Gravity is a conservative, spring is a conservative forces. Friction is not a conservative force since it always opposes motion.

$\oint \vec{F}\,.\,d\vec{r} \neq 0$ [friction]

Potential energy:

$P.E$: Gravity (near earth)

1. Total mechanical energy $(E = K + U = \frac{1}{2}mv^2 + mgz)$
2. $P.E$: spring

Total mechanical energy for the mass-spring system is:

$(E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}mx^2)$

3. $P.E$: Gravitational (general)

Total mechanical energy

$(E = K + U = \frac{1}{2}mv^2 - \frac{GM_E m}{r}$

If several conservative forces acting on an object: $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_C$

$W_{total} = \int \vec{F}_A\,.\,d\vec{r} + \int \vec{F}_B\,.\,d\vec{r} + \int \vec{F}_C\,.\,d\vec{r}$

$U = U_A + U_B + U_C$ [sum all the individual $P.E$ for each force]

6.9 Non-Conservative Force

If non-conservative forces act on an object, then the changes in the $KE + PE$ of the conservative force will be equal to the work done by the friction force

$\Delta K + \Delta U = W_{Friction}$

[$\Delta K \equiv$ change in KE, $\Delta U \equiv$ change in PE]

$E_1 = K_1 + U_1$, $E_2 = K_2 + U_2$

$(E_2 - E_1) = W_{Friction}$

$\frac{1}{2}mv_2^2 + U(x_2) - \frac{1}{2}mv_1^2 + U(x_1) = \int_{x_1}^{x_2} P dx$

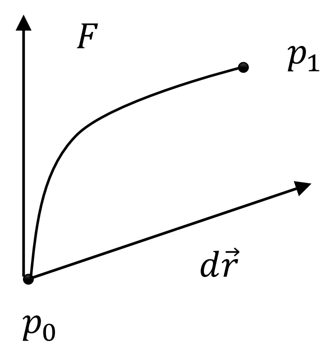
6.10 Force ↔ Potential Energy

We have seen how to calculate the PE given a conservation force

$U(P_1) - U(P_0) = -\int_{P_0}^{P_1} \vec{F} . d\vec{r}$

We can calculate the force given the PE, assume P_0 and P_1 are related by the infinitesimal displacement $d\vec{r}$, then refferentainting expression for $U(P_1)$:

$dU = U(P_1) - U(P_0) = -\vec{F}.d\vec{r}$ [inverse of PE]



$dU = -F_x dx - F_y dy - F_z dz$

Assume displacement is only along x,

$dy = 0$, $dz = 0$

$dU = -F_x dx$ or $F_x = \frac{-dU}{dx}$ (different keeping y, z constat)

$$\begin{Bmatrix} F_x = -\frac{\partial U}{\partial x} \\ F_y = -\frac{\partial U}{\partial y} \\ F_z = -\frac{\partial U}{\partial z} \end{Bmatrix} \text{ combining } F = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right)$$

$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (vector operator]

Example: $U(x,y) = Ax^2y^2$

$$F_x = -\frac{\partial U}{\partial x} = -2Axy^2 \ , \ F_y = -\frac{\partial U}{\partial y} = -2Ax^2y$$

6.11 Energy Curves

Assume we know the PE curve for a particle moving in one- dimension what is the description of the particle motion as a function of time?

Consider conservative forces only. Then the total mechanical energy is a constant of the motion

$$E = K + U = constant$$

$$= \frac{1}{2}mv^2 + U(x) = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + U(x)$$

Solve for $v(x)$

$$v(x) = \frac{dx}{dt} = \sqrt{\frac{2}{m}[E - U(x)]}$$

$$\int_{\hat{x}=x_0}^{x} \frac{d\acute{x}}{\sqrt{\frac{2}{m}[E-U(\acute{x})]}} = \int_{\acute{t}=0}^{t} d\acute{t}$$ where $\hat{x} = x_0$ at $\acute{t} = 0$

6.12 Power

Power is defined as the time rate of doing work. If an external force applied to an object does an amount of work ΔW in the time internal Δt the average power is:

$$\bar{P} = \frac{\Delta W}{\Delta t}$$

Instantaneous power, P, is the limiting value of the average power as Δt approaches zero

