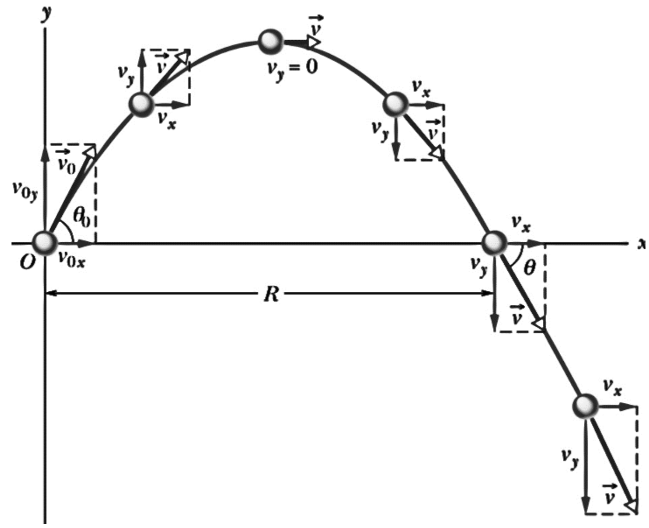



all these specific result (height, time, range) apply only if launch and impact points are at the same height, y. Special cases must be treated carefully

maximum Range occurs at angle $\theta = 45$, the maximum value of $sin2\theta = 1$

$$R = \frac{v_0^2 sin2\theta}{g} = \frac{v_0^2}{g}$$

**Example (1):** The coordinates of a particle moving in the xy-plane are given by

$$x = 1 + 2t^2 \qquad (m)$$

$$y = 2t + t^3 \qquad (m)$$

Find the particle's position, velocity and acceleration at time $t = 2s$

**Solution:**

$t = 25$ , $x = 1 + 2(2)^2 = 9m$

$$y = 2(2) + (2)^3 = 12m$$

$$\vec{r} = 9\hat{\imath} + 12\hat{\jmath}$$

distance from origin

$$r = \sqrt{x^2 + y^2} = \sqrt{9^2 + 12^2} = 15m$$

$$tan\theta = \frac{x}{y} \Longrightarrow \theta = tan^{-1}\frac{y}{x} = tan^{-1}\frac{12}{9} = 53.1^{\circ}$$

The velocity

$$v_x = \frac{dx}{dt} = 4t \left(\frac{m}{s}\right) \qquad v_y = \frac{dy}{dt} = 2 + 3t^2 \left(\frac{m}{s}\right)$$

At $t = 2s$ $\quad v_x(2) = 8\left(\frac{m}{s}\right)$ $\quad v_y(2) = 14\left(\frac{m}{s}\right)$

$\vec{v}(t = 2) = 8\hat{\imath} + 14\hat{\jmath}$

$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 14^2} = 16\left(\frac{m}{s}\right)$

$\theta_v = tan^{-1}\frac{v_y}{v_x} = tan^{-1}\frac{14}{8} \Longrightarrow \theta_v = 60.3^{\circ}$

Acceleration

$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = 4\left(\frac{m}{s^2}\right)$ $\quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} = 6t\left(\frac{m}{s^2}\right)$

At $t = 2s$ $\quad a_x = 4\left(\frac{m}{s^2}\right)$ $\quad a_y = 12\left(\frac{m}{s^2}\right)$

$\vec{a} = 4\hat{\imath} + 12\hat{\jmath}$

$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 12^2} = 12.6\left(\frac{m}{s^2}\right)$

$\theta_a = tan^{-1}\frac{a_y}{a_x} = tan^{-1}\frac{12}{4} \Longrightarrow \theta_a = 71.6^{\circ}$

**Example (2):** if the position of a particle $(\vec{r})$ moving in xy-plane is given by

$\vec{r} = (3t^3 - 5t)\hat{\imath} + (6 - 7t^4)\hat{\jmath}$ Calculate $\vec{r}, \vec{v}$ and $\vec{a}$ at $t = 2s$?

**Solution:**

$\vec{r} = (3t^3 - 5t)\hat{\imath} + (6 - 7t^4)\hat{\jmath}$

$\vec{r} = (3(2)^2 - 5(2))\hat{\imath} + (6 - 7(2)^2)\hat{\jmath}$ $\quad at\ \ t = 2$

$\vec{r} = 4\hat{\imath} - 106\hat{\jmath}$

$\vec{v} = \frac{dr}{dt} = (9t^2 - 5)\hat{\imath} + (-28t^3)\hat{\jmath}$

$\vec{v} = (9(2)^2 - 5)\hat{\imath} + (-28(2)^3)\hat{\jmath}$ $\quad at\ \ t = 2$

$\vec{v} = 31\hat{\imath} - 224\hat{\jmath}$

$\vec{a} = \frac{d^2r}{dt^2} = \frac{dv}{dt} = (18t)\hat{\imath} + (-28 \times 3t^2)\hat{\jmath}$

$\vec{a} = (18 \times 2)\hat{\imath} + (-28 \times 3(2)^2)\hat{\jmath}$ $\qquad at\ \ t = 2$

$\vec{a} = 36\hat{\imath} - 336\hat{\jmath}$

**Example (3):** A particle velocity $\vec{v}_0 = -2\hat{\imath} + 4\hat{\jmath}$ at $t = 0$ undergoes constant $\vec{a}$ of magnitude $a = 3\left(\frac{m}{s^2}\right)$ at angle $\theta = 130$ from the positive direction of the x-axis, what is the particle's velocity $\vec{v}$ at $t = 5\ s$, in unit vector notation, and in magnitude angle notation?

