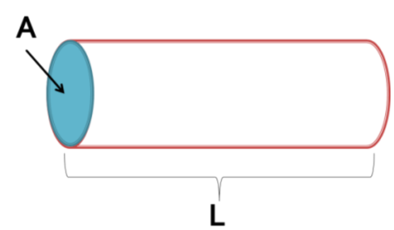
## 1. Introduction to Superconductivity

- In Kamerlingh Onnes laboratory, Leiden University 1911, superconductivity was discovered. This happen when a local high school student employed for the boring job of monitoring experiments has observed that the resistivity of Hg metal vanished abruptly at about 4K.
- Bardeen Cooper and Schrieffer in 1957 have discovered the microscopic mechanism underlying superconductivity, however, all the phenomenological models with predictive power were developed in the 30's and 40's.
- Due to their important interest for promising technological applications, superconductors have been studied intensively. Imagine what if it could be possible if a material which has superconductivity properties at room temperature is discovered.
- Until 1986, critical temperature ($T_c$) at which resistance vanishes is always less than ~ 23K.
- In 1986, Bednorz and Mueller have published a paper, subsequently recognized with the 1987 Nobel Prize, for the discovery of a new class of materials which is presently included materials with $T_c$ of ~135K.

## 2. Conductivity:

Once the electric current passes through a conductor, some of the energy is lost in the form of heat or light. The amount of the lost energy is varied based on the material electrical resistance; some materials have very good conductivity and low resistivity such as gold and copper.




$$\sigma = \frac{1}{\rho} \qquad (1)$$

Where, σ is the conductivity, and ρ is the resistivity.

❖ In general, for a conductor wire with cross section area (A) and length (L), the resistivity or specific resistivity (R) is:

$$R = \rho \frac{L}{A} \qquad (2)$$

From eq.1:

$$\rho = \frac{AR}{L} \qquad (3)$$

$$\because \; R = \frac{V}{I} \qquad (4)$$

And

$$\because \; J = \frac{I}{A} \qquad (5)$$

$$\therefore \sigma = \frac{1}{\rho} = \frac{L}{AR} = \frac{L}{A(\frac{V}{I})} \qquad (6)$$

$$\because \; E = \frac{V}{L} \qquad (7)$$

$$\therefore \sigma = \frac{J}{E} \quad \text{or} \quad J = \sigma E = \frac{E}{\rho} \qquad (8)$$

Where J is the current density, which is the resulted current from the movement of number of charges (n) a cross section of A in one second.

3. ***Superconductivity definition:*** property when the electrical resistance in solids is completely disappeared after they are cooled below a characteristic temperature. This temperature is called transition temperature or critical temperature ($T_c$).

4. **Superconductivity properties:**

Superconducting materials demonstrate the following unusual performances:

a. ***Zero resistance***: Below the critical temperature of a material, the DC electrical resistivity is really zero, not just very small. This is an indication of superconductivity effect (see Figure 1).

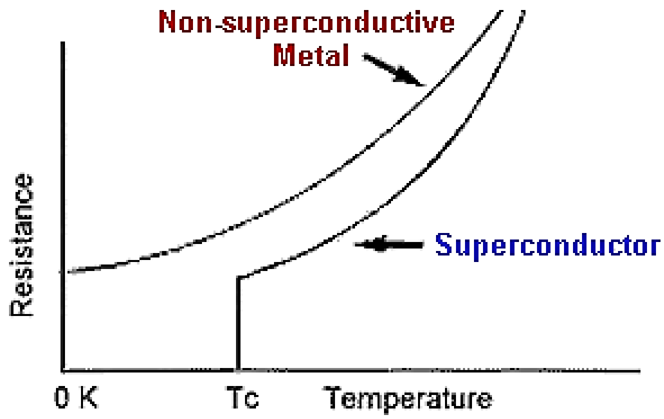



Figure 1: Resistance vs temperature

**b.** ***Persistent currents***: The flow of persistent current in materials with zero resistance material (superconductor) under an applied magnetics flux passes through a continuous loop of this material remains constant.

