حل اسئلة شعبة B

1. $3u_t + 2u_x = \tan t$ with $u(x,0) = \sin 3x$

$$u(x,t) = \mathrm{g}\left(x - \frac{b}{a}t\right) + \frac{1}{a}\int_0^t f\left(x - \frac{b}{a}t + \frac{b}{a}\tau, \tau\right) d\tau$$

$$u(x,t) = \mathrm{g}\left(x - \frac{2}{3}t\right) + \frac{1}{3}\int_0^t \tan\tau\, d\tau = \sin(3x - 2t) + \frac{1}{3}\ln\sec\tau\Big|_0^t$$

$$u(x,t) = \sin(3x - 2t) + \frac{1}{3}\ln\sec t$$

An other way $\quad \dfrac{dt}{3} = \dfrac{dx}{2} = \dfrac{du}{\tan t}$

$$\frac{dt}{3} = \frac{dx}{2} \quad ⇨ \quad \frac{t}{3} + C_1 = \frac{x}{2} \quad ⇨ \quad C = x - \frac{2}{3}t \;:\; C = 2C_1$$

$$\frac{dt}{3} = \frac{du}{\tan t} \quad ⇨ \quad du = \frac{\tan t\, dt}{3} \quad ⇨ \quad u(x,t) = \frac{1}{3}\ln\sec t + K$$

$$u(x,t) = \frac{1}{3}\ln\sec t + g\left(x - \frac{2}{3}t\right) \quad ⇨ \quad u(x,t) = \frac{1}{3}\ln\sec t + \sin(3x - 2t)$$

2. $\dfrac{u_t}{2t} + u_x = u^2$ with $u(x-1,1) = x$

$$2tdt = \frac{dx}{1} = \frac{du}{u^2}$$

$$2tdt = \frac{dx}{1} ⇨ \quad x = t^2 + C \quad ⇨ \quad C = x - t^2$$

$$\frac{dx}{1} = \frac{du}{u^2} \quad ⇨ \frac{dx}{1} = \frac{du}{u^2} \quad ⇨ \; -\frac{1}{u} = x + K$$

$$u(x,t) = \frac{-1}{x + K} = \frac{-1}{x + g(x - t^2)}$$

$$u(x-1,1) = x \quad ⇨ \quad x = \frac{-1}{(x-1) + g(x-1-1)} = \frac{-1}{(x-1) + g(x-2)}$$

$$x(x-1) + xg(x-2) = -1 \quad ⇨ \quad g(x-2) = 1 - x - \frac{1}{x}$$

$$g(x) = 1 - (x+2) - \frac{1}{x+2}$$

$$u(x,t) = \frac{-1}{x + 1 - (x - t^2 + 2) - \dfrac{1}{x - t^2 + 2}}$$

$$u(x,t) = \frac{t^2 - x - 2}{x^2 - xt^2 + 2x + x - t^2 + 2 - (x - t^2 + 2)^2 - 1}$$

$$u(x,t) = \frac{t^2 - x - 2}{x^2 - xt^2 + 3x - t^2 + 1 - (x - t^2 + 2)^2}$$

An other way $\quad 2tdt = \dfrac{dx}{1} = \dfrac{du}{u^2}$

$$2tdt = \frac{dx}{1} \Rightarrow \quad x = t^2 + C \quad \Rightarrow \quad C = x - t^2$$

$$2tdt = \frac{du}{u^2} \quad \Rightarrow -\frac{1}{u} = t^2 + K$$

$$u(x,t) = \frac{-1}{t^2 + K} = \frac{-1}{t^2 + g(x - t^2)}$$

$$(x-1,1) = x \quad \Rightarrow \quad x = \frac{-1}{1 + g(x-1-1)} = \frac{-1}{1 + g(x-2)}$$

$$x + xg(x-2) = -1$$

$$g(x-2) = \frac{-1-x}{x} \quad \Rightarrow \quad g(x) = \frac{-1-(x+2)}{x+2} = \frac{-1}{x+2} - 1$$

