$m_1\vec{v}_1 = m_1\vec{\acute{v}}_1 + m_2\vec{\acute{v}}_2$

$m_1\vec{v}_1 = m_1\vec{\acute{v}}_1 cos\theta + m_2\vec{\acute{v}}_2 cos\emptyset$

$0 = m_1\vec{\acute{v}}_1 sin\theta - m_2\vec{\acute{v}}_2 sin\emptyset$

- Conversation of energy

$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\acute{v}_1^2 + \frac{1}{2}m_2\acute{v}_2^2$

Example (1): A 2 Kg body moving under the effect of the force, if the body at rest at $t = 0$ and $\vec{F} = 2t\hat{\imath} + (3t^2 - 1)\hat{\jmath}N$,

1. What is the momentum and the kinetic energy of the body at $2s$.
2. What is the work done by this force from $t = 0$ to $t = 2s$.

Solution:

1. $\vec{F} = \frac{dp}{dt} \Longrightarrow \int_0^p dp = \int_0^2 Fdt$

   $\vec{p} = \int_0^2 [(2t)\hat{\imath} + (3t^2 - 1)\hat{\jmath}]\, dt = 4\hat{\imath} + 6\hat{\jmath}\ \left(\frac{Kg.m}{sec}\right)$

   $K.E = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = \frac{(4\hat{\imath}+6\hat{\jmath})^2}{2\times2} = 13J$

2. $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = 13 - 0 = 13J$

Example (2): consider a system of three particle: $m_1 = 1gm$ is at $\vec{r}_1 = \hat{\imath} + \hat{\jmath}(cm)$ and has velocity $\vec{v}_1 = 2\hat{\imath}\left(\frac{cm}{s}\right)$, $m_2 = 1gm$ is at $\vec{r}_2 = \hat{\jmath} + \hat{k}(cm)$ and has velocity $\vec{v}_2 = \hat{\jmath}\left(\frac{cm}{s}\right)$, $m_3 = 1gm$ is at $\vec{r}_3 = \hat{k}(cm)$ and has velocity $\vec{v}_2 = \hat{\imath} + \hat{\jmath} + \hat{k}\left(\frac{cm}{s}\right)$

1. Determine the velocity of the center of mass.

2. What is the total linear momentum of this system.

Solution:

1. $M = \sum m_i = m_1 + m_2 + m_3 = 3gm$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{M} = \frac{1\times 2\hat{\imath} + 1\times\hat{\jmath} + 1(\hat{\imath}+\hat{\jmath}+\hat{k})}{3}$$

$$\therefore \vec{v}_{cm} = \hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k} \quad \left(\frac{cm}{s}\right)$$

2. $\vec{p} = M\vec{v}_{cm} = 3\left(\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{1}{3}\hat{k}\right) = 3\hat{\imath} + 2\hat{\jmath} + \hat{k} \quad \left(\frac{gm.cm}{s}\right)$

Example (3): An object of mass $m_1 = 3Kg$, moving with velocity $v_1 = 13\left(\frac{m}{s}\right)$, collide head on with another object of mass $m_2 = 7Kg$ moves with initial velocity $v_2 = 3\left(\frac{m}{s}\right)$. Given that the collision is elastic, what are the final velocities of the two objects?

Solution:

$$\acute{v}_1 = v_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + v_2\left(\frac{2m_1}{m_1 + m_2}\right)$$

$$\acute{v}_1 = 13\left(\frac{3-7}{3+7}\right) + 3\left(\frac{2\times 7}{3+7}\right) = -5.2 + 4.2 = -1\left(\frac{m}{s}\right)$$

$$\acute{v}_2 = v_1\left(\frac{2m_1}{m_1 + m_2}\right) + v_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

$$\acute{v}_2 = 13\left(\frac{2\times 3}{3+7}\right) + 3\left(\frac{7-3}{3+7}\right) = 9\left(\frac{m}{s}\right)$$

Example (4): Two object slide over a frictionless horizontal surface. The first object mass $m_1 = 5Kg$, is propelled with speed $v_1 4.5\left(\frac{m}{s}\right)$ toward the second object, mass $m_2 = 2.5Kg$ which is initially at rest. After the collision, both objects have velocities are direction $\theta = 30^{\circ}$ on either side of the original line of motion of the first object. Is the collision elastic or inelastic?