## Solution:

$v_x = v_{0x} + a_x t$

$v_y = v_{0y} + a_y t$

$a_x = a\ cos\theta = 3cos130 = -1.93\left(\frac{m}{s^2}\right)$

$a_y = asin\theta = 3sin130 = 2.30\left(\frac{m}{s^2}\right)$

At $t = 5\ s$

$v_x = -2 + (-1.93)(5) = -11.65\left(\frac{m}{s}\right)$

$v_y = 4 + 2.30(5) = 15.5\left(\frac{m}{s}\right)$

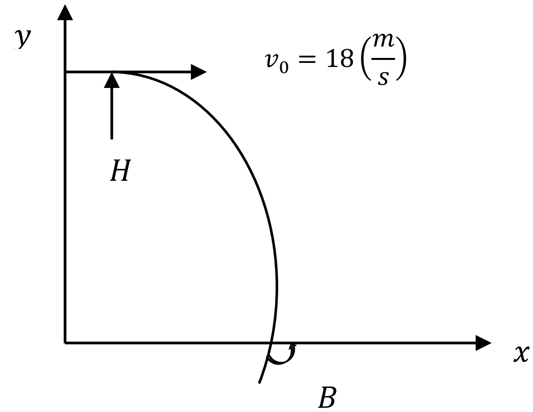
$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = -11.65\hat{\imath} + 15.5\hat{\jmath}$

The magnitude of $\vec{v}$, $v = |v| = \sqrt{v_x^2 + v_y^2} = 19\left(\frac{m}{s}\right)$

$\theta = tan^{-1}\frac{v_y}{v_x} = tan^{-1} = \frac{15.5}{-11.65} \Longrightarrow \theta = 127^{\circ}$

**Example (4):** Ball kicked horizontally at $18\left(\frac{m}{s}\right)$ off a $50m$ high cliff, find

a. Time to impact
b. Speed at impact
c. Impact point
d. Angle at impact




**Solution:**

$x(t) = v_0 cos\theta\ t$

$= v_0 t \qquad [\theta = 0^{\circ}] \qquad (1)$

$y(t) = y_0 + v_{0y} - \frac{1}{2}gt^2$

$= H - \frac{1}{2}gt^2 \qquad (2)$

At impact we must have $y = 0$ , $t = T$ , solving Eq. (2)

$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times50}{9.81}} = 3.19s$

$x(T) = 18 \times 3.19 = 57.42 \qquad (Eq.(1)$

$v_x(T) = \frac{dx}{dt} = v_0 = 18\left(\frac{m}{s}\right)$ [independent of time]

$v_y(T) = \frac{dy}{dt} = -gt = -9.81 \times 3.19 = -31.26\left(\frac{m}{s}\right)$

$tan\beta = \frac{v_y}{v_x} = \frac{-31.26}{18} \Longrightarrow \beta = -60.1$

$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{18^2 + (-31.26)^2} = 36.1\left(\frac{m}{s}\right)$ speed

**Example (5):** Gun fired a bullet with velocity $200\left(\frac{m}{s}\right)$ by an $40^{\circ}$ with horizontal, find a velocity and position of a bullet after $20\ s$ and find range and time required to return to ground?

## Solution:

$$v_{0x} = v_0 cos\theta = 200cos40 = 153.2\left(\frac{m}{s}\right)$$

$$v_{0y} = v_0 sin\theta = 200sin40 = 128.6\left(\frac{m}{s}\right)$$

$$v_{0x} = v_x = 153.2\left(\frac{m}{s}\right)$$

$$v_y = v_{0y} - gt = 128.6 - 9.8t \qquad at\ t = 20\ s$$

$$v_y = 128.6 - 9.8(20)$$

$$v_y = -67.4\left(\frac{m}{s}\right)$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{153.2^2 + (-67.4)^2}$$

$$= 167\left(\frac{m}{s}\right)$$

$$x = 3064m\ ,\ \ y = 612m$$

$$t = \frac{2v_{0y}}{g} = \frac{2(128.6)}{9.8} = 26.24\ s$$

$$R = \frac{v_0^2 sin2\theta}{g} = \frac{(200)^2 sin2(40)}{9.8} = 4021m$$

$$h = \frac{v_0^2 sin^2\theta}{2g} = \frac{(200)^2 (sin40)^2}{2\times 9.8} = 843.7m$$

**Example (6):** A long- jumper leaves the ground at an angle of $20^{\circ}$ above the horizontal and at a speed of $11\left(\frac{m}{s}\right)$

a. How far does he jump in horizontal direction?
b. What is maximum height reached?

**Solution:**

a. $R = \frac{v_0^2 sin2\theta}{g} = \frac{(11)^2 sin2(20)}{9.8} = 7.936m$

b. $h = \frac{v_0^2 sin^2\theta}{2g} = \frac{(11)^2 sin^2 20}{2\times 9.8} = 0.722m$

**Example (7):** A Stone is thrown from the top of a building upward at an angle of $30^{\circ}$ to the horizontal and with an initial speed of $20\left(\frac{m}{s}\right)$. If the height of the building is $45m$,

a. How long is it before the stone hits the ground
b. What the speed of the stone just before it strikes the ground?

**Solution:**

a. $v_{0x} = v_0 cos\theta = 20 \times cos30 = 17.3\left(\frac{m}{s}\right) = v_x$

$v_{0y} = v_0 sin\theta = 20 \times sin30 = 10\left(\frac{m}{s}\right)$

$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \Longrightarrow -45 = 10t - \frac{1}{2}(9.8)t^2 \Longrightarrow t = 4.22s$

b. $v_x = 17.3\left(\frac{m}{s}\right)$ , $v_y = v_{0y} - gt = -31.4\left(\frac{m}{s}\right)$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{17.3^2 + (-31.4)^2} = 35.9\left(\frac{m}{s}\right)$