Lecture #17 / Fatigue in metals:

Fatigue is a degradation of mechanical properties leading to failure of a material or a component under cyclic loading. (This definition excludes the so-called phenomenon of static fatigue, which is sometimes used to describe stress corrosion cracking in glasses and ceramics in the presence of moisture).

In general, fatigue is a problem that affects any structural component or part that moves. Aircraft (principally the wings) in the air, nuclear reactors and turbines under cyclic temperature conditions (i.e., cyclic thermal stresses) are examples in which the fatigue behavior of a material assumes a singular importance. It is estimated that 90% of service failures of metallic components that undergo movement of one form or another can be attributed to fatigue.

• Often, a fatigue fracture surface will show some easily identifiable macroscopic features, such as beach markings.

The main features of this kind of failure are:

a) a fatigue crack initiation site, generally at the surface;
b) a fatigue crack propagation region showing beach markings;
c) a fast-fracture region where the crack length exceeds a critical length.

• Typically, the failure under cyclic loading occurs at much lower stress levels than the strength under monotonic loading with little or no warning (with catastrophic results).

• Three basic factors are necessary to cause fatigue failure:
  - max. tensile stress of sufficiently high value;
  - large enough variation or fluctuation in the applied stress;
  - sufficiently large number of cycles of the applied stress.

In addition: stress concentration, corrosion, temperature, metallurgical structure, residual stresses, and other variable tend to alter the conditions for fatigue.

• The study of cyclic behavior of materials can divided into three classes:
  - Stress-life approach: It is useful when stresses and strains are mostly elastic. The main drawback of this approach is that we are unable to distinguish between the initiation and propagation phases of fatigue life.
  - Strain-life approach: The strain-life approach is useful when there is a significant amount of plastic strain.
  - Fracture mechanics approach: The cyclic stress intensity factor is used as the crack driver to estimate the life for propagating a crack from an initial size to larger size or to the critical size corresponding to failure.

Stress cycles:
cyclic stress range $\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}}$, cyclic stress amplitude $\sigma_a = \Delta \sigma / 2$, mean stress $\sigma_m = (\sigma_{\text{max}} + \sigma_{\text{min}})/2$, stress ratio $R = \sigma_{\text{min}} / \sigma_{\text{max}}$.

Amplitude Ratio, $A = \sigma_a / \sigma_m = (1-R)/(1+R)$

**Fatigue regimes:**

High cyclic fatigue, HCF ($N_f > 10^3$) (High number of cycles with may be low level of stress).
(stress life approach/ The S-N curve)

Low cyclic fatigue, LCF ($N_f < 10^3$) (Low number of cycles with high level of stress mainly of thermal origin as in steam turbine, nuclear pressure vessel,...)
(strain life approach/ The S-N curve)

**The S-N curve (or the stress life approach):** is used for HCF. This approach attempts to keep local stresses so low that the crack initiation stage never begins, i.e., stress and strain everywhere remain in the elastic region and no local yielding occurs to initiate a crack.
A- for \( N < 10^6 \) a fatigue strength \( S_f \) can be defined at any \( N_f \).
- endurance limit \( S_e \), is used only for infinite life strength, \( N > 10^6 \).

For steel \( S_f = 0.5 \sigma_{ult} \) \((\sigma_{ult} < 200\text{Kpsi})\)
\[ S_e = 100 \text{ Kpsi} \] \((\sigma_{ult} \geq 200\text{Kpsi})\)

B- For Al: \( S_f = 5 \times 10^8 = 0.4 \sigma_{ult} \)

### Ferrous metals

- \( S_m \) = Fatigue strength at \( N=10^3 \)
  - \( = 0.9 \sigma_{ult} \) (bending and torsion)
  - \( = 0.75 \sigma_{ult} \) (axial loading)

For ferrous metals:
\( S_f = S_m \) at \( N=N_1 \) and \( S_f = S_e \) at \( N=N_2 = 10^6 \)

But \( \log S_f = \log a + b \log N \), when you subr. the boundary conditions and solve for \( a \& b \) you will get that:
\[ b = \frac{1}{Z} \log \left( \frac{S_m}{S_e} \right) \]

when; \( Z = \log N_1 - \log N_2 = -3 \) So: \( \log a = \log S_m - 3b \)

For non-ferrous metals:
\( S_f = S_m \) at \( N=N_1=10^3 \) and \( S_f = S_f \) at \( N=N_2 \)

\[ b = \frac{1}{Z} \log \left( \frac{S_m}{S_f} \right) \]

when; \( Z = \log N_1 - \log N_2 \)

### Example:
Aluminum bar is subjected to a torsion a torsion load. What is the fatigue strength at \( N= 2\times10^7 \) cycles, if \( \sigma_{ult} = 45\text{kpsi} \) and \( S_{fSE8} = 11.063 \).