$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$ $[P] = \left(\frac{J}{s}\right) = watts$

British: $1hp = 754.7w$

We can express the power in terms of the force acting and the velocity of the object. For a small displacement $d\vec{r}$ the work done

$$dW = \vec{F}.d\vec{r}$$

$$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$$

Energy ↔ Power

$E = \int_{t_1}^{t_2} Pdt = Pt$ [constant P]

$$1Kwh = 3.6 \times 10^6 J$$

$$1Kilocalorie = 4.187 \times 10^3 J$$

$$1Btu = 1.05510^3 J$$

Example (1): The resultant force on an object of mass $F = F_0 - kt$, find the acceleration velocity and position at any time, where F_0 and k are constant and t is the time

Solution: $F = F_0 - kt \Rightarrow a = \frac{F}{m} = \frac{F_0 - kt}{m}$

$$a = \frac{dv}{dt} = \frac{F_0 - kt}{m} \Longrightarrow dv = \frac{(F_0 - kt)}{m} dt$$

$$v = \int dv = \frac{1}{m} \int (F_0 - kt) dt = \frac{1}{m}\left[F_0 t - \frac{kt^2}{2}\right]$$

$$v = \frac{dx}{dt} \Longrightarrow dx = vdt \Longrightarrow x = \int dx = \int vdt$$

$$x = \frac{1}{m}\int \left[F_0 t - \frac{kt^2}{2}\right] dt$$

$$x = \frac{1}{m}\left[F_0 \frac{t^2}{2} - \frac{kt^3}{3}\right]$$

Example (2): A particle of mass 2Kg initially at rest, is acted on by force F=6t, calculated the work done by the force on the particle at 2s.

Solution:

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{6t}{2} = 3t\left(\frac{m}{s^2}\right)$$

$$a = \frac{dv}{dt} = 3t \Rightarrow \int_{v_0}^{v} dv = \int_0^t 3t dt \Rightarrow v - v_0 = 1.5t^2$$

$$v = 1.5t^2 \left(\frac{m}{s}\right) \quad [v_0 = 0]$$

$$v = \frac{dx}{dt} = 1.5t^2 \Rightarrow dx = 1.5t^2 dt$$

$$W = \int F dx = \int_0^t 6t(1.5t^2 dt) = 2.25t^4 J \quad \text{at } t = 2s$$

$$W = 36J$$

Example (3): Calculate the potential energy associated with the following central forces

1. $F = -kr$

2. $F = \frac{k}{r^2}$

Solution:

1. $F = -\frac{dE_p}{dr} = -kr$

$$dE_p = -krdr$$

Integrating, we obtain

$$E_p = -\int krdr = -\frac{1}{2}kr^2 + c$$

Let at $r = 0$, $E_p = 0$ so that $c = 0$

$$E_p = -\frac{1}{2}kr^2 \text{ or } E_p = -\frac{1}{2}k(x^2 + y^2 + z^2)$$

2. $F = -\frac{dE_p}{dr} = \frac{k}{r^2}$

$$dE_p = -k\left(\frac{dr}{r^2}\right)$$

Integrating, we obtain

$$E_p = \int -k\frac{dr}{r^2} = \frac{k}{r} + c$$

Let at $r = \infty$, $E_p = 0$ so that $c = 0$

$$E_p = \frac{k}{r}$$

Example (4): A body whose mass is 2Kg is moving on smooth horizontal surface under the action of a horizontal force $F = 55 + t^2$ calculate the velocity of the mass when $t = 5s$ the body was the rest when $t = 0$

Solution:

$$a = \frac{dv}{dt} = \frac{F}{m} = \left(\frac{55+t^2}{m}\right)dt$$

$$v = \int dv = \frac{1}{m}\int 55 + t^2\, dt = \frac{55t}{m} + \frac{t^3}{3m} + c$$

At $t = 5s \Longrightarrow v = 0 \Longrightarrow t = 0$

$$v = \frac{55\times5}{2} + \frac{(5)^3}{3\times2} = 13533\left(\frac{m}{s}\right)$$