University of Babylon
College of Engineering
Department of Environmental Engineering
Engineering Analysis I (ENAN 103)

Solution of Nonlinear Equation

Newton-Raphson Method

Undergraduate Leve, 3^{th} Stage

Mr. Waleed Ali Tameemi
College of Engineering/ Babylon University
M.Sc. Civil Engineering/ the University of Kansas/ USA

2016-2017

Lecture Outline

1.0 Introduction

2.0 Closed Methods

    2.1 Bisection Method

    2.2 False-Point Position Method

3.0 Open Methods

    3.1 Newton-Raphson Method

    3.2 Secant Method

4.0 Summary

3.0 – Open Methods

These methods are being used in estimating the roots of nonlinear equations. In any of the open methods, only one values of x (one roots) is assumed.

3.1 – Newton-Raphson Method

The following steps are required in estimating the root.

1. Evaluate $f'(x)$.
2. Assume an initial value for x_i.
3. Evaluate $f(x_i)$ and $f'(x_i)$.
4. Calculate the estimated root (x_{i+1}) as following:

   $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

5. Calculate the error value, $f(x_{i+1})$, which represents the corresponding value of x_{i+1}.
6. Compare the error value, $|f(x_{i+1})|$, with the desire accuracy ($\in$).

   If $|f(x_{i+1})| \leq \in$ then x_{i+1} is the required root (accurate enough).

   Otherwise go to step 3.

Ex1: Find a root for $f(x) = x^3 - x - 1$ with accuracy equal to $\in = 0.0001$.

Solution:

Step 1:

$f'(x) = 3x^2 - 1$

Step 2:

Assume $x_0 = 2$

Step 3:

$f(x_0) = f(2) = 2^3 - 2 - 1 = 5$

$f'(x_0) = f'^{(2)} = 3 \times 2^2 - 1 = 11$

Step 4:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{5}{11} = 1.5454$

Step 5:

$f(x_1) = f(1.5454) = 1.5454^3 - 1.5454 - 1 = 1.1457$

Step 6:

$|f(x_1)| > \in \rightarrow (not\ enough\ accurat)$ go to step 3

Step #	$\boldsymbol{x_i}$	$\boldsymbol{x_{i+1}}$	$\boldsymbol{f(x_{i+1})}$	$\boldsymbol{\lvert f(x_r)\rvert - \in}$
0	2	1.545455	1.145755	-1.14476
1	1.545455	1.359615	0.153705	-0.1527

2	1.359615	1.325801	0.004625	-0.00362
3	1.325801	1.324719	4.66E-06	0.000995

The required root is equal to $x = 1.3247$.

Ex2: Find a root for $f(x) = 3\cos(x) - 1 + x$ with accuracy equal to $\in = 0.0001$.

Solution:

Step 1:

$f'^{(x)} = -3\sin(x) + 1$

Step 2:

Assume $x_0 = 2$

Step 3:

$f(x_0) = f(2) = 3\cos(2) - 1 + 2 = -0.24844$

$f'(x_0) = f'^{(2)} == -3\sin(2) + 2 = -1.72789$

Step 4:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-0.24844}{-1.72789} = 1.8562$$

Step 5:

$f(x_1) = f(1.8562) = 3\cos(1.8562) - 1 + 1.8562 = 0.01153$

Step 6:

$|f(x_1)| > \in \rightarrow (not\ enough\ accurat)$ go to step 3

Step #	x_i	x_{i+1}	$f(x_{i+1})$	$\|f(x_r)\| - \in$
1	2	1.856218	0.011532	-0.01053
2	1.856218	1.862356	1.6E-05	0.000984
3	1.862356	1.862365	3.16E-11	0.001

The required root is equal to $x = 1.8624$.

Ex3: Find a root for $f(x) = x^3 + 2x^2 - 2$ with accuracy equal to $\in = 0.000001$.

Solution:

Step 1:

$f'^{(x)} = 3x^2 + 4x$

Step 2:

Assume $x_0 = 1$

Step 3:

$f(x_0) = f(1) = x^3 + 2x^2 - 2 = 1$

$f'(x_0) = f'(1) = 3x^2 + 4x = 7$

Step 4:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1}{7}$ =0.857143

Step 5:

$f(x_1) = f(0.857143) = x^3 + 2x^2 - 2 \ = 0.099125$

Step 6:

$|f(x_1)| \ > \in \rightarrow (not\ enough\ accurat)$ go to step 3

Step #	x_i	x_{i+1}	$f(x_{i+1})$	$\|f(x_r)\| - \in$
1	1	0.857143	0.099125	-0.09912
2	0.857143	0.839545	0.00141	-0.00141
3	0.839545	0.839287	3E-07	7E-07

The required root is equal to $x = 0.839287$.

Homework 3

1- Find a root for $f(x) = 3x + \sin x - e^x$ with accuracy equal to $\in = 0.00001$.

   Start with: $x = 0$

2- Find a root for $f(x) = x^3 + x^2 - 3x - 3$ with accuracy equal to $\in =$ 0.0001.

   Start with: $x = 1$