$$u(x,t) = \frac{-1}{t^2 - \dfrac{1}{(x-t^2)+2} - 1}$$

3. $u_{yy}(x,y) = \sin(x-2y)$ with $u(x,\pi) = (3/4)\sin x$, $u_y(x,\pi) = -(1/2)\cos x$

$$u_y(x,y) = (1/2)\cos(x-2y) + f(x)$$

$$u_y(x,\pi) = -(1/2)\cos x \quad \Rightarrow \quad -(1/2)\cos x = (1/2)\cos(x-2\pi) + f(x)$$

$$-(1/2)\cos x = (2)\cos x + f(x) \quad \Rightarrow \quad f(x) = -\cos x$$

$$u_y(x,y) = (1/2)\cos(x-2y) - \cos x$$

$$u(x,y) = -(1/4)\sin(x-2y) - y\cos x + h(x)$$

$$u(x,\pi) = (3/4)\sin x \quad \Rightarrow \quad (3/4)\sin x = -(1/4)\sin(x-2\pi) - \pi\cos x + h(x)$$

$$h(x) = \sin x + \pi\cos x$$

$$u(x,y) = -(1/4)\sin(x-2y) - y\cos x + \sin x + \pi\cos x$$

4. $u_{xy}(x,y) = 3x^2 - 3y^2 - 4xy$ with $u(1,y) = y$, $u_x(x,1) = 3x^2$

$$u_x(x,y) = 3x^2y - y^3 - 2xy^2 + f(x)$$

$$u_x(x,1) = 3x^2 \quad \Rightarrow \quad 3x^2 = 3x^2 - 1 - 2x + f(x)$$

$$f(x) = 2x + 1$$

$$u_x(x,y) = 3x^2y - y^3 - 2xy^2 + 2x + 1$$

$$u(x,y) = x^3y - xy^3 - x^2y^2 + x^2 + x + h(y)$$

$$u(1,y) = y \quad \Rightarrow \quad y = y - y^3 - y^2 + 1 + 1 + h(y)$$

$$h(y) = y^3 + y^2 - 2$$

$$u(x,y) = x^3y - xy^3 - x^2y^2 + x^2 + x + y^3 + y^2 - 2$$

حل اسئلة شعبة A

1. $2u_t + 3u_x = \cos^2 t \quad \text{with} \quad u(x,0) = \tan 2x$

$$u(x,t) = g\left(x - \frac{b}{a}t\right) + \frac{1}{a}\int_0^t f\left(x - \frac{b}{a}t + \frac{b}{a}\tau, \tau\right) d\tau$$

$$u(x,t) = g\left(x - \frac{3}{2}t\right) + \frac{1}{2}\int_0^t \cos^2 \tau \, d\tau$$

$$u(x,t) = \tan 2\left(x - \frac{3}{2}t\right) + \frac{1}{4}\int_0^t (1 + \cos 2\tau)\, d\tau$$

$$u(x,t) = \tan(2x - 3t) + \frac{1}{4}\left(\tau + \frac{1}{2}\sin 2\tau\right)\Big|_0^t$$

$$u(x,t) = \tan(2x - 3t) + \frac{1}{4}\left(t + \frac{1}{2}\sin 2t\right)$$

An other way $\quad \dfrac{dt}{2} = \dfrac{dx}{3} = \dfrac{du}{\cos^2 t}$

$$\frac{dt}{2} = \frac{dx}{3} \quad ⇨ \quad \frac{t}{2} + C_1 = \frac{x}{3} \quad ⇨ \quad C = x - \frac{3}{2}t \ : \ C = 3C_1$$

$$\frac{dt}{2} = \frac{du}{\cos^2 t} \quad ⇨ \quad du = \frac{\cos^2 t\, dt}{2} \quad ⇨ \quad u = \frac{1}{4}\int (1 + \cos 2t) dt$$

$$u(x,t) = \frac{1}{4}\left(t + \frac{1}{2}\sin 2t\right) + K = \frac{1}{4}\left(t + \frac{1}{2}\sin 2t\right) + g\left(x - \frac{3}{2}t\right)$$