Solution: \( S_m = \text{Fatigue strength at } N=10^3 \)
- \( = 0.9 \sigma_{ult} \) (bending and torsion)
\[ Z = \log N_1 - \log N_2 = \log 10^3 - \log 5 \times 10^8 = -5.699 \]

\[ b = \frac{1}{Z} \log \left( \frac{S_m}{S_f} \right) = \frac{1}{5.699} \log \left( \frac{40.5}{11.063} \right) = -0.0989 \]

\[ \log a = \log S_m - 3b \]
so: \( a = 80193 \)

\[ S_f = 80193(2E7)^{-0.0989} = 15.209\text{kpsi} \]
In LCF fatigue life is treated in terms of strain versus number of cycles to failure. For low-cycle fatigue, crack initiation is rapid and crack propagation accounts for most of the life, whereas for high-cycle fatigue most of the life is spent in crack initiation. This leads to the conclusion that under constant strain-amplitude cycling, a ductile material is desirable for low-cycle fatigue and a high-strength material is desirable for high-cycle fatigue. High-strength materials generally have low ductility (low value of δ) and ductile materials generally have low strength.

**Effect of Mean Stress on Fatigue Life:**

The mean stress $\sigma_m$ can have an important effect on the fatigue strength of a material. For a given stress amplitude $\sigma_a$, as the mean stress increases, the fatigue life decreases.

Various empirical expressions have been proposed which take into account the effect of mean stress on fatigue life. Some of these are the following:

- **Goodman’s relationship:**

  $$\sigma_a = \sigma_0 \left[ 1 - \frac{\sigma_m}{\sigma_{ult}} \right].$$

- **Gerber’s relationship:**

  $$\sigma_a = \sigma_0 \left[ 1 - \left( \frac{\sigma_m}{\sigma_{ult}} \right)^2 \right].$$

- **Soderberg’s relationship:**

  $$\sigma_a = \sigma_0 (1 - \frac{\sigma_m}{\sigma_{yp}}).$$

In all these relationships, $\sigma_0$ is the fatigue strength in terms of stress amplitude when $\sigma_m = 0$.

**Cumulative Damage:**

It is of great importance to be able to predict the life of a component subjected to variable-amplitude conditions, starting from data obtained in simple constant-amplitude tests. The cumulative-damage theories consider fatigue to be a process of accumulating damage in a material until a certain maximum tolerable damage is reached.
At $\sigma_a = \sigma_1$ $N = 50$ and $N_f = 150$ and at $\sigma_a = \sigma_2$ $N = 150$ and $N_f = 300$

From A to E the material will exhaust $(1/3)$ of its life available at $\sigma_1$.

If the stress level is to be changed to $\sigma_2$, then the percentage of life already exhausted at $\sigma_1$ is equivalent to the percentage of life exhausted at $\sigma_2$. That is, the material will be at F (when $N = 100$ as $100/300 = 1/3$), so:

$$\frac{n_1}{N_2} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = 1$$

Called Miner’s rule or linear cumulative damage theory.

Assumptions:
1- $\sigma_m = 0$
2- Change from low to high or from high to low stress do not have an effect on fatigue life.

**Example 1:**
A microalloyed steel was subjected to two fatigue tests at $\pm 400$ MPa and $\pm 250$ MPa. Failure occurred after $2 \times 10^4$ and $1.2 \times 10^6$ cycles, respectively, at these two stress levels. Making appropriate assumptions, estimate the fatigue life at $\pm 300$ MPa of a part made from this steel that has already undergone $2.5 \times 10^4$ cycles at $\pm 350$ MPa.

