University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Numerical Integration

Simpson's 1/3 Rule

Undergraduate Level, 3th Stage

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Lecture Outline

3.0 – Simpson's Rule Method

3.1 – Simpson's 1/3 Rule

The integration of a function can be approximately calculated using the 1/3 rule of Simpson and as follows:

The number of segments has to be even ($n = even\ \#$),

$$\int_a^b \boldsymbol{f(x)dx} = \frac{\Delta x}{3}[y_0 + y_n + 4\sum_{i=1}^{n-1} y_{i_{odd}} + 2\sum_{i=2}^{n-1} y_{i_{even}}]$$

$$\Delta x = \frac{b-a}{n}$$

$$\in = \left|\frac{True\ Value - Approximate\ Value}{True\ Value}\right| \times 100\%$$

where:

Δx: segment length,

n: the number of segments (for Simpson's 1/3 Rule method, n must be an even number),

∈: the error value.

$x_0 = a$	$y_0 = f(x_0)$
$x_1 = x_0 + \Delta x$	$y_1 = f(x_1)$
$x_n = x_{n-1} + \Delta x = b$	$y_n = f(x_n)$

Ex1: Calculate the value of the given integration.

$$\int_{8}^{30} \{2000 \ln[\frac{140000}{140000 - 2100x}] - 9.8x\}dx$$

The number of segments (n) is equal to 4.

Compare your solution with the exact solution ($I = 11061$).

Solution:

Four segments, n=4, even number

$a = 8 \quad , \quad b = 30$

$$\Delta x = \frac{b-a}{n} = \frac{30-8}{4} = 5.5$$

$$y_i = f(x_i) = 2000 \ln[\frac{140000}{140000 - 2100x}] - 9.8x_i$$

i	x	y
0	$x_0 = a = 8$	$2000 \times \ln[\frac{140000}{140000 - 2100 \times (8)}] - 9.8 \times (8) = 177.3$
1	8+5.5=13.5	$2000 \times \ln[\frac{140000}{140000 - 2100 \times (13.5)}] - 9.8 \times (13.5) = 320.2$
2	13.5+5.5=19	$2000 \times \ln[\frac{140000}{140000 - 2100 \times (19)}] - 9.8 \times (19) = 484.7$
3	19+5.5=24.5	$2000 \times \ln[\frac{140000}{140000 - 2100 \times (24.5)}] - 9.8 \times (24.5) = 676.1$
4	24.4+5.5=30=b	$2000 \times \ln[\frac{140000}{140000 - 2100 \times (30)}] - 9.8 \times (30) = 901.7$

$$I = \int_a^b f(x)dx = \frac{\Delta x}{3}[y_0 + y_n + 4\sum_{i=1}^{n-1} y_{i_{odd}} + 2\sum_{i=2}^{n-1} y_{i_{even}}]$$

$$= \frac{5.5}{3}[177.3 + 901.7 + 4 \times (320.2 + 676.1) + 2 \times (484.75)] = 11078.3$$

Compare with the true value:

$$\epsilon = \left|\frac{True\ Value - Approximate\ Value}{True\ Value}\right| \times 100\% = \left|\frac{11061 - 11078.3}{11061}\right| \times 100\%$$

$$= 0.16\%$$

Ex2: Calculate the value of the given integration.

$$\int_0^{10} \frac{300x}{1+e^x} \text{dx}$$

The number of segments (n) is equal to 4.

Compare your solution with the exact solution (I = 246.59).

Solution:

Four segments, n=4, even number

$a = 0$, $b = 10$

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{4} = 2.5$$

$$y_i = f(x_i) = \int_0^{10} \frac{300x}{1+e^x} \text{dx}$$

i	x	y

0	$x_0 = a = 0$	$\frac{300 \times (0)}{1 + e^{(0)}} = 0$
1	0+2.5=2.5	$\frac{300 \times (2.5)}{1 + e^{(2.5)}} = 56.89$
2	2.5+2.5=5	$\frac{300 \times (5)}{1 + e^{(5)}} = 10.04$
3	5+2.5=7.5	$\frac{300 \times (7.5)}{1 + e^{(7.5)}} = 1.24$
4	7.5+2.5=10=b	$\frac{300 \times (10)}{1 + e^{(10)}} = 0.14$

$$I = \int_a^b f(x)dx = \frac{\Delta x}{3}[y_0 + y_n + 4\sum_{i=1}^{n-1} y_{i_{odd}} + 2\sum_{i=2}^{n-1} y_{i_{even}}]$$

$$= \frac{2.5}{3}[0 + 0.14 + 4 \times (56.89 + 1.24) + 2 \times (10.04)] = 210.62$$

Compare with the true value:

$$\in = \left|\frac{True\ Value - Approximate\ Value}{True\ Value}\right| \times 100\% = \left|\frac{246.59 - 210.62}{246.59}\right| \times 100\%$$

$$= 14.59\%$$

Ex3: Calculate the value of the given integration.

$$\int_1^5 \sqrt{1 + x^2}\mathrm{dx}$$

The number of segments (n) is equal to 8.

Solution:

Eight segments (n=8, even #)

$a = 1 \quad , \quad b = 5$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{8} = 0.5$$

$$y_i = f(x_i) = \sqrt{1+x^2}$$

i	x	y
0	$x_0 = a$ =1	$\sqrt{1+(1)^2} = 1.41$
1	1+0.5=1.5	$\sqrt{1+(1.5)^2} = 1.80$
2	1.5+0.5=2	$\sqrt{1+(2)^2} = 2.24$
3	2+0.5=2.5	$\sqrt{1+(2.5)^2} = 2.69$
4	2.5+0.5=3	$\sqrt{1+(3)^2} = 3.26$
5	3+0.5=3.5	$\sqrt{1+(3.5)^2} = 3.64$
6	3.5+0.5=4	$\sqrt{1+(4)^2} = 4.12$
7	4+0.5=4.5	$\sqrt{1+(4.5)^2} = 4.61$
8	4.5+0.5=5=b	$\sqrt{1+(5)^2} = 5.10$

$$I = \int_a^b f(x)dx = \frac{\Delta x}{3}[y_0 + y_n + 4\sum_{i=1}^{n-1} y_{i_{odd}} + 2\sum_{i=2}^{n-1} y_{i_{even}}]$$
$$= \frac{0.5}{3}[1.41 + 5.10 + 4 \times (1.80 + 2.69 + 3.64 + 4.61) + 2 \times (2.24 + 3.26 + 4.12)] = 12.79$$

Homework 13

Use the 1/3 Simpson's Rule method to estimate the value of the following integration.

$$\int_{0}^{4} xe^{2x} dx$$

The number of segments (n) is equal 8.

Compare your solution with the exact solution ($I = 5216.92$).