University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Fraction

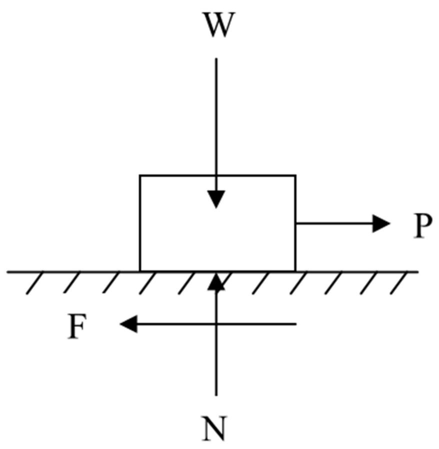
Undergraduate Level, 1st Stage

Mr. Waleed Ali Tameemi
College of Engineering/ Babylon University
M.Sc. Civil Engineering/ the University of Kansas/ USA

2016-2017

1.0 Fraction

Fraction or fractional force can be define as the force that tend to oppose the movement or the tendency of movement of a rigid body on a non-smooth surface. When the external force on a body is larger than the fractional force, the body will move (**Limiting Fraction**). Whereas, when the applied force is less than the fractional force, the body remains at rest (**Static Fraction**)



Where:

W: the weight of the rigid body,

P: the external force,

F: the fraction Force,

N: the surface reaction.

<u>Law of Fraction</u>

- Experiments show that there is a relationship between the fractional force (F) and the reaction force (N) and as follows:

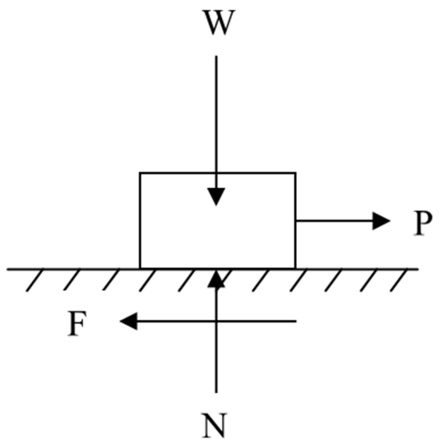
Coefficient of fraction $\mu = \frac{F}{N}$

- The direction of the fractional force is always opposite to the direction of movement.
- The magnitude of the fractional force depends on the roughness/smoothness of the surface. As the roughness of the surface increase the fraction increase and as the smoothness of the surface increase the fraction decrease.

Example 1

If the body of weight (W= 200 N) shown below is resting on a rough surface with coefficient of fraction ($\mu = 0.25$), calculate:

1. The normal reaction (N)
2. The fractional force (F)
3. The Maximum pulling force (P) keeps the body in equilibrium.



Solution:

1. The normal reaction (N)

$\sum F_y = 0$ $\quad\quad\quad\quad \uparrow \ (+v \ .)$

$N - W = 0$

$N = W = 200\ N\ \uparrow$ $\quad\quad\quad\quad$ *(1)*

2. The fractional force (F)

By using the fraction Law:

Coefficient of fraction $\mu = \frac{F}{N} \leftrightarrow 0.25 = \frac{F}{200}$

$$F = 200 \times 0.25 = 50\ N \ \leftarrow$$

3. The Maximum pulling force (P) keeps the body in equilibrium

$\sum F_x = 0 \qquad\qquad \rightarrow \ (+ve.)$

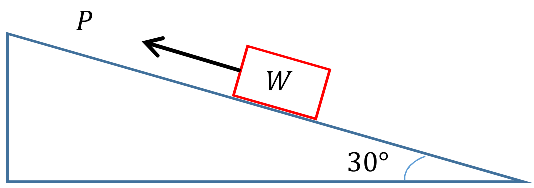
$P - F = 0$

$P = 50\ N$

Example 2

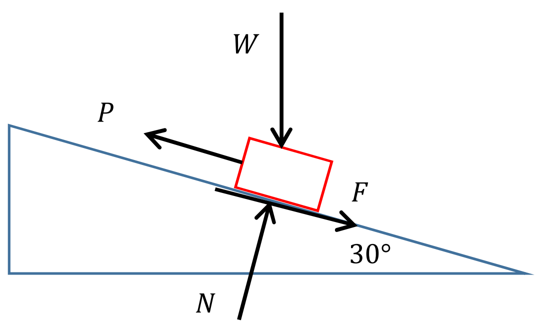
If the body (weight W= 500 N) shown below is resting on an inclined (with angle $\emptyset = 30°$) rough surface with coefficient of fraction ($\mu = 0.25$), calculate:

1. The reaction (N) that is normal to the inclined surface.
2. The fractional force (F).
3. The Maximum pulling force (P) keeps the body in equilibrium.



