University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Numerical Integration

Unequal Segments Trapezoidal Method

Undergraduate Level, 3th Stage

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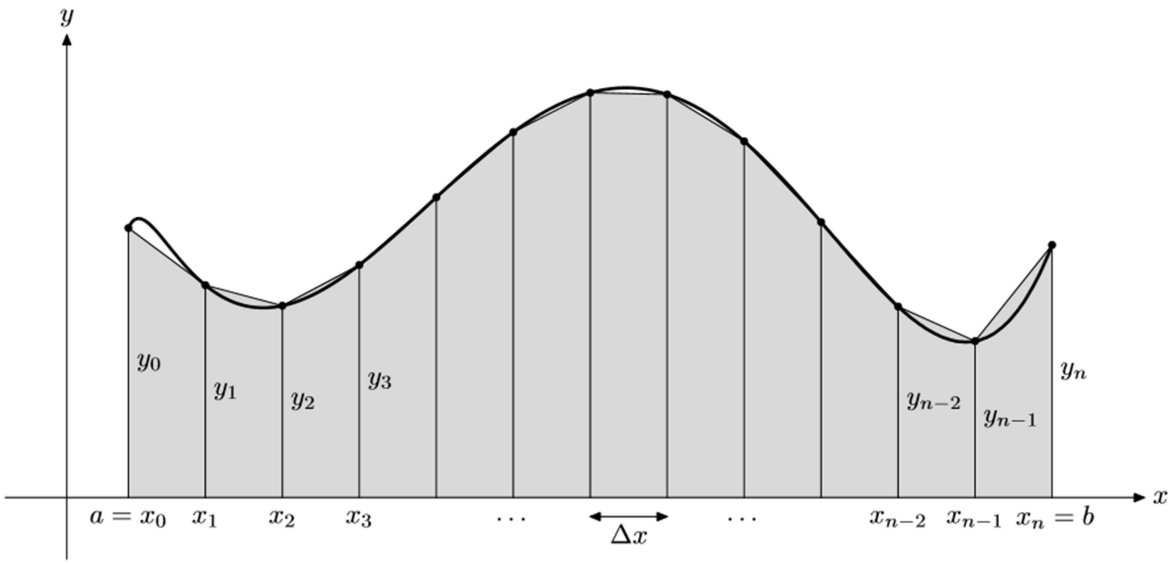
Lecture Outline

2.1 – Unequal Segments Trapezoidal Method

The integration of a function with unequal segments can be approximately calculated using the trapezoid method as follows:



$$\Delta x_1 + \Delta x_2 + \cdots + \Delta x_n = (b - a)$$

$$I = \int_a^b f(x)dx = \Delta x_1 \frac{y_0 + y_1}{2} + \Delta x_2 \frac{y_1 + y_2}{2} + \cdots + \Delta x_n \frac{y_{n-1} + y_n}{2}$$

$$\Delta x_1 + \Delta x_2 + \cdots + \Delta x_n = (b - a)$$

$$\in = \left| \frac{True\ Value - Approximate\ Value}{True\ Value} \right| \times 100\%$$

where:

$\in$: the error value.

$x_0 = a$ $\qquad$ $y_0 = f(x_0)$

$x_1 = x_0 + \Delta x_1$ $\qquad$ $y_1 = f(x_1)$

$x_n = x_{n-1} + \Delta x_n = b$ $\qquad\qquad$ $y_n = f(x_n)$

Ex1: Use the trapezoidal method to evaluate the value of the following integration.

$$\int_0^4 xe^{2x}dx$$

$$\Delta x_1 = 2, \quad \Delta x_2 = 1, \quad \Delta x_3 = 0.5, and \quad \Delta x_4 = 0.5$$

Compare your solution with the exact solution ($I = 5216.92$).

Solution:

$a = 0$, $b = 4$

Four unequal segments

$y_i = f(x_i) = xe^{2x}$

i	x	y
0	$x_0 = a = 0$	$(0) \times e^{2\times(0)} = 0$
1	$0 + \Delta x_1 = 2$	$(2) \times e^{2\times(2)} = 109.2$
2	$2 + \Delta x_2 = 3$	$(3) \times e^{2\times(3)} = 1210.3$
3	$3 + \Delta x_3 = 3.5$	$(3.5) \times e^{2\times(3.5)} = 3838.2$
4	$3.5 + \Delta x_3 = 4 = b$	$(4) \times e^{2\times(4)} = 11923.8$

$$I = \int_0^4 xe^{2x}dx = \Delta x_1 \frac{y_0 + y_1}{2} + \Delta x_2 \frac{y_1 + y_2}{2} + \cdots + \Delta x_n \frac{y_{n-1} + y_n}{2}$$

$$= 2 \times \frac{0 + 109.2}{2} + 1 \times \frac{109.2 + 1210.3}{2} + 0.5 \times \frac{1210.3 + 3838.2}{2} + 0.5 \times \frac{3838.2 + 11923.8}{2} = 5971.55$$

Compare with the true value:

$$\in = \left|\frac{True\ Value - Approximate\ Value}{True\ Value}\right| \times 100\% = \left|\frac{5216.92 - 5971.55}{5216.92}\right| \times 100\%$$

$$= 14.5\%$$

Ex2: Use the trapezoidal method to evaluate the value of the following integration.

$$\int_0^{10} \frac{300x}{1 + e^x} \mathrm{dx}$$

Use two unequal segments : $\Delta x_1 = 4, \quad \Delta x_2 = 6$

Compare your solution with the exact solution (I = 246.59).

Solution:

$a = 0 \quad , \quad b = 10$

Two un equal segments

$$y_i = f(x_i) = \int_0^{10} \frac{300x}{1 + e^x} \mathrm{dx}$$

i	x	y
0	$x_0 = a = 0$	$\frac{300 \times (0)}{1 + e^{(0)}} = 0$
1	$0 + \Delta x_1 = 0 + 4 = \mathbf{4}$	$\frac{300 \times (4)}{1 + e^{(4)}} = 21.58$
2	$4 + \Delta x_2 = 4 + 6 = \mathbf{10} = b$	$\frac{300 \times (10)}{1 + e^{(10)}} = 0.14$

$$I = \int_0^{10} \frac{300x}{1 + e^x} \mathrm{dx} = \Delta x_1 \frac{y_0 + y_1}{2} + \Delta x_2 \frac{y_1 + y_2}{2} + \cdots + \Delta x_n \frac{y_{n-1} + y_n}{2}$$

$$= 4 \times \frac{0 + 21.58}{2} + 6 \times \frac{21.58 + 0.14}{2} = 138.32$$

Compare with the true value:

$$\in = \left| \frac{True\ Value - Approximate\ Value}{True\ Value} \right| \times 100\% = \left| \frac{246.59 - 138.32}{246.59} \right| \times 100\% = 43.5\%$$

Homework 12

1. Use the unequal trapezoidal method to estimate the value of the following integration.

$$\int_0^2 x^2 dx$$

The number of segments (n) is equal to 3 as:

$(\Delta x_1 = 0.5, \quad \Delta x_2 = 0.5, and \quad \Delta x_3 = 1)$

Compare your solution with the exact solution.

2. Use the unequal trapezoidal method to estimate the value of the following integration.

$$\int_0^4 xe^{2x} dx$$

The number of segments (n) is equal to 3 as:

$(\Delta x_1 = 1, \quad \Delta x_2 = 1, and \quad \Delta x_3 = 2)$

Compare your solution with the exact solution $(I = 5216.92)$.