**c.** ***Perfect diamagnetism:*** A superconductor expels a weak magnetic field nearly completely from its interior (screening currents flow to compensate the field within a surface layer of a few 100 or 1000 A, and the field at the sample surface drops to zero over this layer).

**d.** ***Energy gap:*** Most thermodynamic properties of a superconductor are found to vary as $e^{-\Delta/k_B T}$, indicating the existence of a gap, or energy interval with no allowed Eigen energies (special energies) in the energy spectrum.

❖ *In quantum physics, energy level splitting of a quantum system occurs when a degenerate energy level of two or more states is split because corresponding Hamiltonian's* ***eigenvalues*** *become different.*

Idea: If there is an energy gap, only small numbers of particles have enough thermal energy to be promoted to the available unoccupied states above the energy gap.

In addition, this gap is visible in electromagnetic absorption: send in a photon at low temperatures (strictly speaking, $T = 0$), and no absorption is possible until the photon energy reaches $2\Delta$, i.e. until the energy required to break a pair is available.

## 5. Effect of trapped magnetic flux:

Consider a ring made out of superconductive material with a fixed area (A) (see Figure 2a).

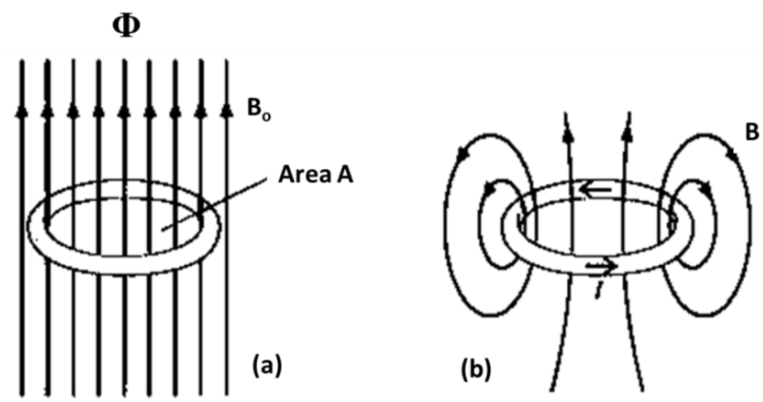
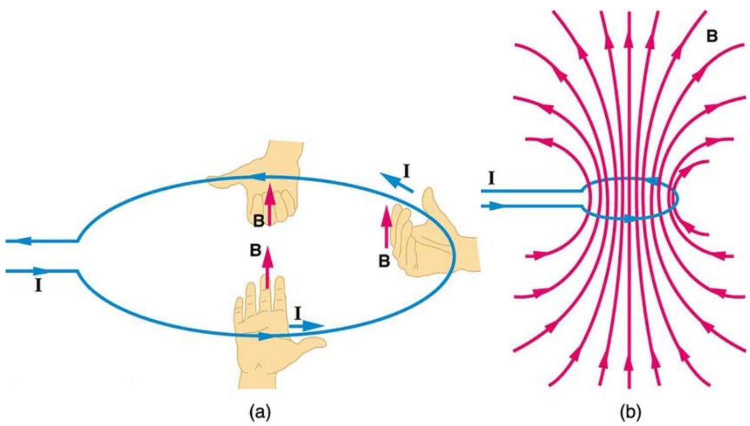



Figure 2: (a) A ring cooled below its critical temperature in an applied field $B_o$ and (b) Removing the applied field, a persistent current maintains the flux through the ring at the same value.