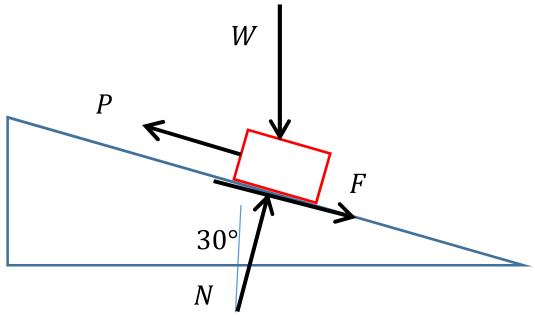
Solution:

Draw free-body diagram for the force system:



Since the reaction force (N) is normal (perpendicular) to the inclined surface and the vertical axis (y-axis) is perpendicular to the horizontal axis (x-axis), then the angle of the inclined surface with the horizontal axis (30°) is equal to the angle of the reaction force (N) with the vertical axis (30°).

In other words,



By using the fraction Law:

Coefficient of fraction $\mu = \frac{F}{N} \leftrightarrow 0.25 = \frac{F}{N}$

$$F = 0.25N \quad (1)$$

$\sum F_y = 0 \qquad \uparrow \ (+ve.)$

$$N\cos 30 + P\sin 30 - W - F\sin 30 = 0 \quad (2)$$

Substitute W and F values into equation # 2

$$N\cos 30 + P\sin 30 - 500 - 0.25N \times \sin 30 = 0$$

$$N \cos 30 + P \sin 30 - 500 - 0.25N \times \sin 30 = 0$$

$$P = \frac{500 - 0.74N}{sin30} = 1000 - 1.48N \qquad (3)$$

$$\sum F_x = 0 \qquad \rightarrow (+ve.)$$

$$N \sin 30 - P \cos 30 + F \cos 30 = 0 \qquad (4)$$

Substitute P and F values into equation # 4 and solve for N

$$N \sin 30 - (1000 - 1.48N) \times \cos 30 + (0.25N) \times \cos 30 = 0$$

$$0.5N - 866 + 1.28N + 0.22N = 0$$

$$0.5N + 1.28N + 0.22N = 866$$

$$2N = 866$$

$N = 433$ Newton [The reaction (N) that is normal to the inclined surface]

Substitute N value into equation #1 and solve for F:

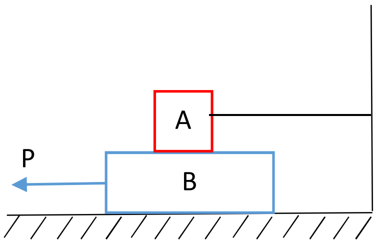
$F = 0.25 \times 433 = 108.25$ Newton [The fractional force (F)]

Substitute N value into equation #3 and solve for P:

$P = 1000 - 1.48 \times 433 = 359$ Newton [The Maximum pulling force (P) keeps the body in equilibrium].

Example 3

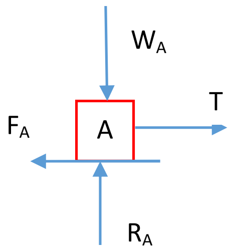
Body A which is tied to a rigid wall using a steel cable (weight 500N) is located on body B (weight 1000N) that rest on rough surface (coefficient of fraction is 1/4). If the fraction coefficient between body A and B is equal to 1/3, calculate horizontal force P.



Solution

Body A

Draw free-body diagram for body A:



$\sum F_y = 0 \qquad \uparrow \ (+ve.)$

$R_A - W_A = 0$

$R_A = W_A = 500\ N \ \uparrow$

By using the fraction Law:

Coefficient of fraction $\qquad \mu = \frac{F_A}{R_A} \leftrightarrow \frac{1}{3} = \frac{F_A}{500}$

$F_A = \frac{500}{3} = 166.7\ N \quad \leftarrow$

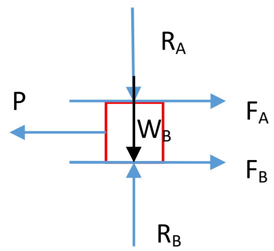
$\sum F_x = 0 \qquad\qquad \rightarrow\ (+ve.)$

$T - F_A = 0$

$T = F_A = 166.7\ N \quad \rightarrow$

<u>Body B</u>

Draw free-body diagram for body B:



$\sum F_y = 0 \qquad\qquad \uparrow\ (+ve.)$

$R_B - R_A - W_B = 0$

$R_B = W_B + R_A = 1000 + 500 = 1500\ N \ \uparrow$

<u>By using the fraction Law:</u>

Coefficient of fraction $\qquad \mu = \frac{F_A}{R_A} \leftrightarrow \frac{1}{4} = \frac{F_B}{1500}$

$F_A = \frac{1500}{4} = 750\ N \quad \rightarrow$

$\sum F_x = 0 \qquad\qquad \rightarrow\ (+ve.)$

$$F_A + F_B - P = 0$$

$$P = F_A + F_B = 166.7 + 750 = 916.7\ N \quad \leftarrow$$