Work and Energy

6.1 Work and Energy

In this section we will see how to relate force particle motion in a second way. The scalar produced of force and displacement defines ***work***. The product of mass and the square of a particle's velocity gives twice the ***kinetic energy***. Combining work and kinetic energy we drive the work-energy principle. This principle plays a role which is analogous to hat of Newton's second law.

Energy is the capacity that an object has for preforming work.

Kinetic energy is energy an object possesses because of its motion

Work is energy transferred to or form an object via force acting on the object

6.2 Work in One Dimension (1-D)

Force F_x acting on a particle moving along x does an amount of work:

$W = F_x \Delta x$

W work done by the force F_x (work is a scalar quantity)

Δx displacement of particle

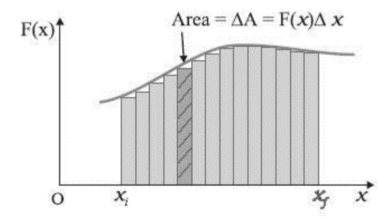
$[W] = N.m = Joule$

$W > 0$ when force and displacement are in the same direction

$W < 0$ force and displacement are opposed

6.3 Variable Force / Work

Force is a function of position, spring, gravity, etc. The total work done on the particle when moving from x_i to x_f is the sum of all the work done during successive infinitesimal displacement, that is



$$W = \lim_{\Delta x \to 0} \sum F_x(x_i)\Delta x$$

$$W = \int_{x_i}^{x_f} F_x dx \qquad \text{[definite integral]}$$

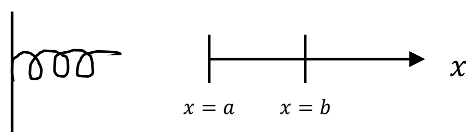
Work≡ area bounded by the curve $F_x(x)$

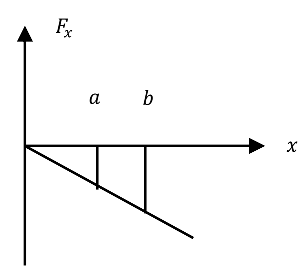
Example Spring Force:

$$F = -kx$$

How much work is needed to move a spring (fixed at one end) from $x = a$ to

$x = b$?





$$W = \int_a^b F_x \, dx$$

$$= \int_a^b (-kx)dx = \left.\frac{-kx^2}{2}\right|_a^b = -\frac{k}{2}(b^2 - a^2)$$

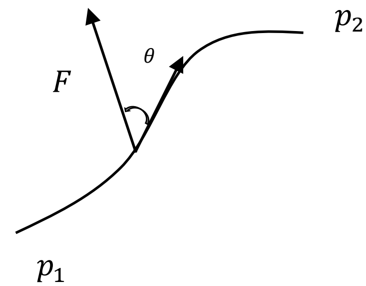
6.4 Work in Three Dimension (3-D)

In general:

$$W = \vec{F}.\Delta r$$

$\vec{F}$: constant force

Δr : displacement



$W = F\Delta r cos\theta$

$W = 0$ $if \vec{F} \perp \Delta\vec{r}$

$W = F_x\Delta x + F_y\Delta y + F_z\Delta z$ (using component)

If $F = F_x$ is not constant,

$\Delta W = F\Delta r cos\theta$

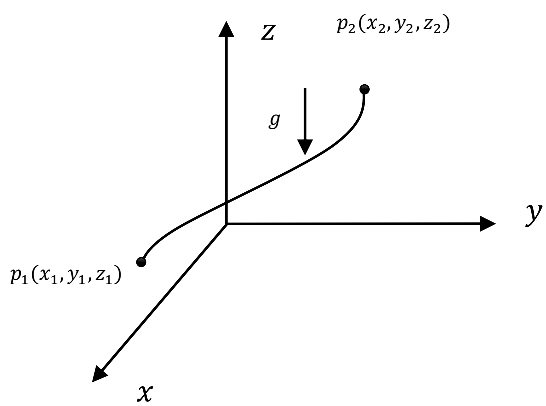
Limiting case $\Delta r \to 0$

$$W = \int_{p_1}^{p_2} \vec{F}.d\vec{r}$$

$$W = \int_{p_1}^{p_2} F_x dx + F_y dy + F_z dz \quad , \quad \begin{Bmatrix} \vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} \\ d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k} \end{Bmatrix}$$

$$W = \int_{p_1}^{p_2} F_x dx + \int_{p_1}^{p_2} F_y dy + \int_{p_1}^{p_2} F_z dz$$