- At $T>T_c$ the material is in normal state.
- When the external magnetic field ($B_o$) is applied perpendicularly on the ring plane, it penetrates through the ring.
- Reduce the temperature so that $T<T_c$.
- Remove the external magnetic field.
- The magnetic field which is penetrating through the opening of the ring remains there (the magnetic flux remains trapped in the ring opening).
- This effect can be explained in terms of Faraday's law of induction. The magnetic flux $\Phi$ through the ring is $AB_o$
- If the ring is cooled below its $T_c$ under this applied field, therefore, the flux passing through it is unchanged.
- If the applied magnetic field is changed, then a current will be induced in the ring according to Lenz's law. ***There is no magic involved***.
- The trapped magnetic field passing through the ring is due to the current induced in the ring when the external magnetic field was turned off.

- The induced current is called the persistent current; it does not decay because the resistance of the ring is zero. Actually no decrease of current was observed over the period of three years!
- Theoretically, the relaxation time of current carriers in the superconductor is greater than the age of universe.

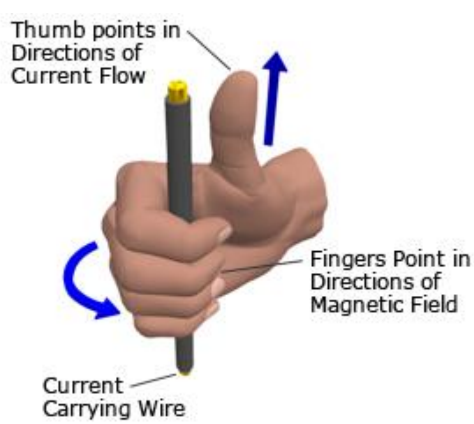
***Faraday's law of induction:*** This law describes how an electric current produces a magnetic field and, conversely, how a changing magnetic field generates an electric current in a conductor.




In a current-carrying circular loop, (a) the right-hand rule gives the direction of the magnetic field inside and outside the loop. (b) More detailed mapping of the field.

***Lenz's law:*** An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

In a perfect conductor, an arbitrarily large current can be induced, and the resulting magnetic field exactly cancels the applied field.



- Lenz's law is shown by the negative sign in Faraday's law of induction:

$$E = -\frac{d\Phi}{dt} \qquad (9)$$

Where E is the electric filed along the closed loop and Φ is the magnetic flux through the opening of the ring.

Before the external magnetic field was turned off there was a magnetic flux through the ring:

$$\Phi = AB_o \qquad (10)$$

*Below $T_c$ the **resistivity** of superconductor becomes equal to **zero**. Therefore at $T<T_c$ the **electric field** inside the superconductor must be and is **zero** as well.*

In view of this:

$$E = 0 \qquad (11)$$

Therefore, the right side of Faraday's equation:

$$\frac{d\Phi}{dt} = 0 \qquad (12)$$

From equation 9:

$$E = -AB \qquad (13)$$

$$\therefore E + AB = \text{constant} \qquad (14)$$

where $E = LI$ (L is the self-inductance of the ring), LI is the amount of flux passing through the ring generated by the current I flowing in the ring. And the induced current is called the *persistent current*.

The magnetic flux Φ through the ring must remain constant. For this reason the magnetic flux remains trapped in the opening of the ring after the external magnetic field has been turned off.

- *Note that there is **no ohmic term**, IR, on the left-hand side of this equation, because we are assuming that R = 0.*

**Therefore, (LI + AB) is the total magnetic flux through the ring.**

- The total flux threading a circuit with zero resistance must therefore remain constant (it cannot change).

- If the applied magnetic field is changed, an induced current is set up that creates a flux to compensate exactly for the change in the flux from the applied magnetic field.

- Because the circuit has no resistance, the induced current can flow forever, and the original amount of flux through the ring can be maintained forever.

- This is true even if the external field is removed altogether; the flux through the ring is maintained by a persistent induced current.

- Note that constant flux through the ring does not mean that the magnetic field is unchanged. In Figure 2a there is a uniform field within the ring, whereas in Figure 2b the field is produced by a current flowing in the ring and will be much larger close to the ring than at its center.