$$u(x,t) = \frac{1}{4}\left(t + \frac{1}{2}\sin 2t\right) + \tan(2x - 3t)$$

2. $\dfrac{u_t}{2t} + u_x = u^2 \quad \text{with} \quad u(x+1,1) = 1$

$$2tdt = \frac{dx}{1} = \frac{du}{u^2}$$

$$2tdt = \frac{dx}{1} \quad ⇨ \quad x = t^2 + C \quad ⇨ \quad C = x - t^2$$

$$\frac{dx}{1} = \frac{du}{u^2} \quad ⇨ \frac{dx}{1} = \frac{du}{u^2} \quad ⇨ \quad -\frac{1}{u} = x + K$$

$$u(x,t) = \frac{-1}{x + K} = \frac{-1}{x + g(x - t^2)}$$

$$u(x+1,1) = 1 ⇨ \quad 1 = \frac{-1}{(x+1) + g(x+1-1)} = \frac{-1}{(x+1) + g(x)}$$

$$(x+1) + g(x) = -1 \quad ⇨ \quad g(x) = -x - 2$$

$$u(x,t) = \frac{-1}{x - (x - t^2) - 2} = \frac{-1}{t^2 - 2}$$

An other way $\quad 2tdt = \frac{dx}{1} = \frac{du}{u^2}$

$$2tdt = \frac{dx}{1} \Rightarrow \quad x = t^2 + C \quad \Rightarrow \quad C = x - t^2$$

$$2tdt = \frac{du}{u^2} \quad \Rightarrow -\frac{1}{u} = t^2 + K$$

$$u(x,t) = \frac{-1}{t^2 + K} = \frac{-1}{t^2 + g(x - t^2)}$$

$$u(x+1,1) = 1 \Rightarrow \ 1 = \frac{-1}{1 + g(x+1-1)} = \frac{-1}{1 + g(x)} \quad \Rightarrow \quad g(x) = -2$$

$$u(x,t) = \frac{-1}{t^2 - 2}$$

3. $u_{yy}(x,y) = e^{x-2y}$ with $\quad u(x,0) = \frac{3}{4}e^x \ , u_y(x,0) = \frac{1}{2}e^x$

$$u_y(x,y) = -\frac{1}{2}e^{x-2y} + f(x)$$

$$u_y(x,0) = \frac{1}{2}e^x \quad \Rightarrow \quad \frac{1}{2}e^x = -\frac{1}{2}e^x + f(x) \quad \Rightarrow \quad f(x) = e^x$$

$$u_y(x,y) = -\frac{1}{2}e^{x-2y} + e^x$$

$$u(x,y) = \frac{1}{4}e^{x-2y} + ye^x + h(x)$$

$$u(x,0) = \frac{3}{4}e^x \quad \Rightarrow \quad \frac{3}{4}e^x = \frac{1}{4}e^x + h(x) \quad \Rightarrow \quad h(x) = \frac{1}{2}e^x$$

$$u(x,y) = \frac{1}{4}e^{x-2y} + ye^x + \frac{1}{2}e^x$$

4. $u_{xy}(x,y) = 3y^2 - 3x^2 - 4xy$ with $\quad u(x,1) = x \ , u_y(1,y) = 3y^2$

$$u_y(x,y) = 3y^2x - x^3 - 2x^2y + f(y)$$

$$u_y(1,y) = 3y^2 \quad \Rightarrow \ 3y^2 = 3y^2 - 1 - 2y + f(y)$$

$$f(y) = 2y + 1$$

$$u_y(x,y) = 3y^2x - x^3 - 2x^2y + 2y + 1$$

$$u(x,y) = y^3x - x^3y - x^2y^2 + y^2 + y + h(x)$$

$$u(x,1) = x \quad \Rightarrow \quad x = x - x^3 - x^2 + 1 + 1 + h(x)$$

$$h(x) = x^3 + x^2 - 2$$

$$u(x,y) = y^3x - x^3y - x^2y^2 + y^2 + y + x^3 + x^2 - 2$$