Example: Gravitational Force:



$F_x = 0$, $F_y = 0$ and $F_z = -mg$

$$W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

$$W = \int_{z_1}^{z_2} -mgdz = -mg(z_2 - z_1)$$

$= -mg\Delta z$ (Δz) change in height

Work done by gravity depends on the vertical separation between p_1 and p_2

6.5 Kinetic Energy

Want to develop a relationship between the work done and the change of speed of a particle

$$W = \int_{p_1}^{p_2} F_x dx + F_y dy + F_z dz$$

$$F_x = ma_x = m\frac{dv_x}{dt}$$

$$W = \int_{p_1}^{p_2} F_x dx = \int_{p_1}^{p_2} m\frac{dv_x}{dt}\, dx$$ [v_x is a function of time]

$$\frac{dv_x}{dt} = \frac{dv_x}{dx}.\left(\frac{dx}{dt}\right) = v_x\frac{dv_x}{dx}$$

$$\therefore \int_{p_1}^{p_2} m\frac{dv_x}{dt}\, dx = \int_{p_1}^{p_2} mv_x\frac{dv_x}{dx}.\, dx = \int_{p_1}^{p_2} mv_x dv_x = \frac{1}{2}mv_x^2\Big|_{v_{x1}}^{v_{x2}}$$

$$= \frac{1}{2}m(v_{x2}^2 - v_{x1}^2)$$

$$W = \frac{1}{2}m\left[v_{x2}^2 + v_{y2}^2 + v_{z2}^2 - (v_{x1}^2 + v_{y1}^2 + v_{z1}^2)\right] = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Define $K.E = \frac{1}{2}mv^2 \equiv$ kinetic energy of particle

$W = K_2 - K_1$ or $W = \Delta K$ (Work - Energy Theorem)

For a particle $\vec{p} = m\vec{v}$

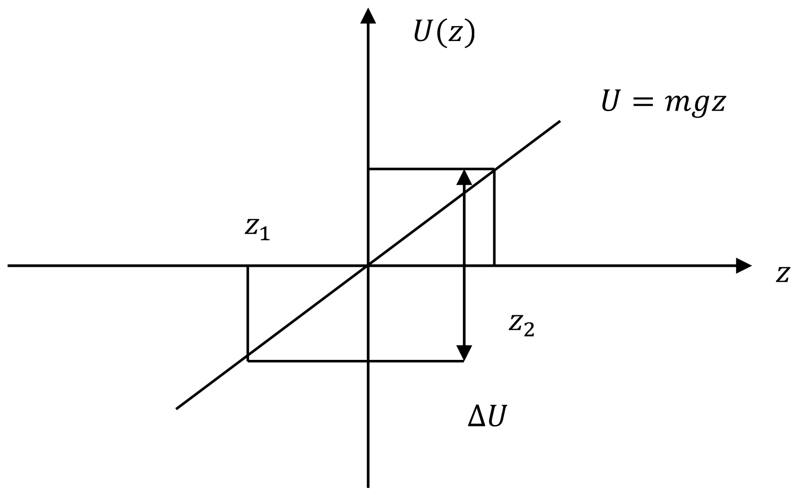
$$\therefore K = \frac{1}{2m}\, p^2$$

6.6 Gravitational Potential Energy

K.E represented the capacity of a particle to do work by virtue of its velocity

P.E represented the capacity of a particle to do work by virtue of its position in space

Consider a constant force of gravity $F_z = -mg$ acting on a particle which undergoes a displacement from (x_1, y_1, z_1) to (x_2, y_2, z_2) the force does an amount of work;



$$W_{gr} = -mg(z_2 - z_1) = \int_{z_1}^{z_2} \vec{F} . d\vec{z} = -U(z_2) + U(z_1) = -\Delta z$$

Where $U = mgz$ is called gravitational potential energy. The change in potential energy between the points z_1 and z_1 is the negative of the work done by gravity on the particle.

