displacement of the particle as it moves from A to B in the time interval.

$\Delta t = t_f - t_i$ is equal to the vector

$\Delta r = r_f - r_i$




The instantaneous velocity v is given by:

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

The instantaneous acceleration a is :

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

## 3-2 Two- Dimension motion with constant acceleration

The position vector for particle moving in the xy-plane can be written:

$$r = x\hat{i} + y\hat{j}$$

The velocity of the particle can be obtained from:

$$v = \frac{dr}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$v = v_x\hat{i} + v_y\hat{j}$$

We can apple the eq's of motion to the x and y component of the velocity vector, substituting

$$v_{xf} = v_{xi} + a_x t \quad , \quad v_{yf} = v_{yi} + a_y t$$

We obtain the final velocity at any time t:

$$v_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y)\hat{j}$$
$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x\hat{i} + a_y\hat{j})\, t$$
$$v_f = v_i + a\, t$$

Similarly, from equation $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We know that the x and y coordinates of a particle moving with constant acceleration are:

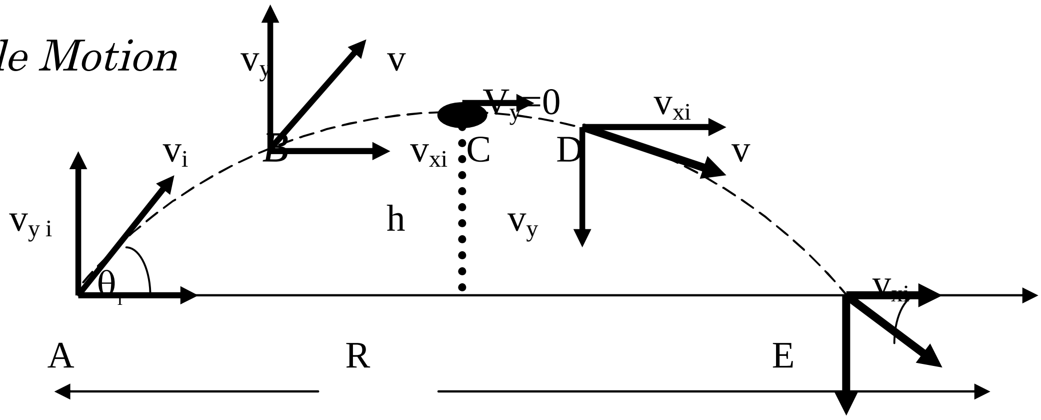
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad , \; y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Obtain:

$$r_f = (x_i + v_{xi}t + \frac{1}{2}a_x t^2)\hat{i} + (y_i + v_{yi}t + \frac{1}{2}a_y t^2)\hat{j}$$
$$= (x_i\hat{i} + y_i\hat{j}) + (v_{xi}\hat{i} + v_{yi}\hat{j})\, t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})\, t^2$$
$$r_f = r_i + v_i\, t + \frac{1}{2}a\, t^2$$

This equation tells us that the displacement vector $\Delta r = r_f + r_i$ is the vector sum of a displacement $v_i t$ arising from the initial velocity of the particle and a displacement $\frac{1}{2}a_x t^2$ resulting from the uniform acceleration of particle

## *3-3 Projectile Motion*




The two assumption with projectile motion are:

(1) The free fell acceleration g is constant over the range of motion and is direction downward.

(2) The effect of air resistance is negligible.

The path of projectile is a parabola.

At t=0, the projectile leaves the origin ($x_i = y_i = 0$) with speed $v_i$ and makes an angle $\theta_i$ with the horizontal

$$\therefore \cos\theta_i = \frac{v_{xi}}{v_i} \Rightarrow v_{xi} = v_i \cos\theta_i \quad \text{.................}(1)$$

$$\sin\theta_i = \frac{v_{yi}}{v_i} \Rightarrow v_{yi} = v_i \sin\theta_i \quad \text{...............}(2)$$

$$\because x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \text{................}(3)$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \quad \text{................}(4)$$

Substituting eq.(1) into eq.(3) with $x_i = 0$ and $a_x = 0$, we find that:

$$x_f = v_{xi}t = (v_i \cos\theta_i)t \quad \text{..................}(5)$$

Substituting eq.(2) into eq.(4) and using $y = 0$ and $a_y = -g$, we obtain:

$$y_f = (v_i \sin\theta_i)t - \frac{1}{2}g\,t^2 \qquad \text{....................(6)}$$

From eq.(5)obtain:

$$x_f = v_i \cos\theta_i\, t \;\Rightarrow t = \frac{x_f}{v_i \cos\theta_i} \qquad \text{..................(7)}$$

Now substitute eq.(7) into eq.(6),this gives:

$$y = \left(\frac{v_i \sin\theta_i}{v_i \cos\theta_i}\right)x - \frac{1}{2}g\left(\frac{x^2}{v_i^2 \cos^2\theta_i}\right) \qquad \text{................(8)}$$

$$y = (\tan\theta_i)x - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x^2 \qquad \text{................(9)}$$

Where $0 < \theta < \frac{\pi}{2}$

This equation is equivalent to parabola form $y = ax - bx^2$

From the expression above, we see that the projectile motion is the superposition of two motion:

(1) Constant velocity motion in the horizontal direction.

(2) Free fall motion in the vertical direction.

From the diagram, the peak point c, which has Cartesian coordinates $(\frac{R}{2}, h)$, and the point E, which has coordinates (R,0).

The distance (R) is called the horizontal range of the projectile, and (h) it's maximum height.

At the peak, $v_{y0} = 0$, then

Use $v_{yf} = v_{yi} + a_y t$ obtain:

$$0 = v_i \sin\theta_i - g\,t_c \Rightarrow t_c = \frac{v_i \sin\theta_i}{g} \quad \text{..............(10)}$$

Substituting eq.(10) into eq.(4) and replacing $y_f = y_c$ with (h) , we obtain:

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{.......................(11)}$$

Now, use eq.(3) and setting $R \cong x_f$ at $t = t_c$ , we find

$$R = (v_i \cos\theta_i)\,2t_c = (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g}$$

$$\therefore R = \frac{v_i^2 \sin 2\theta}{g} \quad \text{.......................(12)}$$

The maximum value of R from eq.(12) is

$$R_{max} = \frac{v_i^2}{g}$$

where $\sin 2\theta = 1$ when $2\theta = 90^0$ therefore R is maximum when $\theta_i = 45^0$.