# complex Numbers

Lecturer

**Baseem altwajre**

# Imaginary Unit

- Until now, you have always been told that you can't take the square root of a negative number. If you use imaginary units, you can!
- The imaginary unit is **i**.
- $\mathbf{i} = \sqrt{-1}$
- It is used to write the square root of a negative number.

# Property of the square root of negative numbers

- If r is a positive real number, then

$$\sqrt{-r} = i\sqrt{r}$$

Examples:

$\sqrt{-3} = i\sqrt{3}$

$\sqrt{-4} = i\sqrt{4} = 2i$

If $i = \sqrt{-1}$, then

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

*etc.*

*For larger exponents, divide the exponent by 4, then use the remainder as your exponent instead.

Example: $i^{23} = ?$

$\frac{23}{4} = 5$ with a remainder of 3

So, use $i^3$ which $= -i$

$i^{23} = -i$

# Examples

1. $(i\sqrt{3})^2$

$= i^2(\sqrt{3})^2$

$= -1(\sqrt{3} * \sqrt{3})$

$= -1(3)$

$= -3$

2. Solve $3x^2 + 10 = -26$

$3x^2 = -36$

$x^2 = -12$

$\sqrt{x^2} = \sqrt{-12}$

$x = \pm i\sqrt{12}$

$x = \pm 2i\sqrt{3}$

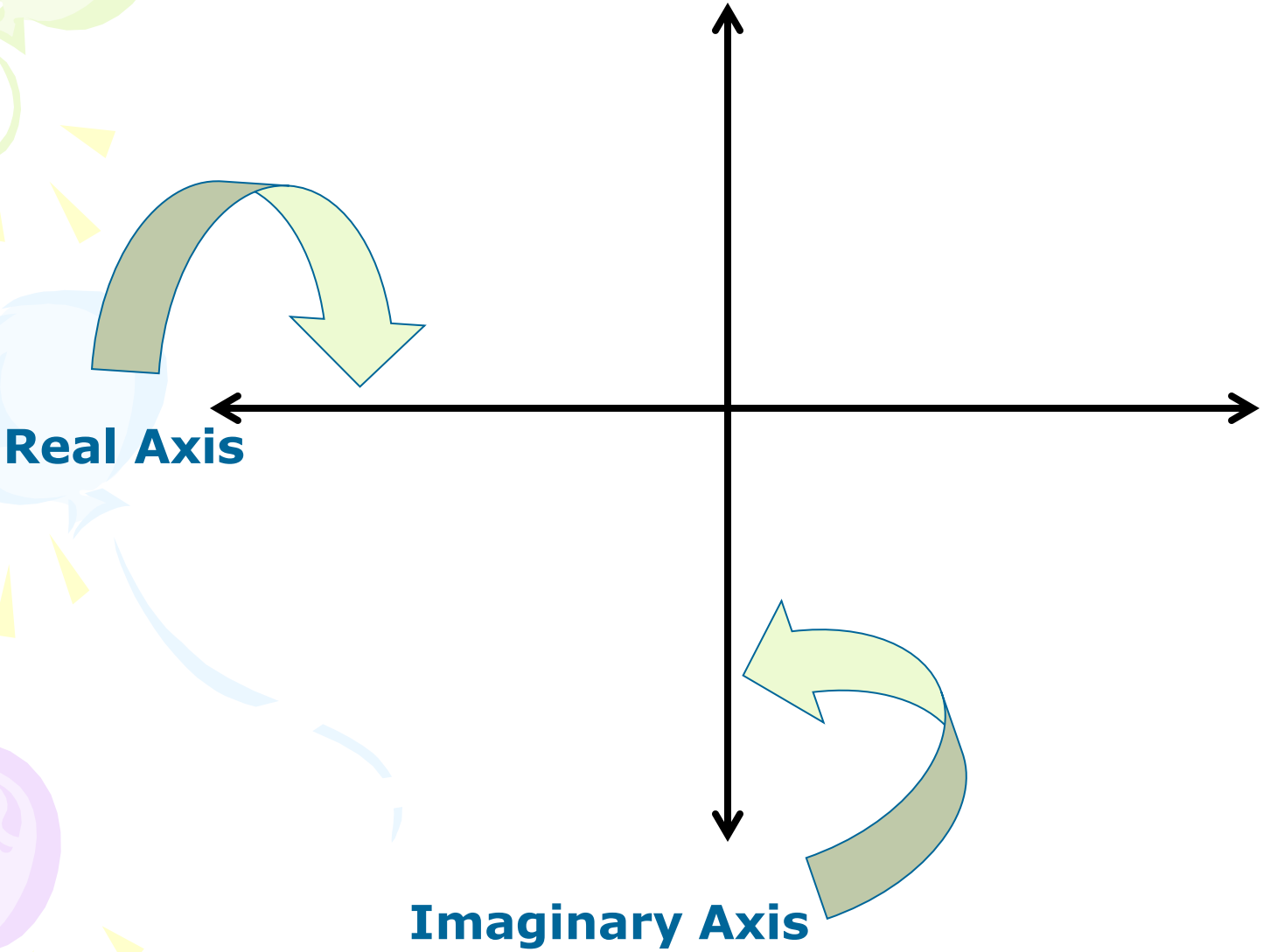
# Complex Numbers

- A complex number has a real part & an imaginary part.
- Standard form is:

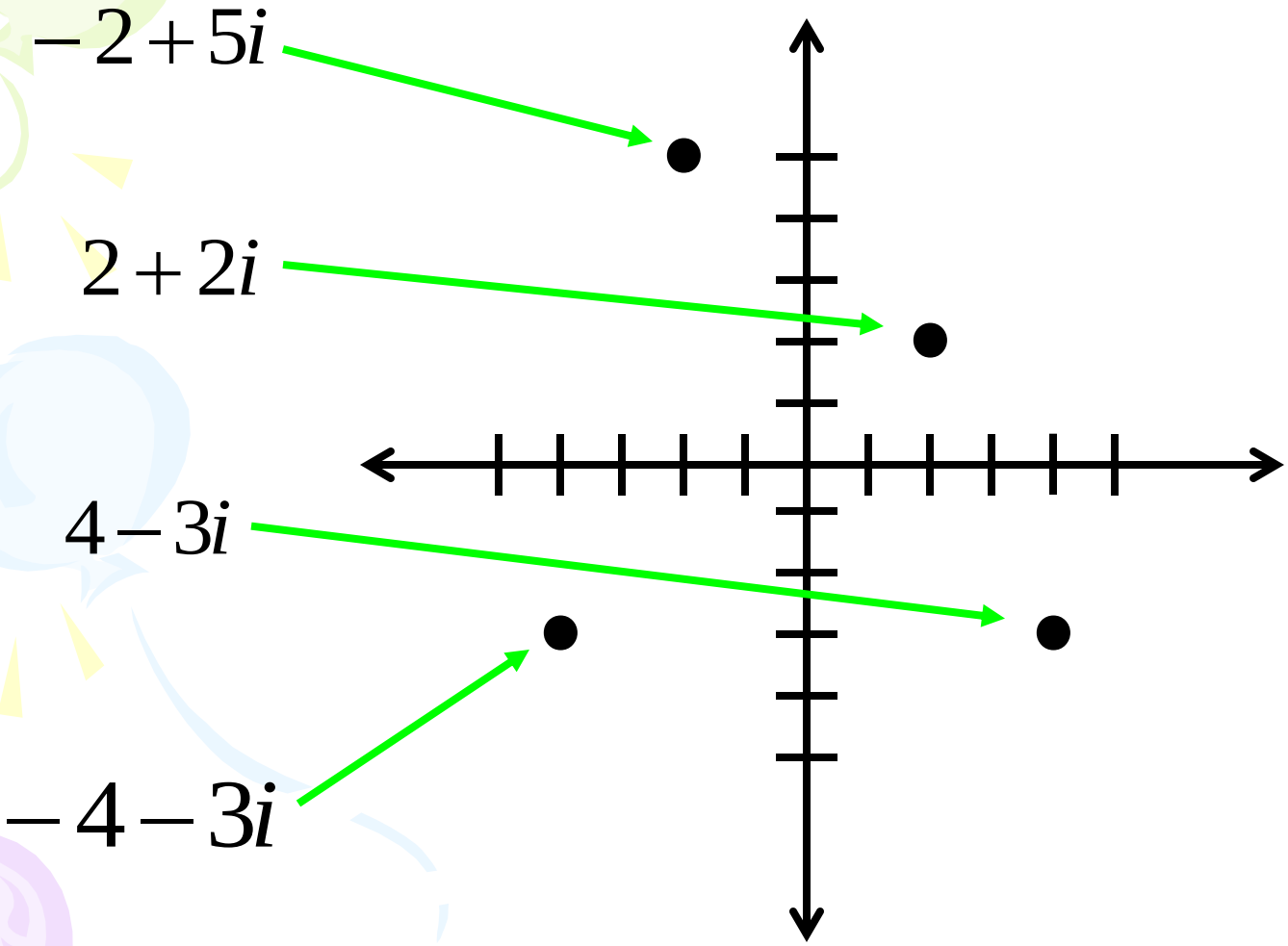
$$a+bi$$


**Example: 5+4i**

# The Complex plane



# Graphing in the complex plane



# Adding and Subtracting (add or subtract the real parts, then add or subtract the imaginary parts)

Ex: $(-1+2i)+(3+3i)$

$=(-1+3)+(2i+3i)$

$=2+5i$

Ex: $2i-(3+i)+(2-3i)$

$=(-3+2)+(2i-i-3i)$

$=-1-2i$

Ex: $(2-3i)-(3-7i)$

$=(2-3)+(-3i--7i)$

$=-1+4i$

# Multiplying
# Treat the i's like variables, then change any that are not to the first power

Ex: $-i(3+i)$

$= -3i - i^2$

$= -3i - (-1)$

$= 1 - 3i$

Ex: $(2+3i)(-6-2i)$

$= -12 - 4i - 18i - 6i^2$

$= -12 - 22i - 6(-1)$

$= -12 - 22i + 6$

$= -6 - 22i$

$$Ex: \frac{3+11i}{-1-2i} * \frac{-1+2i}{-1+2i}$$

$$= \frac{(3+11i)(-1+2i)}{(-1-2i)(-1+2i)}$$

$$= \frac{-3+6i-11i+22i^2}{1-2i+2i-4i^2}$$

$$= \frac{-3-5i+22(-1)}{1-4(-1)}$$

$$= \frac{-3-5i-22}{1+4}$$

$$= \frac{-25-5i}{5}$$

$$= \frac{-25}{5} - \frac{5i}{5}$$

$$= -5-i$$

# Absolute Value of a Complex Number

- The distance the complex number is from the origin on the complex plane.
- If you have a complex number $(a+bi)$ the absolute value can be found using: $\sqrt{a^2+b^2}$

# Examples

1. $|-2+5i|$

$$= \sqrt{(-2)^2 + (5)^2}$$

$$= \sqrt{4+25}$$

$$= \sqrt{29}$$

2. $|-6i|$

$$= \sqrt{(0)^2 + (-6)^2}$$

$$= \sqrt{0+36}$$

$$= \sqrt{36}$$

$$= 6$$

Which of these 2 complex numbers is closest to the origin? -2+5i

# Assignment