Gravitational potential energy: capacity of a particle to do work by virtue of its height above the surface of an attracting mass (earth). If the only force acting is gravity, then using the work-energy theorem,

$W_{gr} = k_2 - k_1$ and $W_{gr} = -U(z_2) + U(z_1)$

$\therefore k_1 + U(z_1) = k_2 + -U(z_2)$

$\therefore E = k + U(z) = constant$

Mechanical energy represents the total capacity of a particle to do work by virtue of both its velocity and its position

$\therefore E == \frac{1}{2}mv^2 + mgz = constant$ (Law of conservation of mechanical energy)

Consider two different position:

$\frac{1}{2}mv_1^2 + mgz_1 + \frac{1}{2}mv_2^2 + mgz_2$

$v_1^2 + 2gz_1 = v_2^2 + 2gz_2$

$v_1^2 - v_2^2 = 2g(z_1 - z_2)$

$= -2g(z_2 - z_1)$

If there are forces acts besides gravity: friction, etc. let W_{other} represent the work done by all forces other than gravity. Then the work done by all other forces acting on the body, with exception of the gravitational force equals the change in the total mechanical energy of the body:

$W_{other} = (k_2 + U_2) - (k_1 + U_1) = E_2 - E_1 = \Delta E$

6.7 Conservation of Energy

The more general law of conservation of energy was established by including other forms of energy: thermal energy, electrical, chemical and nuclear.

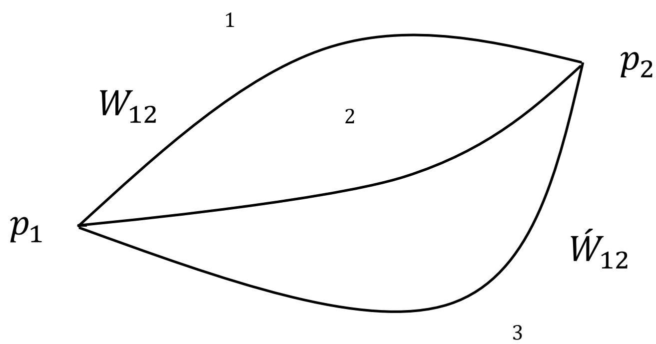
The changes in all forms of energy:

$\Delta KE + \Delta U + \Delta(all\ other\ forms) \equiv 0$

This is the law of conservation of energy and is one of the most important principle of physics.

"The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains a constant"

6.8 Conservation Forces



Consider a particle which moves from p_1 to p_2 with a force $\vec{F}$ acting on it. Assume $\vec{F}$ is a function only of position but doesn't depends explicitly on time but we assume that at any given location the force is the same of matter when the particle arrived there. The work done by the force $\vec{F}$ on the particle is moving from 1→2 along path 1

$W_{12} = \int_{p_1}^{p_2} \vec{F}.d\vec{r}$

$\acute{W}_{12} = \int_{p_1}^{p_2} \vec{F}.d\vec{r}$ [work done in moving from 1→2 along path 2]

Definition: $\vec{F}$ is a conservation force if the work depends only on the position of the points p_1 and p_2 but not on path between p_1 and p_2

$W_{12} = \acute{W}_{12}$ for any path

Closed Path-Round trip:

Particle moves from $p_1$to p_2 and then back to p_1 if the force is conservation the total work is exactly zero for a round trip a closed path

$W_{12} + \acute{W}_{12} = W_{12} - \acute{W}_{12} = 0$

$\oint \vec{F}.d\vec{r} = 0$ [Line integral around a closed loop]