**Solution:**
We assume an equation of the form (other forms can also be assumed):

$$\frac{\Delta \sigma}{\Delta \sigma}^{N_f} = \text{const.}$$

So: $a = 0.115$ and const., $c = 2.498$ MPa

At $\pm 350$ MPa:

$$N_f = \left(\frac{c}{\Delta \sigma}\right)^{1/a} = \left(\frac{2.498}{700}\right)^{1/0.115} = 6.4 \times 10^4$$

For $2.5 \times 10^4$ cycles:

At $\pm 300$ MPa:

$$N_f = \left(\frac{c}{\Delta \sigma}\right)^{1/a} = \left(\frac{2.498}{600}\right)^{1/0.115} = 2.45 \times 10^5$$

So:

$$\frac{2.5}{6.4} + \frac{n_2}{2.45 \times 10^5} = 1$$

$n_2 = 1.49 \times 10^5$ cycles
Fatigue Crack Nucleation:

Fatigue cracks nucleate at singularities or discontinuities in most materials. Discontinuities may be on the surface or in the interior of the material. The singularities can be structural (such as inclusions or second-phase particles) or geometrical (such as scratches or steps). Plastic deformation is easier at surfaces and slip steps form on the surface. Slip steps alone can be responsible for initiating cracks, or they can interact with existing structural or geometric defects to produce cracks. Channels through which the dislocations move and produce intrusions and extrusions at the surface.

During the loading part of the cycle, slip occurs on a favorably oriented plane, and during the unloading part of the cycle, reverse slip occurs on a parallel plane, because the slip on the original plane is inhibited owing to hardening or, perhaps, the oxidation of the newly formed free surface.

The first cyclic slip may create an extrusion or an intrusion at the surface. An intrusion may grow and form a crack by continued plastic deformation during subsequent cycles.

Such intrusions/extrusions can form in ductile metals under the action of thermal fatigue as well.

Twin boundaries can be important crack nucleation sites in hexagonal close-packed materials such as magnesium, titanium, etc., and their alloys.

Inclusions and second-phase particles are commonly the dominant nucleation sites in materials of commercial purity – for example, aluminum, high-strength steels.

Grain boundaries can become important nucleation sites at large strain amplitudes and at temperatures greater than about $0.5T_{\text{melting}}$ or in the presence of impurities that produce grain-boundary embrittlement as $O_2$ in iron.
**Fatigue Crack Propagation:**

At large stress amplitudes, a very large fraction (around 90%) of fatigue life is spent in the growth or propagation of a crack. For a component that contains a notch, this fraction becomes even larger.

**stage I:** A few cracks nucleate at the surface and start propagating in a crystallographic shear mode on planes oriented at approximately 45° to the stress axis. During this stage, cracking occurs along the crystallographic slip planes and the crack growth is on the order of a few micrometers or less per cycle. These crystallographic cracks in this mode.

**stage II:** crack starts propagating in a direction perpendicular to the stress axis in the tensile mode. Typically, the fracture surface shows striation markings. The ratio of the extent of stage I to stage II decreases with an increase in stress amplitude. The stress concentration at the tip of the crack causes local plastic deformation in a zone in front of the crack. With crack growth, the plastic zone increases in size until it becomes comparable to the thickness of the specimen. When this occurs, the plane-strain condition at the crack front in stage II does not exist anymore, the crack plane undergoes a rotation, and the final part of rupture occurs in plane-stress or shear mode. This corresponds to **Stage III,** characterized by rapid crack propagation.

Microscopic observations of fatigue fracture surfaces frequently show striations in stage II. Frequently, each striation is thought to represent one load cycle, and indeed, it has been observed by means of programmed amplitude fatigue tests that in many materials these striations do represent the crack front position in each cycle. Note the smaller striation spacing in the early stage, indicating a lower crack propagation rate than in the late stage.

**Propagation rate:**

Under cyclic loading, a dominant crack grows, as a function of the number of cycles, from an initial size $a_0$ to a critical size $a_c$, corresponding to failure. The basic problem is thus reduced to one of characterizing the growth kinetics of the dominant crack in terms of an appropriate driving force.
Since crack growth starts from the most highly stressed region at the crack tip, the driving force can be characterized in terms of the stress intensity factors at the tip - that is, the range of the stress intensity factor $\Delta K = K_{\text{max}} - K_{\text{min}}$, where $K_{\text{max}}$ and $K_{\text{min}}$ are the maximum and minimum stress intensity factors corresponding to the maximum and minimum loads (i.e., stresses) respectively.