Solution:

$$m_1\vec{v}_1 = m_1\vec{\acute{v}}_1 cos\theta + m_2\vec{\acute{v}}_2 cos\emptyset \qquad (1)$$

$$m_1\vec{\acute{v}}_1 sin\theta = m_2\vec{\acute{v}}_2 sin\emptyset \qquad (2)$$

By combining (1) and (2)

$$\acute{v}_1 = \frac{v_1}{2cos\theta} = \frac{4.5}{2\ cos30} = 2.5981 \left(\frac{m}{s}\right)$$

$$\acute{v}_2 = \frac{m_1}{m_2}\acute{v}_1 = \frac{5\times 2.5981}{2.5} = 5.1962 \left(\frac{m}{s}\right)$$

$$E_i = \frac{1}{2}m_1\acute{v}_1^2 = \frac{1}{2}\times 5\times (2.5981)^2 = 50.63J$$

$$E_f = \frac{1}{2}m_1\acute{v}_1^2 + \frac{1}{2}m_2\acute{v}_2^2 = \frac{1}{2}\times 5\times (2.5981)^2 + \frac{1}{2}\times 2.5\times (5.1962)^2 = 50.63J$$

$E_i = E_f$ The collision is elastic

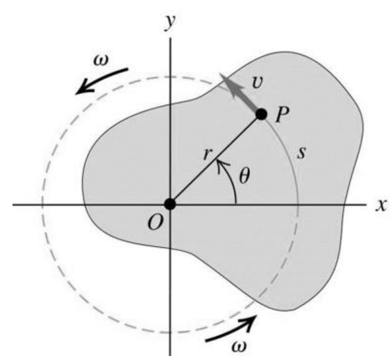
Rigid Body Kinematic

8.1 Rigid body

Object in the real world are not point- like particles that we have been dealing with up to now. A real object has a mass distribution associated whit its size and shape. The motion of the real object involves both:

- Translational motion of the center of mass.
- Rotational motion about an axis (usually take to be an axis through the center of mass or some other fixed axis). We will restrict our discussions to that of the rigid bodies. A rigid body is one in which the relative coordinates connecting all the constituent particles remain constant.

8.2 Rotation about a Fixed Axis



Consider motion around the z-axis. Reference point p represent the rotational motion of the body and of its angular position. Given a reference point p, its angular position is measured by the angle θ , between the position vector $\vec{r}$ and the x-axis. As the particle moves from the positive x- axis $\theta = 0$ to the point p it move through an arc length

$$S = r\theta,\ \theta(rad) = \frac{\pi}{180}\theta(degree)$$

The rotational motion of a body is described by the rate of the change of θ. In general the position angle is a function of time $[\theta = \theta(t)]$. Suppose the particle moves from P to Q. the reference OP makes an angle θ_1 at the time t_1, and an angle θ_2 at the time t_2 . Define the average angular velocity of the body $\overline{\omega}$, in the time internal $\Delta t = t_2 - t_1$ as the ratio of angular displacement

$\Delta\emptyset = \theta_2 - \theta_1$ to Δt

$$\overline{\omega} = \frac{\theta - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \qquad \left(\frac{rad}{s}\right) or\ (s)^{-1}$$

Analogous to linear velocity, the instantaneous angular velocity is defined as

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad (s)^{-1}$$

If the angular velocity, ω is a constant $\omega = \omega_0$ the rate of rotation is often given in terms of the frequency, or number of revolution per unit time

1 revolution $= \Delta\theta = 2\pi\ radians$

Time per revolution or period $T = \frac{2\pi}{\omega_0}$ $\qquad (s)$

Frequency of revolution is $\nu = \frac{1}{T} = \frac{\omega_0}{2\pi}$ $\qquad (Hz)$

If the angular velocity of the body is changing with time (i.e. ω is not constant), then the acceleration.

If the angular velocities are ω_1 and ω_2 at the times t_1 and t_2 the average angular acceleration

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous acceleration is the limit of this ratio as $\Delta t \to 0$

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\vec{\omega}}{dt}\ ,\ \omega = \frac{d\theta}{dt} \text{ we also have } \vec{\alpha} = \frac{d^2\theta}{dt^2}$$

The direction of $\vec{\alpha}$ is along the same axis as $\vec{\omega}$. If the axis of rotation is changing then $\vec{\alpha}$ is not the same direction as $\vec{\omega}$.

8.3 Rotational Motion with Constant Angular Acceleration

- Assume motion along a fixed axis
- Ignore vector notation (sign designates direction)
- Results also hold for axis in linear translation

$$\frac{d\omega}{dt} = \alpha$$

$$\int d\omega = \int \alpha dt$$

$$\omega = \alpha t + c$$

If $\omega = \omega_0$ at $t = 0 \Longrightarrow c = \omega_0$

$$\omega = \omega_0 + \alpha t \qquad (1)$$

$$\frac{d\theta}{dt} = \omega = \omega_0 + \alpha t$$

$$\int d\theta \int \omega_0 dt + \int \alpha t\, dt$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 + c$$

$\theta = \theta_0$ at $t = 0, \Longrightarrow c = \theta_0$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad (2)$$

Solve Eq. (1) for t substitute in Eq. (2)

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \qquad (3)$$