- Opposite to the direction of the motion
- μ_k depends on the nature of the materials, μ_k independent of v over wide range
- Friction force on each interacting body is opposite in direction to the motion of that body relative to the other.

Static Friction

Frictional forces also acts between surfaces that at rest (no relative motion)

- Object at the rest require a non-zero force to start them moving

  $f_s \leq \mu_s N$

  μ_s coefficient of static friction

- Force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) up to a maximum due to $\mu_s N$. Equality sign holds when motion is about to start
- Proportional to normal force
- Independent of area
- Opposite to the lateral push that tries to move the body
- Usually $\mu_s > \mu_k$, so that once block starts moving it will take less force to keep it from acceleration
- μ_s depends on nature and condition of surfaces

5.13 Problem- Solving Strategy

1. Draw a diagram indicating all key features in the problem
2. Draw one or more free body diagram for the objects. For the chosen object include all the forces acting on it. Do not include any internal forces. Do not include any forces exerted by the body on some other body
3. Select a coordinates system and show it in the free- body diagram. Determine components of the forces with reference to these axis as x-axis can choose different reference frame for each body all must be "inertial"
4. If there are geometrical relationship between two or more bodies- relate these algebraically
5. Write down Newton's Eq. of motion for each body and solve for unknown.

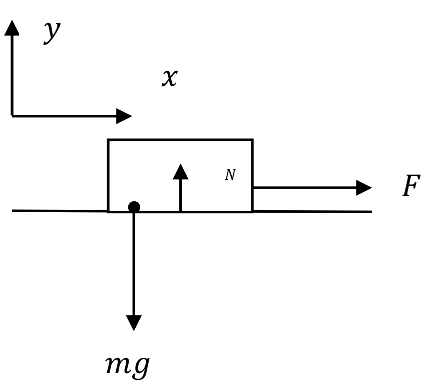
$\vec{F} = m\vec{a}$

$\sum F_x = ma_x$

$\sum F_y = ma_y$ $\quad\quad \begin{Bmatrix} \vec{F} = constant \\ \vec{a} = constant \end{Bmatrix}$

$\sum F_z = ma_z$

Check special cases and extreme value of quantities, compare with intuitive expectations. Does the result make sense?



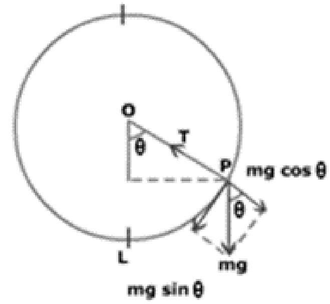
Example: y-axis $N - mg = ma_y$

$$N = mg$$

x-axis $F = ma_x$

$$\therefore a_x = \frac{F}{m}$$

5.15 Motion in a Vertical Circle



For a particle moving with speed v in a circle of radius R, the centripetal acceleration is:

$$a_c = \frac{v^2}{R}$$

Let ball whirled in a vertical circle about point o, then motion circular but is not uniform speed increases on the way down, decreases on the way up.

v changes continuously around the path, we must $a_\perp$ and $a_\parallel$. Forces and ball are gravity and tension

$$F_\parallel = mgsin\theta \qquad (1)$$

$$F_\perp = T - mgcos\theta \qquad (2)$$

The tangential acceleration

$$a_\parallel = \frac{F_\parallel}{m} = gsin\theta \qquad (3)$$

$$a_\perp = \frac{F_\perp}{m} = \frac{T - mgcos\theta}{m} = \frac{v^2}{R} \qquad (4)$$

Solve (4) for $T = m\left(\frac{v^2}{R} + gcos\theta\right)$, lowest point $\theta = 0$

$F_\parallel = 0$, $a_\parallel = 0$, acceleration is purely radial

$$T = m\left(\frac{v^2}{R} + g\right)$$

The highest point $\theta = 180 \Longrightarrow T = m\left(\frac{v^2}{m} - g\right)$

If speed equals a critical value v_c, tension vanishes $T = 0$

$$0 = m\left(\frac{v_c^2}{R} - g\right) \Longrightarrow v_c^2 = \sqrt{Rg}$$

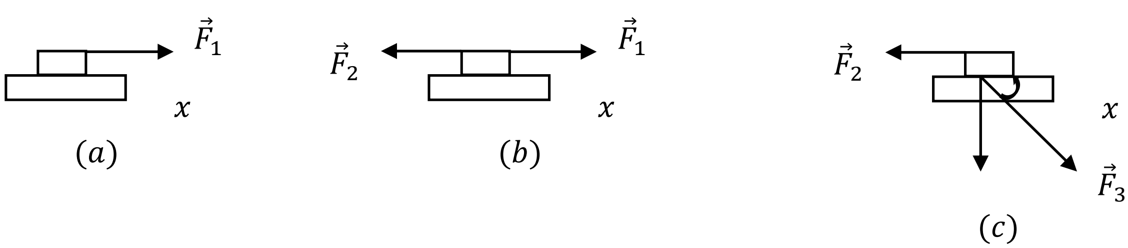
Example (1): A satellite in a circular orbit around earth, at an altitude (h) of 520Km and with a constant speed v of $7.6\left(\frac{Km}{s}\right)$. Satellite's mass is 790Km, what is his acceleration, and what force does earth exert on satellite?