The crack growth rate per cycle, $da/dN$, can then be expressed as a function of the cyclic stress intensity factor at the crack tip, $\Delta K$. An empirical mathematical equation (known as the Paris-Erdogan relationship) describing the crack growth process and the appropriate boundary conditions will be:

$$\frac{da}{dN} = c (\Delta K)^m$$

Here, $a$ is the crack length, $N$ is the number of cycles, $\Delta K$ is the cyclic stress intensity factor, and $C$ and $m$ are empirical constants that depend on the material, environment, and test conditions, such as the load ratio $R$, the test temperature, the wave form, etc.

$$\Delta K = Y \Delta \sigma \sqrt{\pi a}$$

So:  

$$\Delta N = \int_{a_f}^{a_i} \frac{da}{C(\Delta K)^m} = \int_{a_f}^{a_i} \frac{da}{C(Y\Delta \sigma \sqrt{\pi})^m a^{m/2}} = \frac{a_i^{(1-m)/2} - a_f^{(1-m)/2}}{C(Y\Delta \sigma \sqrt{\pi})^m (\frac{m}{2}-1)}$$

$$Y = 1.12 \quad \text{for long crack, the crack is at the edge.}$$

$$Y = \sqrt{\frac{\pi a}{2b}} \quad \text{for long crack, center-cracked plate (2b=width of the plate)}$$

**Example:**

When subjected to fatigue under a $\Delta \sigma = 140$ MPa, an alloy showed the following Paris-type fatigue crack propagation relationship:

$$\frac{da}{dN} (\text{m/cycle}) = 0.66 \times 10^{-8} (\Delta K)^{2.25}$$

where $\Delta K$ is in MPa m$^{1/2}$. Estimate the number of cycles required for the crack to grow from 2 mm to 8.8 mm.

**Solution:**

$$\frac{da}{dN} = 0.66 \times 10^{-8} (1.12 \Delta \sigma \sqrt{\pi a})^{2.25},$$

$$a^{-1.25} da = 0.66 \times 10^{-8} (1.12)^{2.25} \times (140)^{2.25} \pi^{1.125} dN.$$

Integrating, get:

$$\int_{a_f}^{a_i} a^{-1.25} da = 2.0815 \times 10^{-3} \int_{0}^{N_f} dN$$

$$N_f \times 2.0815 \times 10^{-3} = \left[ a^{-1.25} \right]_{0.0002}^{0.0008}$$

$$N_f = 1.4129 \times 10^3 \text{ cycles} = 1413 \text{ cycles}$$
EFFECT OF STRESS CONCENTRATION ON FATIGUE:

Fatigue strength is reduced by stress raisers:
- notch or hole, fillets, keyways, screw threads, etc
- surface roughness
- metallurgical (porosity, inclusions, local overheating, …).

Stress raisers introduce three effects:
1. Increase or concentrate the stress at its root and this may lead to local yielding.
2. Stress gradient is set up from the root toward the center of the specimen.
3. A triaxial state of stress is produced.

The ratio of maximum stress to normal stress is the theoretical stress-concentration factor $K_t$.

$$\sigma_{\text{max}} = K_t * \sigma_{\text{min}}$$

Values for $K_t$ may be determined basing on the theory of elasticity or may be obtained experimentally.
- Under static load ($K_t = 1$) for ductile materials and ($K_t > 1$) for brittle materials.
- Under cyclic loading, the effect of stress concentration is of concern for brittle and ductile materials.

The effect of notches on fatigue strength is determined by comparing the S-N curves of notch and untouched specimens. The effectiveness of the notch in expressed the fatigue limit is expressed by the fatigue-strength reduction factor, $K_f$, so as:

$$K_f = \frac{s_0(s_f)_{\text{unnotched}}}{s_0(s_f)_{\text{notched}}}$$

Values of $K_f$ have been found to vary with: severity of the notch, the type of notch, the material, the type of loading, the stress level.

In general $K_f$ is usually less than $K_t$, and $K_f/K_t$ decreases as $K_t$ increases, thus, very sharp notches (high $K_t$) have less effect on fatigue strength than would be expected from their high value of $K_t$.

The effect of the notch on the fatigue strength depends also on the nature of the materials. The more ductile the material, the less notch-sensitive it is. Brittle materials are more notch-sensitive. Since ductility and brittleness are roughly related to strength and hardness, low strength and soft materials tend to be less notch sensitive than high strength and hard materials.

