اجوبة شعبة A

(1) Find f_{xx} , f_{xy} , f_{yy} for $f(x,y) = xy^3 + \dfrac{y^2}{x^2} = xy^3 + y^2x^{-2}$ [5]

$$f_x = y^3 - 2y^2x^{-3} \quad \Rightarrow \quad f_{xx} = 6y^2x^{-4}$$

$$f_{xy} = 3y^2 - 4yx^{-3}$$

$$f_y = 3xy^2 + 2yx^{-2} \quad \Rightarrow \quad f_{yy} = 6xy + 2x^{-2}$$

for $F(x,y,z) = x^2yi + xy^2j + ye^{xz}k$. (2) Find $\nabla \times F$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2y & xy^2 & ye^{xz} \end{vmatrix}$$

$$= \left(\frac{\partial ye^{xz}}{\partial y} - \frac{\partial xy^2}{\partial z}\right)i - \left(\frac{\partial ye^{xz}}{\partial x} - \frac{\partial x^2y}{\partial z}\right)j + \left(\frac{\partial xy^2}{\partial x} - \frac{\partial x^2y}{\partial y}\right)k$$

$$= e^{xz}i - yze^{xz}j + (y^2 - x^2)k$$

(3) $$\int_0^2\int_0^{y^2} \sqrt{y^3+1}\, dxdy = \int_0^2 x\Big|_0^{y^2} \sqrt{y^3+1}dy = \int_0^2 y^2\sqrt{y^3+1}\, dy$$

$$= \frac{1}{3} \times \frac{2}{3}(y^3+1)^{3/2}\Big|_0^2 = \frac{2}{9}(27-1) = \frac{52}{9}$$

(4) $$\iint_D (x+3y)\; dA = \int_0^{\pi/4}\int_0^2 (r\cos\theta + 3r\sin\theta)rdrd\theta = \int_0^{\pi/4}\int_0^2 (\cos\theta + 3\sin\theta)r^2drd\theta$$

$$= \int_0^{\pi/4} (\cos\theta + 3\sin\theta)\frac{r^3}{3}\Big|_0^2 d\theta$$

$$= \int_0^{\pi/4} \frac{8}{3}(\cos\theta + 3\sin\theta)d\theta = \frac{8}{3}(\sin\theta - 3\cos\theta)\Big|_0^{\pi/4}$$

$$= \frac{8}{3}\left(\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}} - 0 + 3\right) = \frac{8}{3}(3 - \sqrt{2})$$

(1) Find f_{xx}, f_{xy}, f_{yy} for $f(x,y) = x^2y^2 + \dfrac{y^3}{x^2} = x^2y^2 + y^3x^{-2}$ [6]

$f_x = 2xy^2 - 2y^3x^{-3} \quad \Rightarrow \quad f_{xx} = 2y^2 + 6y^3x^{-4}$

$f_{xy} = 4xy - 6y^2x^{-3}$

$f_y = 2x^2y + 3y^2x^{-2} \quad \Rightarrow \quad f_{yy} = 2x^2 + 6yx^{-2}$

for $F(x,y,z) = xy^2i + x^2yj + xe^{yz}k$.(2) Find $\nabla \times F$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xy^2 & x^2y & xe^{yz} \end{vmatrix}$$

$$= \left(\frac{\partial xe^{yz}}{\partial y} - \frac{\partial x^2y}{\partial z}\right)i - \left(\frac{\partial xe^{yz}}{\partial x} - \frac{\partial xy^2}{\partial z}\right)j + \left(\frac{\partial x^2y}{\partial x} - \frac{\partial xy^2}{\partial y}\right)k$$

$$= xze^{yz}i - e^{yz}j + (2xy - 2xy)k = xze^{yz}i - e^{yz}j$$

اجوبة شعبة B

(1) Find f_{xx}, f_{xy}, f_{yy} for $f(x,y) = \dfrac{x^2}{y^2} + x^3y^2 = x^2y^{-2} + x^3y^2$ [7]

$f_x = 2xy^{-2} + 3x^2y^2 \quad \Rightarrow \quad f_{xx} = 2y^{-2} + 6xy^2$

$f_{xy} = -4xy^{-3} + 6x^2y$

$f_y = -2x^2y^{-3} + 2x^3y \quad \Rightarrow \quad f_{yy} = 6x^2y^{-4} + 2x^3$

for $F(x,y,z) = z\sin(xy)i + xy^2j + xyz^2k$.(2) Find $\nabla \times F$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ z\sin(xy) & xy^2 & xyz^2 \end{vmatrix}$$

$$= \left(\frac{\partial xyz^2}{\partial y} - \frac{\partial xy^2}{\partial z}\right)i - \left(\frac{\partial xyz^2}{\partial x} - \frac{\partial z\sin(xy)}{\partial z}\right)j + \left(\frac{\partial xy^2}{\partial x} - \frac{\partial z\sin(xy)}{\partial y}\right)k$$

$$= xz^2i - (yz^2 - \sin(xy))j + (y^2 - xz\cos(xy))k$$

(3) $$\int_0^2\int_0^{x^2}\sqrt{x^3+1}\,dy\,dx=\int_0^2 y\Big|_0^{x^2}\sqrt{x^3+1}\,dx=\int_0^2 x^2\sqrt{x^3+1}\,dx$$

$$=\frac{1}{3}\times\frac{2}{3}(x^3+1)^{3/2}\Big|_0^2=\frac{2}{9}(27-1)=\frac{52}{9}$$

(4) $$\iint_D (x+3y)\,dA=\int_0^{\pi}\int_0^1(r\cos\theta+3r\sin\theta)r\,dr\,d\theta=\int_0^{\pi}\int_0^1(\cos\theta+3\sin\theta)r^2\,dr\,d\theta$$

$$=\int_0^{\pi}(\cos\theta+3\sin\theta)\frac{r^3}{3}\Big|_0^1\,d\theta$$

$$=\int_0^{\pi}\frac{1}{3}(\cos\theta+3\sin\theta)\,d\theta=\frac{1}{3}(\sin\theta-3\cos\theta)\Big|_0^{\pi}$$

$$=\frac{1}{3}(0+3-0+3)=2$$

(1) Find f_{xx}, f_{xy}, f_{yy} for $f(x,y)=\dfrac{x^3}{y^2}+x^2y^3=x^3y^{-2}+x^2y^3$ [8]

$f_x=3x^2y^{-2}+2xy^3 \quad \Rightarrow \quad f_{xx}=6xy^{-2}+2y^3$

$f_{xy}=-6x^2y^{-3}+6xy^2$

$f_y=-2x^3y^{-3}+3x^2y^2 \quad \Rightarrow \quad f_{yy}=6x^3y^{-4}+6x^2y$

for $F(x,y,z)=x\sin(yz)i+x^2yzj+yz^2k$. (2) Find $\nabla\times F$

$$\nabla\times F=\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x\sin(yz) & x^2yz & yz^2 \end{vmatrix}$$

$$=\left(\frac{\partial yz^2}{\partial y}-\frac{\partial x^2yz}{\partial z}\right)i-\left(\frac{\partial yz^2}{\partial x}-\frac{\partial x\sin(yz)}{\partial z}\right)j+\left(\frac{\partial x^2yz}{\partial x}-\frac{\partial x\sin(yz)}{\partial y}\right)k$$

$$=(z^2-x^2y)i-(-xy\cos(yz))j+(2xyz-xz\cos(yz))k$$

$$=(z^2-x^2y)i+xy\cos(yz)\,j+(2xyz-xz\cos(yz))k$$