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<u>Polar form of C-R equations</u>:-

We can write C-R eq. by polar form such that

$$u_r = \frac{1}{r} v_\theta \quad \& \quad u_\theta = -r v_r$$

2.8 <u>Harmonic Functions</u>:-

Let $h(x, y)$ be a real function depend on two variable x, y and it has domain in $xy - plane$.we are said to be a function h is harmonic function in this domain, if the first and second partial derivatives of its are continuous function and satisfied this equation.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad (1)$$

We are said to be this equation by Laplace's equation we can rewrite equation (1) as follow:-

$$\nabla^2 h = 0 \text{ Where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

<u>EX</u>:-Prove that $f(x, y) = e^x \cos y$ is harmonic fun.

$$\frac{\partial f}{\partial x} = e^x \cos y \quad , \quad \frac{\partial f}{\partial y} = -e^x \sin y$$

$$\frac{\partial^2 f}{\partial x^2} = e^x \cos y \quad , \frac{\partial^2 f}{\partial y^2} = -e^x \cos y$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = e^x \cos y + (-e^x \cos y)$$

$$= e^x \cos y - e^x \cos y = 0$$

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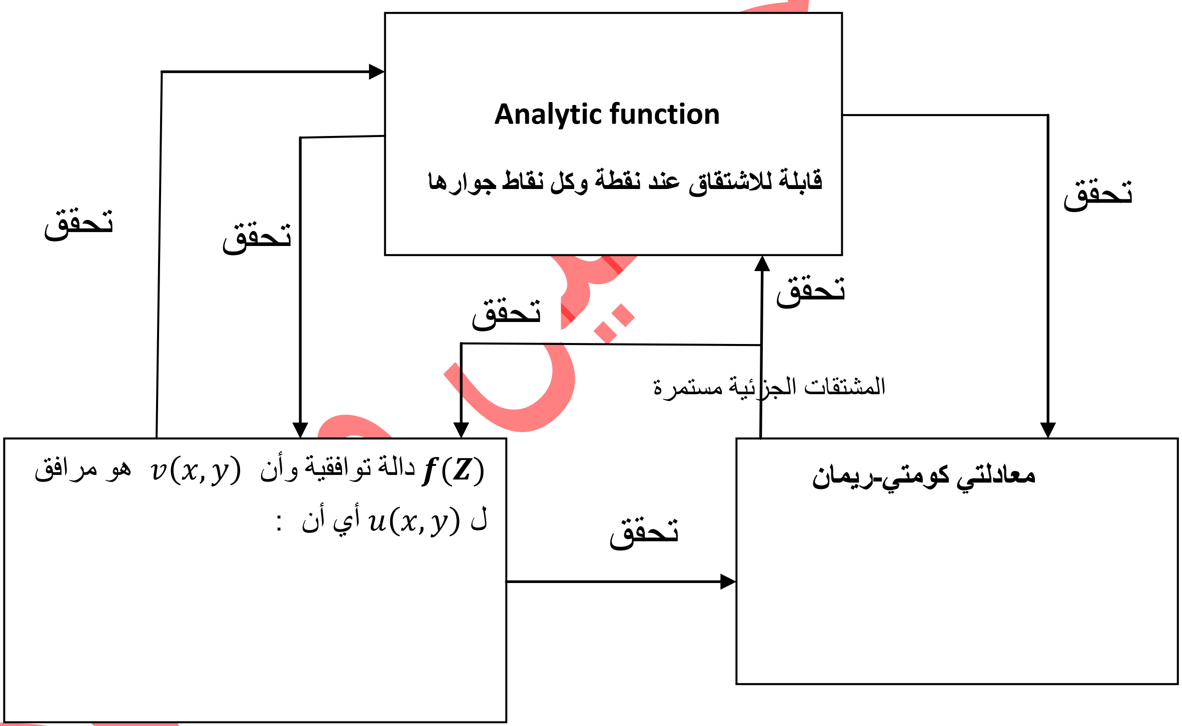
Theorem:- If $f(Z) = u(x,y) + iv(x,y)$ is analytic functions in region D then u, v are harmonic functions in region .

2.9 Harmonic Conjugate:-

I f the two function $u(x,y)$ and $v(x,y)$ are harmonic fun. In region D then $f(Z) = u(x,y) + iv(x,y)$ is an analytic function in this region and $v(x,y)$ is said to be a harmonic conjugate of the function u .



Remark:-

1) If $f(Z)$ is a harmonic (analytic) function then
$$u_{xx} + u_{yy} = 0 \quad \& \quad v_{xx} + v_{yy=0}$$
2) The word "conjugate" in above definition is different conjugate of the complex number $\bar{Z}$.

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3) If $v(x,y)$ a harmonic conjugate of $u(x,y)$, then $-u(x,y)$ is a harmonic conjugate of $v(x,y)$ where the functions $v(x,y)$ and

$-u(x,y)$ are real and imaginary parts of function$-if(z)$,that is $v(x,y)$ is a h.c.(harmonic conjugate) of $u(x)$**,** then
$f(Z) = u(x,y) + iv(x,y)$ and $-u(x,y)$ is a h.c. Of $v(x,y)$, then
$-if(Z) = -i\big(u(x,y) + iv(x,y)\big) = v(x,y) - iu(x,y).$