So:

$$K_f = 1 + q(K_t - 1)$$

Where $q$- notch-sensitivity factor. It is not a true material constant since it varies with the severity and type of notch, the size of specimen, and the type of loading. It can be obtained from special charts.

$$q = \frac{K_f - 1}{K_t - 1}$$

Notch sensitivity increases with tensile strength. Thus, it is possible in certain circumstances to decrease fatigue performance by increasing the hardness or tensile strength of a material.
Size Effect:
Changing the size of a fatigue specimen usually results in a variation in two factors. First, increasing the diameter increases the volume or surface area of the specimen. The change in amount of surface is of significance since fatigue failures usually start at the surface. Second, for plain or notched specimens loaded in bending or torsion, an increase in diameter usually decreases the stress gradient across the diameter and increases the volume of material which is highly stressed.

Surface Effects:
Practically all fatigue failures start at the surface. For many common types of loading, like bending and torsion, the maximum stress occurs at the surface so that it is logical that failure should start there. However, in axial loading the fatigue failure nearly always begins at the surface. There is ample evidence that fatigue properties are very sensitive to surface condition. The factors which affect the surface of fatigue specimen can be divided roughly into three categories:
- surface roughness or stress raisers at the surface.
- Changes in the fatigue strength of the surface metal.
- Changes in the residual stress condition of the surface. In addition, the surface is subjected to oxidation and corrosion.

Different surface finishes produced by different machining procedures can appreciably affect fatigue performance, and anything that changes the fatigue strength of the surface material will greatly alter the fatigue properties.
- Decarburization of the surface of heat-treated steel is particularly detrimental to fatigue performance. Similarly, the fatigue strength of aluminum alloy sheet is reduced when a soft aluminum coating is applied to the stronger age-hardenable aluminum-alloy sheet.
- Electroplating of the surface generally decreases the fatigue limit of steel. The particular plating condition used to produce an electroplated surface can have an appreciable effect on the fatigue properties, since large changes in the residual stress, adhesion, porosity, and hardness of the plate can be produced.

Surface Residual Stress:
The formation favorable compressive residual-stress pattern at the surface is probably the most effective method of increasing fatigue performance. Residual stresses arise when plastic deformation is not uniform throughout the entire cross section of the part being deformed. The maximum value of residual stress which can be produced is equal to the elastic limit of the metal.
It should also be apparent that the improvements in fatigue performance which result from the introduction of surface compressive residual stress will be greater when the loading is one in which a stress gradient exists than when no stress gradient is present.

Methods for introducing residual-stress are:
- Thermal treatments/ Case hardening (carburizing, Nitriding, Flame hardening, .....
- Surface treatments/ Cold forming (rolling, hammering, using a pressure or mandrel,......), Shot peening.
- Mechanical prestressing (as for the leaf springs)

Distributions may be modified by plastic deformation or by thermal activation. When gross plastic deformation occurs, the stress drops to the yield stress. Thus, periods of overloading in the fatigue cycle or testing at high stresses in the low-cycle region may alter the residual-stress distribution by plastic deformation This is referred to as fading of residual stress. Residual stresses have their greatest influence near the fatigue limit, where little fading occurs. Conversely, the fatigue life at high applied stresses depends little on residual stresses.

**Thermal Fatigue:**

Fatigue failure can be produced by fluctuating thermal stresses under conditions where no stresses are produced mechanical causes. For the simple case of a bar with fixed end supports, the thermal stress developed by a temperature change $\Delta T$ is: $\sigma = \alpha E \Delta T$

If failure occurs by one application of thermal stress, the condition is called thermal shock. However, if failure occurs after repeated applications of thermal stress, of a lower magnitude, it is called thermal fatigue.

**EFFECT OF METALLURGICAL VARIABLES ON FATIGUE:**
The fatigue properties of metals are quite structure-sensitive.
- There is good evidence that high fatigue resistance can be achieved by homogenizing slip deformation so that local concentrations of plastic deformation are avoided. This is in agreement with the observation that fatigue strength is directly proportional to the difficulty of dislocation cross slip. Materials with high stacking-fault energy permit dislocations to cross slip easily around obstacles, which promotes slip-band formation and large plastic zones at the tips of cracks. Both of these phenomena promote the initiation and propagation of fatigue cracks.
- The dependence of fatigue life on grain size varies also depending on the deformation mode. Grain size has its greatest effect on fatigue life in the low-stress, high-cycle regime in which stage I cracking predominates. In high stacking-fault-energy materials (such as aluminum and copper) cell structures develop readily and these control the stage I crack propagation.
- Second-phase particles
- Microstructural orientation