Solution:

$$a_c = \frac{v^2}{R} = \frac{v^2}{R+h} = \frac{(7.6\times10^3)^2}{6.37\times10^6+0.52\times10^6} = 8.38\left(\frac{m}{s^2}\right)$$

$$F = ma_c = 790 \times 8.38 = 6620N$$

Example (2): In Figures a to c, one or two forces act on a puck that moves over frictionless ice along on x-axis, in one dimension motion. The puck's mass $m = 0.20Kg$. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitude $F_1 = 4N,\quad F_2 = 2N$. Forces $\vec{F}_3$ is directed at angle $\theta = 30$ and has magnitude $F_2 = 1N$. In each situation, what is the acceleration of the puck?



In Fig (a) $F_1 = ma_x \Rightarrow a_x = \frac{F_1}{m} = \frac{4}{0.20} = 20\left(\frac{m}{s^2}\right)$

In Fig (b) $F_1 - F_2 = ma_x \Rightarrow a_x = \frac{F_1-F_2}{m} = \frac{4-2}{0.20} = 10\left(\frac{m}{s^2}\right)$

In Fig (c) $F_{3x} - F_2 = ma_x \Rightarrow F_3cos\theta - F_2 == ma_x \Rightarrow a_x = -5.7\left(\frac{m}{s^2}\right)$

Example (3): $F = 2N$ is applied to block A which is contact with B. Identify all reaction-action pairs

$m_A = 1Kg$

$m_B = 2Kg$

$\vec{a} =?$ [Acceleration]

$\vec{F}_{AB} =?$ [Contact force]

Choose coordinate system, x-axis positive to night. Apply Newton's 2nd law for motion of B

$$F_{AB} = m_B a_B \qquad (1)$$

Forces acting an A

$$F - F_{AB} = m_A a_A \qquad (2)$$

Since both block stay in contact

$\therefore a_A = a_B = a$ [constrain in problem]

Add Eq. (1) +(2)

$$F - F_{AB} + F_{AB} = (m_A + m_B)a$$

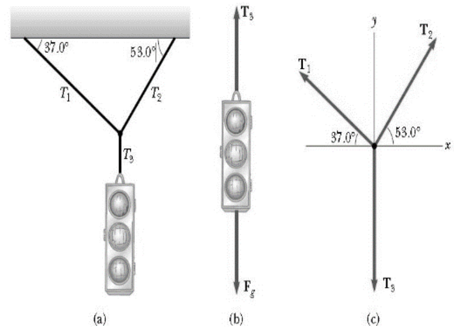
$|F_{AB}| = |F_{BA}|$ magnitude equal due to Newton 3rd law

$\therefore F = (m_A + m_B)a$ single mass $M = (m_A + m_B)$ an which only external force F is important

$$a = \frac{F}{m_A + m_B} = \frac{2N}{(1+2)Kg} = \frac{2}{3}\left(\frac{m}{s^2}\right)$$

Contact Force: $F_{AB} = m_B a_B = m_B a = 2 \times \frac{2}{3} = \frac{4}{3} N \neq F$

Example (4): A traffic light weighing 125N hangs from a cable to two other cables fastened to a support. The upper cables makes angle 37.0° and 53.0° with the horizontal. Find the tension in the three cables?



Solution: In Fig (b) the force T_3 exerted by the vertical cable supports the light, and so Next, $T_3 - F_g = 125\, N$. We choose the coordinate axes shown in Fig (c) and resolve the forces acting on the knot into their components:

$$\sum F_x = -T_1 cos 37.0^{\circ} + T_2 cos 53.0^{\circ} = 0 \qquad (1)$$

$$\sum F_y = T_1 \sin 37.0^{\circ} + T_2 \sin 53.0^{\circ} + (-125) = 0 \qquad (2)$$

From (1) we see that the horizontal components of T_1 and T_2 must be equal in magnitude, and from (2) we see that the sum of the vertical components of T_1 and T_2 must balance the weight of the light. We solve (1) for T_2 in terms of T_1 to obtain

$$T_2 = T_1 \left(\frac{\cos 37.0^{\circ}}{\cos 53.0^{\circ}}\right) = 1.33 T_1$$

This value for T_2 is substituted into (2) to yield

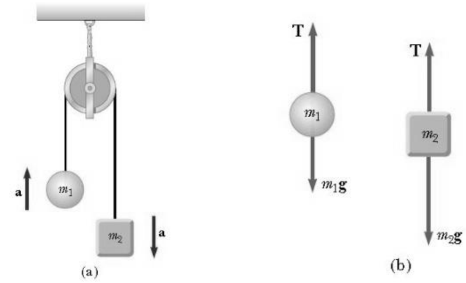
$$T_1 \sin 37.0^{\circ} + (1.33 T_1)(\sin 53.0^{\circ}) - 125 = 0$$

$$T_1 = 75.1 N$$

$$T_2 = 1.33 T_1 = 99.9 N$$

Example (5): When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as shown in Fig (a), the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to measure the freefall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

Solution: When the Newton's 2nd law is applied to object 1, we obtain



$\sum F_y = T - m_1 g = m_1 a_1$ (1)

Similarly, for object 2 we find

$\sum F_y = m_2 g - T = m_2 a_2$ (2)

When (2) is added to (1), T drops out and we get

$-m_1 g + m_2 g = m_1 a_1 + m_2 a_2$

When (3) is substituted into (1), we obtain

$a_y = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$ (3)

$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$ (4)